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Semester - V

An Introduction to

Financial Economics

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Dr. Sk. Raju Ali

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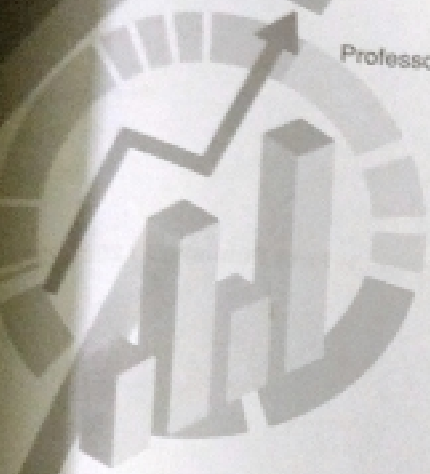
An Introduction to Financial Economics

SEMESTER-V



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Syllabus : University of Calcutta

Discipline Specific Elective-B(1) :

ECO-A-DSE-5-B(1)-TH-TU

Financial Economics [FE]

Total Marks : 100 [Theory(Th) 65 + Tutorial(Tu) 15 + Internal Assessment 10 + Attendance : 10]

Total Credits : [5(Th) + 1(Tu)] = 6 .

No. of Lecture hours : 75, No. of Tutorial contact hours : 15

[Semester-V]

ECO-A-DSE-5-B(1)-TH

1. **Investment Theory and Portfolio Analysis 35 lecture hours**
   - Deterministic cash-flow streams : Basic theory of interest; discounting and present value ; internal rate of return ; evaluation criteria ; fixed-income securities; bond prices and yields ; interest rate sensitivity and duration ; immunisation ; the term structure of interest rates ; yield curves ; spot rates and forward rates.
   - Single-period random cash flows : Random asset returns ; portfolios of assets ; portfolio mean and variance; feasible combinations of mean and variance; mean-variance portfolio analysis : the Markowitz model and the two-fund theorem ; risk-free assets and the one-fund theorem.
   - CAPM : The capital market line ; the capital asset pricing model ; the beta of an asset and of a portfolio ; security market line; use of the CAPM model in investment analysis and as a pricing formula.
2. **Options and Derivatives 20 lecture hours**
   - Introduction to derivatives and options ; forward and futures contracts ; options ; other derivatives ; forward and future prices ; stock index futures ; interest rate futures ; the use of futures for hedging ; duration-based hedging strategies ; option markets ; call and put options ; factors affecting option prices ; put-call parity ; option trading strategies : spreads ; straddles ; strips and straps ; strangles ; the principle of arbitrage ; discrete processes and the binomial tree model ; risk-neutral valuation.
3. **Corporate Finance 20 lecture hours**
   - Patterns of corporate financing : common stock; debt ; preferences ; convertibles ; Capital structure and the cost of capital ; corporate debt and dividend policy ; the Modigliani-Miller theorem.

ECO-A-DSE-5-B(1)-TU

Tutorial Contact hours : 15

Contents

1

Deterministic Cash-flow & Project Evaluation

.1. Introduction

he continuous growth and diversity in business activities on the one hand and the growing importance f financial assets with new analytical tools in the field of finance on the other have brought forward e importance of financial economics in general and the investment theory and portfolio analysis in articular. As a starting point of this analysis, we shall deal with the concepts of deterministic cash-low stream, basic theory of interest, Net Present Value (NPV) and Internal Rate of Return (IRR) riteria for project evaluation in this chapter.

.2. Deterministic cash flow stream

ormally an investment is defined in terms of a cash flow stream or sequence generated out of that vestment during any particular time period. So, it is the flow of returns (cash) to an investment ndertaken by any investor. Such cash flows usually occur at some definite intervals and these flows clude both positive and negative cash flows. If there remains no uncertainty in these cash flows uch as interest receipts from banks (on fixed deposits) at regular intervals], we call it deterministic ash flow.

f the investor spends, say, ₹ 1 crore for initiating any investment project then it would imply a negative ash flow at the beginning of the investment project. However, in subsequent years the project may enerate positive cash flow to the tune of, say ₹ 2,00,000 p.m.

hen any investment generates future income streams then there is a question of determining the resent value of the future income stream. Interest rate on investment is generally used to determine e present value of the future cash flow. Hence, interest rate is often called as the time value of oney.

.2.1. Time Value of money

n simple terms, the time value of money means the difference in the value of money when it is eceived at different points of time. Normally, the value of a certain amount of money is more if it is eceived today rather than at some future date.

e can give three specific reasons for the differences in the value of money at different points of time :

(i) Inflationary pressure in an economy ;

(ii) Preference for current cousumption over future consumption by any individual ;

(iii) Possibilities of investment opportunity before the investors to put their money in projects with an assured return.

f the inflationary pressure increases over time, the value of money or the purchasing power of money ith fall. Similarly, if individuals prefer present consumption more than the future consumption, hen any postponement of present consumption would mean that the money which has not been

used for present consumption, should fetch sufficient return so that the future consumption can be increased sufficiently to compensate for the present sacrifice.

Further, money has a time value because any individual having some investible fund at present can invest it in some project that ensures a fixed rate of return on the principal amount per time period. Thus, in financial analysis, the concept of time value of money is used to make a comparison between the cash flow at different points of time. The future value of an amount can be estimated by considering either a simple interest rate or a compound interest rate on the principal amount invested at present. Similarly, the present value of the future cash-flow stream can also be estimated using a definite discount rate. Again,

(i) the risks involved in any investment project, and

(ii) the time for which the present consumption is being deferred, would also determine the time value of money.

If an investment project is more risky, the investor would naturally expect more return from that project. Any uncertainty in getting the return makes the project more risky. Hence, to make an equivalence between the money available at present and the money available in future from a risky venture, adequate returns are to be added with future stock of money.

Further, an individual can be induced to defer his/her present consumption if the amount of money he lends at present can bring in returns sufficient to compensate for that sacrifice for a long time.

1.3. Basic Theory of Interest

Let us first start with the notion of simple interest.

- **Simple Interest :** In this case, an investment generates an interest income equal to 'r' (interest rate) times the orginal investment every year. Further, there may be 'f' fraction of 1 year (say, 6 months = 0·5 year) and in that case the interest income will be 'rf' times the original investment.

- **General Rule :** If 'A' amount is invested at a simple interest rate of 'r' for 'n' number of years then the total value (V) to be received after 'n' number of years will be $V = A(1 + rn)$ .... (1.1)

Example 1.1

If A = ₹ 100, r = 10% = 0·1 and n = 5 years.

Then $V = 100\ (1 + 0{\cdot}1 \times 5) = 100 \times 1{\cdot}5$

= ₹ 150

Let t = fractional years (say, 5·5 years)

In this case, the total value received at a simple rate of interest (r) after 't' time period will be

$V = A(1 + rt)$ .... (1.2)

Example 1.2

If A = ₹ 100, r = 10% = 0·1 and t = 5·5 years.

Then $V = 100\ (1 + 0{\cdot}1 \times 5{\cdot}5)$

$= 100\ (1{\cdot}55)$

= ₹ 155

- **Compound Interest rate :**

In case of compound interest rate, the interest rate is compounded yearly. Here, if an amount A is invested at a compound interest rate 'r' for 'n' number of years then the value received after that period is estimated by the following formula :

$V = A(1 + r)^n$ .... (1.3)

In this case, we get a geometric growth of 'A'.

If $n = 4$ years then

$A(1 + r)$ = value after 1 year.

$[A(1 + r)](1 + r) = A(1 + r)^2$ = value after 2 years.

$[A(1 + r)^2](1 + r) = A(1 + r)^3$ = value after 3 years.

and $[A(1 + r)^3](1 + r) = A(1 + r)^4$ = value after 4 years.

Example 1.3

If A = ₹ 100, $r = 10\% = 0{\cdot}1$

and $n = 4$ years.

Then $V = 100(1 + 0{\cdot}1)^4$

= ₹ 146·41

Here, the investor earns interest on interest, *i.e.*, interest is reinvested. This feature is called as 'compounding'. In case of simple interest, the interest amount is not reinvested. In that case, interest is earned in each period only on the original principal amount 'A'. In our compound interest formula (4), the expression $(1 + r)^n$ is sometimes called as 'future value interest factor' or 'compound value interest factor'.

In our example, this future value interest factor is $(1 + 0{\cdot}1)^4 = 1{\cdot}4641$

and $V = 146{\cdot}41$

$= 100\,(1{\cdot}1) \times 1{\cdot}1 \times 1{\cdot}1 \times 1{\cdot}1$

$= 100 \times (1{\cdot}1)^4$

$= 100 \times 1{\cdot}4641$

= ₹ 146·41

So, in case of compound interest the future value of ₹ 1 invested for 'n' years at an interest rate of 'r' per year is ₹ $1 \times (1 + r)^n$.

In this compounding process, the value that the investor receives after 'n' years has four parts :

(a) The original principal amount.

(b) The interest earned on the original principal amount (viz. the simple interest) per year.

(c) The interest on interest (viz. the compound interest) earned per year.

(d) The accumulated sum at the end of the period.

This can also be presented in a tabular form (Table - 1.1).

Table - 1.1

Future Value of ₹ 100 invested for 4 years at an interest rate of 10% p.a.

Year	Value at the beginning of each year (₹)	Simple interest (₹)	Compound interest (₹)	Total interest (₹)	Value at the end of each year (₹)
1	100	10	—	10	110
2	110	10	1·00	11	121
3	121	10	2·10	12·10	133·10
4	133·10	10	3·31	13·31	146·41
Total		40	6·41	46·41	

Table - 1.1 suggests that the simple interest amount remains constant for each year but the amount of compound interest gets inflated every year. This case also be shown with the help of a diagram (Fig. - 1.1)

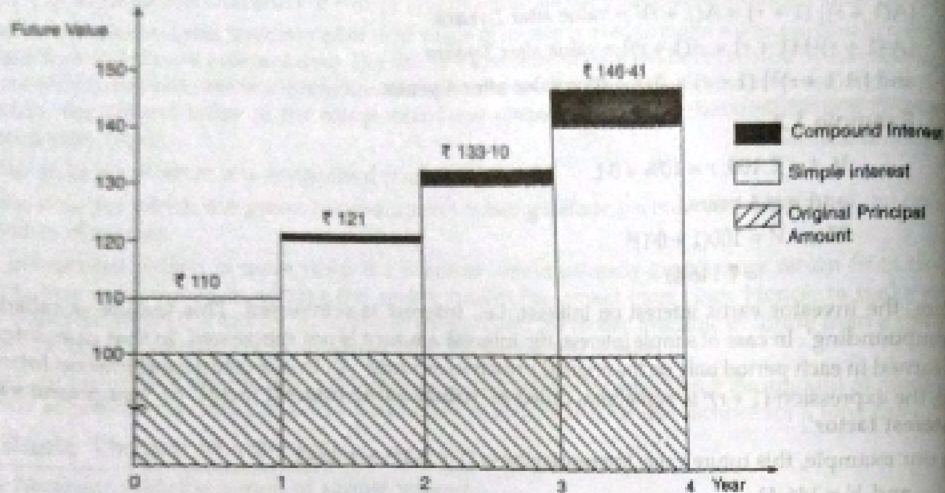

Fig. - 1.1

- **7 - 10 RULE :**

This rule suggests that if some amount is invested at an interest rate of, say, 7 per cent per annum gets doubled in approximately 10 years, then that amount invested at an interest rate of 10 per cent would be doubled in approximately 7 years. Generally if an amount '*A*', invested at an interest rate of '*r*' p.a., gets doubled in '*n*' years

then $A(1+r)^n = 2A$

or, $(1+r)^n = 2$

or, $\ln(1+r)^n = \ln 2$

or, $n.\ln(1+r) = 0{\cdot}69$

It can be shown that for a very small value of '*r*' (normally less than 20% p.a.), an amount '*A*' is doubled in approximately $\frac{72}{i}$ years, where $i = 100r$.

Example 1.4

If ₹ 1 is invested at an annual fixed interest rate of 10% = 0·1

then it would take $\frac{72}{10} = 7{\cdot}2$ years (approx) for ₹ 1 to grow ₹ 2. [In reality, however, $(1{\cdot}10)^{7{\cdot}3} = 2$, i.e., 7·3 years would be required to make ₹ 1 to grow upto ₹ 2 at 10% interest rate p.a.]. Similarly, it can be stated that if ₹ 1 is invested at an annual fixed interest rate of 7% then it would take $\frac{72}{7} = 10{\cdot}28$ (approx) years for ₹ 1 to grow to ₹ 2.

1.3.1. Nominal and real interest rate

When the interest rate is expressed in money terms, it is called nominal interest rate. For example, consider a person who has earned ₹ 100 as interest income by lending ₹ 1,000 for one year to another

person. So, in this case, the nominal interest rate would be $\frac{100}{1,000} \times 100 = 10\%$ per annum.

However, the real interest rate means the purchasing power of the nominal interest rate. If there is an increase in the level of commodity prices, this purchasing power will fall and *vice versa*.

Let us suppose that an individual has lent ₹ 1,000 for one year at an interest rate of 10% per annum. So, after one year his interest income will be ₹ 100, and he will get ₹ 1100 as the principal amount along with interest earned.

Let us also suppose that the consumer price index shows 8% increase (*i.e.*, an increase from 100 to 108 during that year), i.e., average price of consumer goods has increased by 8 per cent. Now, if we deflate the nominal income of that individual by the price index, we get the real income. So, real

income $= \dfrac{1,100}{\frac{108}{100}} = 1,000 \times \dfrac{100}{108} =$ ₹ 1018·52

It implies that compared to the base year, his present income of ₹ 1,100 can only purchase goods worth ₹ 1,018·52. Hence, he is actually earning an interest income of ₹ 18·52 on ₹ 1,000, i.e. the interest

rate $= \dfrac{18 \cdot 52}{1,000} \times 100 = 1 \cdot 852\%$ or almost equal to 1·85%. This shows the real interest rate.

Generally, the real interest rate is estimated as follows :

Real interest rate = Nominal interest rate − inflation rate

$= 10\% - 8\% = 2\%$.

Alternatively, it can be estimated as follows :

$$1 + \text{real interest rage} = \frac{(1 + \text{nominal interest rate})}{(1 + \text{inflation rate})}$$

$$\text{or, real interest rate} = \frac{(1 + \text{nominal interest rate})}{(1 + \text{inflation rate})} - 1$$

So, in our example,

$$\text{real interest rate} = \frac{(1 + 0 \cdot 10)}{(1 + 0 \cdot 08)} - 1 = \frac{1 \cdot 1}{1 \cdot 08} - 1$$

$$= 1 \cdot 0185 - 1 = 0 \cdot 0185 = 1 \cdot 85\%$$

Thus, real interest rate falls with an increase in inflation rate and *vice versa*.

Example 1.4(a)

Suppose you got ₹ 1,070 on maturity of a deposit of ₹ 1,000 for one year. If the inflation rate for that year was 5%, what was the rate of interest you actually received on your deposit ?

[C.U., B.Sc. (H), Sem-V, 2021]

Solution :

Here, ₹ $1,070 = 1,000\,(1 + r)$

or, $\dfrac{1,070}{1,000} = (1 + r)$

or, $r = 1 \cdot 07 - 1 = 0 \cdot 07$ or 7%

So, real rate of interest = 7% − 5% = 2%

where 5% = inflation rate.

1.4. Cash flow streams : Future Value

An investment project can generate **unequal cash flows** or a stream of cash flow. Let the cash flows be $x_0, x_1, \ldots, x_n$ for 'n' number of periods. Let us assume that the cash flows occur at the end of each period where a 'period' means a length of a time cycle (say, 1 year). Some cash flows may be zero or even negative. When the investor takes a loan from a bank to finance the investment project, it can be considered as a negative cash flow. However, positive cash flows may imply the deposits with a bank. Here, we can think of a banking institution where interest on deposit is equal to the interest on loan. This is called as an **'ideal bank'**. However, in this case, interest rate on, say, 1 year deposit might be different from interest on 2 years' deposit. If in an ideal bank, **the interest rate remains independent of the length of time** for which it applies then that given interest is compounded according to the normal rules. In that case, it is called as a **constant ideal bank.**

Now, given the cash flow stream for 'n' number of years where cash flows occur at the end of each period (say, at the end of a year), the initial cash flow x_0 will grow to $x_0(1+r)^n$ after 'n' number years at an annual interest rate 'r'. Therefore, the next cash flow x_1 will grow to $x_1(1+r)^{n-1}$ at the end of n^{th} year. However, the final cash flow x_n will not earn any interest since it occurs at the end of n^{th} period. Therefore, the total value generated at the end of n^{th} period (say, $n = 10$ years) would be termed as **future value** (FV)

where $FV = x_0(1+r)^n + x_1(1+r)^{n-1} + \ldots + x_n$

When the interest is compounded annually (i.e., at the end of each year), it is termed as **annual compounding.**

Here, the value $(1+r)^n$ is known as **Compound Value Interest Factor** (CVIF). For instance, if $r = 10\%$ and $n = 10$ years

then $CVIF = (1 + 0{\cdot}1)^{10} = 2{\cdot}594.$

Example 1.5

Let us consider an unequal cash flow for 5 years where the cash flows occur at the end of each year and the investor can invest the respective amount at an annual interest rate of 10%. The future value of this cash flow stream can be estimated as follows :

Let $x_0 = (-)$ ₹ 1000, $x_1 =$ ₹ 1,500

$x_2 =$ ₹ 2000, $x_3 =$ ₹ 2,500 and

$x_4 =$ ₹ 3,000. [Here $n = 4$]

[*Note :* The minus sign signifies cash outflow at the initial period]

$$\begin{aligned}\therefore FV &= (-)1000\,(1+0{\cdot}1)^4 + 1500\,(1+0{\cdot}1)^3 + 2000\,(1+0{\cdot}1)^2 + 2500\,(1+0{\cdot}1) + 3000\\ &= (-)1000\,(1{\cdot}464) + 1500\,(1{\cdot}331) + 2000\,(1{\cdot}21) + 2500\,(1{\cdot}1) + 3000\\ &= (-)1464 + 1996{\cdot}50 + 2420 + 2750 + 3000\\ &= ₹\ 10{,}166{\cdot}50\end{aligned}$$

1.4.1. Semi-annual compounding

When the interest rate is compounded twice (i.e., after every 6 months) within a given year, it is called **semi-annual compounding.** Thus, in this case, there are two compounding periods within a given year. Here, the interest rate is compounded after every 6 months at a rate of $\frac{1}{2}$ of the annual interest rate.

Example 1.6

If an investor invests a sum of ₹ 10,000 at an annual interest rate of 10% compounded semi-annually for 2 years then he would earn $\frac{10}{2}\% = 5\%$ interest compounded over four periods as shown in the following table :

Periods	Amt. at the beginning (₹)	Interest (₹)	Amt. at the end (₹)
1. 6 months	10,000	500	10,500
2. 12 months	10,500	525	11,025
3. 18 months	11,025	551·25	11,576·25
4. 24 months	11,576·25	578·81	12,155·06

Such semi-annual compounding of interest can easily be calculated using the following formula :

$$V = A\left(1+\frac{r}{m}\right)^{mt} \quad \text{......... (1.4)}$$

Where A = Principal amount

r = interest rate p.a.

m = Frequency of compounding per year

t = No. of years for which compounding has to be done.

V = Value received at the end of the period.

For example, if $A =$ ₹ 10,000, $r = 10\%$, $m = 2$ and $t = 2$ years, then

$$\begin{aligned}V &= 10{,}000\left[1+\frac{0{\cdot}1}{2}\right]^{2\times 2}\\ &= 10{,}000\,(1{\cdot}05)^4\\ &= 10{,}000\,(1{\cdot}2155) = ₹\ 12{,}155\end{aligned}$$

1.4.2. Quarterly compounding

When interest rate is compounded after every three months (i.e., there will be four compounding periods in a year) at a rate of $\frac{1}{4}$ th of annual interest rate, then it is called as **quarterly compounding.**

Here also we can use the formula $V = A\left(1+\frac{r}{m}\right)^{mt}$ to determine the value received (V) at the end of the period.

For example, if $A =$ ₹ 5,000, $r = 10\%$, $m = 4$ and $t = 2$ years then

$$\begin{aligned}V &= 5000\left[1+\frac{0{\cdot}1}{4}\right]^{4\times 2}\\ &= 5000\,(1{\cdot}025)^8\\ &= 5000\,(1{\cdot}2184) = ₹\ 6092.\end{aligned}$$

1.4.3. Monthly compounding

When the interest rate is compounded at the end of every month within a given year, it is called **monthly compounding.** So, in this case there will be 12 compounding periods within a given year. We can use the same formula (1.4) as before to determine the value received at the end of the investment period.

Example 1.7

If A = ₹ 5,000, $r = 10\%$

$m = 12$ and $t = 2$ years then

$$V = A\left[1+\frac{r}{m}\right]^{mt}$$

$$= 5000\left[1+\frac{0.1}{12}\right]^{12\times 2}$$

$$= 5000(1.00833)^{24}$$

$$= 5000(1.2203)$$

$$= ₹\ 6101.50$$

1.4.4. Continuous compounding

When any amount is invested at an annual interest rate which is compounded for 'm' times in a year and m tends to infinity (∞), then it is called as continuous compounding.

Let us consider our previous formula (1.4) :

$$V = A\left[1+\frac{r}{m}\right]^{mt}$$

It can be expressed as

$$V = A\left[1+\left(\frac{r}{m}\right)^{\frac{m}{r}}\right]^{rt}$$

Now, if $m \to \infty$

then $\frac{m}{r} \to \infty$

or, $\frac{r}{m} \to 0$

However, $\underset{m\to\infty}{Lt}\left(1+\frac{r}{m}\right)^{\frac{m}{r}} = e$

Since the irrational number e (where e = 2.7818 .... is the base of natural logarithm) is defined as

$$e = \underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^{x}$$

[Please note that if $x = \frac{m}{r}$ *then* $\frac{1}{x} = \frac{r}{m}$*]*

$$\therefore \underset{m\to\infty}{Lt} A\left[1+\left(\frac{r}{m}\right)^{\frac{m}{r}}\right]^{rt}$$

$V = Ae^{rt}$ .... **(1.5)**

It implies that V grows exponentially at the rate of r.

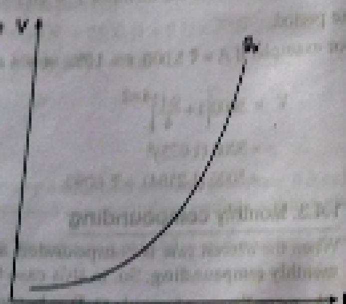

Fig.-1.2

[Here, $\log V = \log A + rt \log e$]

or, $\frac{d(\log V)}{dt} = r$ [∵ $\log e = 1$]

or, $\frac{dV}{dt}\cdot\frac{1}{V} = r$]

In Fig.-1.2, g_v *curve shows the exponential growth of value in case of continuous compounding of interest rate.*

1.4.5. Effective interest rate

It is observed that in case of multi-period compounding (i.e., semi-annual or quarterly compounding) the amount grows faster than the nominal interest rate. The interest rate realised in case of multi-period compounding is called as the ***effective rate of interest.***

We have seen that for any $r > 0$

$$\left(1+\frac{r}{m}\right)^{m} > (1+r) \quad \text{[considering } t = 1]$$

Let $r = 10\%$ and $m = 4$

$$\therefore \left(1+\frac{0.1}{4}\right)^{4} = 1.1038 > (1+0.1) = 1.1$$

When a year is divided into 'm' periods (say, $m = 12$) then the interest rate for each of the m periods would be $\left(\frac{r}{m}\right)$. In that case, after a full year $V = A\left(1+\frac{r}{m}\right)^{m}$

The effective interest rate (r_E) satisfies the relation $1+r_E = \left[1+\left(\frac{r}{m}\right)\right]^{m}$ .... (1.6)

In case of continuous compounding a full year is divided into smaller segments, and we get

$$\underset{m\to\infty}{Lt}\left[1+\left(\frac{r}{m}\right)\right]^{m} = e^{r}$$

∴ $1 + r_E = e^r$ in case of continuous compounding.

or, $r_E = e^r - 1$ .... (1.7)

Now, if $r = 10\%$ then $e^{0.1} = 1.105$

[Students can calculate this value using scientific calculator (Mode : COMPLEX)].

$\therefore r_E = 1.105 - 1 = 0.105 = 10.5\%$

Again, if $r = 8\%$, $m = 2$ (i.e, semi-annual compounding)

Then, $1 + r_E = \left(1+\frac{0.08}{2}\right)^{2}$

or, $r_E = (1.04)^2 - 1$

$= 1.0816 - 1$

$= 0.0816$

or 8.16%

Therefore $r_E > r$.

1.5. Compounded sum of an Annuity

A stream of equal annual cash flows (inflows or outflows) is regarded as an annuity. Here, the investor may be interested to know the future compounded value of an annuity on which interest is paid at a specified rate. When the cash flows occur at the end of each period, the annuity is called a deferred annuity. However, if the cash flows occur at the beginning of each period, it is called an annuity due. The deferred annuity is a contract with an insurance company that promises to pay the owner of the policy holder (annuitant) a regular income at equal time intervals at some future date. If the annuity is fixed then the annuitant receives a guaranteed rate of return per period (at the end of each period) on the money in the account. [However, there can also be indexed annuities and variable annuities. In these cases, the return is based on the performance of a particular market index, e.g., NIFTY. Similarly, the variable annuity payment is based on the performance of a portfolio of mutual funds or some other portfolios as chosen by the annuity owner.]

Annuities are primarily bought by individuals who want to receive stable retirement income. In case of 'annuity due', the payment is due immediately at the beginning of each period. For example, the payment of insurance premium, the payment of rent for a factory shed (where the landlord demands the payment of rent at the beginning of a period) can be considered as annuity due.

1.5.1. Future Value of a deferred annuity

The estimation of future value of a deferred annuity can be analysed with the help of a simple example. An individual may invest a fixed sum of ₹ 1000 at an interest rate of 10% p.a at the end of each year for 5 years.

So, in this case, the future compounded value of an annuity of ₹ 1000 will be :

$1000(1+0{\cdot}1)^4 + 1000(1+0{\cdot}1)^3 + 1000(1+0{\cdot}1)^2 + 1000(1+0{\cdot}1) + 1000$

The last cash flow (viz, ₹ 1000 at the end of 5th year) will not earn any interest.

$$\begin{aligned}\therefore V &= 1000(1{\cdot}464) + 1000(1{\cdot}331) + 1000(1{\cdot}21) + 1000(1{\cdot}1) + 1000\\ &= 1000(1{\cdot}464 + 1{\cdot}331 + 1{\cdot}21 + 1{\cdot}1 + 1)\\ &= 1000(6{\cdot}105)\\ &= ₹\ 6{,}105\end{aligned}$$

Here, 6·105 = Compound Value Interest Factor (CVIF) for annuity of ₹ 1 at 10% interest rate p.a. for 5 years.

This future value can also be calculated using the formula :

$$V = A\left\{\frac{(1+r)^n - 1}{r}\right\} \text{ .... } (1.8)$$

where A = fixed sum of money invested at the end of each year.

r = given intest rate p.a.

n = number of years.

$$\therefore V = 1000\left\{\frac{(1+0{\cdot}1)^5 - 1}{0{\cdot}1}\right\}$$

$$= 1000\left\{\frac{1{\cdot}6105 - 1}{0{\cdot}1}\right\}$$

$$= 1000\left(\frac{0{\cdot}6105}{0{\cdot}1}\right)$$

$$= 1000 \times 6{\cdot}105$$

$$= ₹\ 6{,}105$$

- **Proof :** *The formula (1.8) used for the estimation of the future compounded value of a deferred annuity, viz.*

$$V = A\left\{\frac{(1+r)^n - 1}{r}\right\}$$

can be proved as follows :

Here, we get a G.P. series with subsequent value increased by a factor $(1+r)$

So, sum of this G.P. series will be :

$$V = 1 + (1+r) + (1+r)^2 + \text{.....} + (1+r)^{n-1} \text{ .... } (1)$$

The first term of this series shows ₹ 1 received at the end of n^{th} period, and hence, it does not earn any interest income.

Now, multiplying both the sides of (1) by $(1+r)$, we get

$$(1+r)V = (1+r) + (1+r)^2 + (1+r)^3 + \text{.....} + (1+r)^{n-1} + (1+r)^n \text{ .... } (2)$$

Now, subtracting (1) from (2), we get

$$rV = (1+r)^n - 1$$

$$\text{or, } V = \left\{\frac{(1+r)^n - 1}{r}\right\} \text{ .... } (3)$$

The result (3) shows the future compounded value of a deferred annuity of ₹ 1.

1.5.2. Future Value of an annuity due

When the cash flows occur regularly at the beginning of each period, the annuity is called an annuity due.

In this case, the future compounded value of the annuity due can be calculated using the following formula :

$$V = A\left\{\frac{(1+r)^n - 1}{r}\right\}(1+r) \text{ .... } (1.9)$$

- **Proof :** In this case, the cash flow generates the following G.P. series

$$(1+r) + (1+r)^2 + (1+r)^3 + \text{.....} + (1+r)^n$$

and the future value becomes

$$V = (1+r) + (1+r)^2 + (1+r)^3 + \text{.....} + (1+r)^n \text{ .... } (1)$$

Multiplying both the sides of (5) by $(1+r)$, we get

$$(1+r)V = (1+r)^2 + (1+r)^3 + (1+r)^4 + \text{.....} + (1+r)^{n-1} \text{ .... } (2)$$

Now, subtracting (1) from (2), we get

$$rV = -(1+r) + (1+r)^{n+1}$$

$$= (1+r)\left[(1+r)^n - 1\right]$$

$$\text{or, } V = \left\{\frac{(1+r)^n - 1}{r}\right\}(1+r) \text{ .... (3)}$$

The result (3) shows the future compounded value of an annuity due of ₹ 1.

Example 1.8

An individual invests a fixed sum of ₹ 1000 at the beginning of each year for 5 years at an interest rate of 10% p.a. In this case, the future compounded sum of ₹ 1000 at the end of 5th year can be calculated as follows :

$$V = 1000\left\{\frac{(1+0.1)^5 - 1}{0.1}\right\}(1+0.1)$$

$$= 1000 \times 6.106 \times 1.1$$

$$= 1000 \times 6.7155$$

$$= ₹\ 6{,}715.50$$

1.6. Discounting and Present Value

Now, we shall discuss the process of estimating the present value of future cash-flow stream.

1.6.1. Annual discounting and Present Value of a Single Cash Flow

The discounting technique is used to measure the present value of the future income stream generated from an investment project. An investment project generally creates cash flows in future during the life-span of the said project. From the view point of time value of money, it is natural that the cash flow received at some future date would be less worthy since the interest on that amount is sacrificed for that period by the investor. Given any positive interest rate, the present value of a future income would always be less than its future value in absolute terms.

For example, if a sum of ₹ 100 earns interest rate at the rate of 10% p.a., then after 1 year it would become

$$₹\ 110 = 100 + 10\% \text{ of } 100$$

$$= 100\ (1 + 0.1).$$

Thus, the present value of ₹ 110 which is received after 1 year would be

$$\frac{110}{(1+0.1)} = \frac{110}{1.1} = ₹\ 100$$

The general rule is

$$P = \frac{V}{(1+r)^n} = V\left\{\frac{1}{(1+r)^n}\right\} \text{ .... (1.10)}$$

where V = Amount of cash flow received in some future time period.

n = number of years.

P = present value.

r = given interest rate or the discount rate.

Here, we observe that the present value is just the reciprocal of the future value (or the compound value) as we have discussed before.

Example 1.9

An individual investor is interested in estimating the present value of ₹ 5,000 to be received after 10 years from the present period (assuming a given interest rate of 10% p.a.).

Here, we get $P = 5000\left\{\frac{1}{(1+0.1)^{10}}\right\}$

$$= 5000 \times \frac{1}{2.594} = 5000\ (0.3855)$$

$$= ₹\ 1{,}927.50$$

It implies that the investor will remain indifferent between a sum of ₹ 1,927.50 received at present and a sum of ₹ 5,000 to be received after 10 years (assuming a yearly interest rate of 10%).

Here, the market rate of interest is considered as the 'discounting factor' to determine the present value of any future cash flow.

In our formula (1.10), the expression $\left\{\frac{1}{(1+r)^n}\right\}$ is considered as Present Value Interest Factor (PVIF).

1.6.2. Multi-period discounting and Present Value of a Single Cash Flow

Just like multi-period compounding in determining the future value of a cash flow, here we have multi-period discounting. When discounting is done more than once in a year, it is called multi-period discounting. In this case we use the following formula :

$$P = V\left\{\frac{1}{(1+\frac{r}{m})^{mt}}\right\} \text{ .... (1.11)}$$

Here, P = Present value.

V = Amount of cash flow received in some future time period.

m = Number of times for which discounting is done per year, i.e., frequency of discounting per year.

t = Number of years.

Thus, when such discounting is done twice a year (i.e., after every 6 months), it is called **semi-annual discounting**. In this case equation (1.11) is expressed as follows :

$$P = V\left\{\frac{1}{(1+\frac{r}{2})^{2t}}\right\} \text{ .... (1.12)}$$

Example 1.10

If we want to find out the present value of ₹ 5,000 at an interest rate of 10% receivable at the end of the 3 years, and discounting is done half yearly, then the present value of that future cash flow will be

$$P = 5000\left\{\frac{1}{\left(1+\frac{0.1}{2}\right)^{2\times 3}}\right\}$$

$$= 5000\left\{\frac{1}{(1.05)^{6}}\right\}$$

$$= \frac{5000}{1.3404} = ₹\ 3{,}731$$

Similarly, if the discounting is done quarterly, then it is called quarterly discounting of the cash flow in determining the present value, and here $m = 4$. So, in case of quarterly discounting

$$P = V\left\{\frac{1}{\left(1+\frac{r}{4}\right)^{4t}}\right\} \text{ ------ (1.13)}$$

Example 1.11

If we want to determine the present value of ₹ 5,000 at an interest rate of 10% receivable at the end of 5 years, and discounting is done quarterly then the present value of that amount will be

$$P = 5000\left\{\frac{1}{\left(1+\frac{0.1}{4}\right)^{4\times 5}}\right\}$$

$$= 5000\left\{\frac{1}{(1.025)^{20}}\right\} = \frac{5000}{1.6386}$$

$$= ₹\ 3{,}051.38$$

In a similar fashion there can be monthly discounting of the future value in determining its present value. In this case $m = 12$, and we use the following formula :

$$P = V\left\{\frac{1}{\left(1+\frac{r}{12}\right)^{12t}}\right\} \text{ ------ (1.14)}$$

If we consider our previous example

$$\text{Then } P = 5000\left\{\frac{1}{\left(1+\frac{0.1}{12}\right)^{12\times 5}}\right\}$$

$$= 5000\left\{\frac{1}{(1.0083)^{60}}\right\}$$

$$= \frac{5000}{1.6420} = ₹\ 3{,}045.06$$

In case of continuous discounting, the present value of a given cash flow is determined as follows : We have already explained that in case of continuous compounding the future value of a cash flow is determined by the formula $V = Ae^{rt}$

Therefore, the present value of 'V' will be just a reciprocal of that result, i.e.,

$$A = P = \frac{V}{e^{rt}} = Ve^{-rt} \text{ .... (1.15)}$$

where V = Future value of a given cash flow (A)

$$\text{Here, } P = V \cdot \lim_{m\to\infty}\left\{\frac{1}{\left(1+\frac{r}{m}\right)^{mt}}\right\}$$

$$= V \cdot \lim_{m\to\infty}\left[\frac{1}{\left\{\left(1+\left(\frac{r}{m}\right)\right)^{\frac{m}{r}}\right\}^{rt}}\right]$$

$$= V \cdot \left(\frac{1}{e^{rt}}\right)$$

$$= Ve^{-rt}$$

Example 1.12

If we want to find out the present value of ₹ 5,000 at an interest rate of 10% receivable at the end of 2 years, and discounting is done continuously then the present value of that future cash flow will be

$$P = \frac{5000}{e^{0.1\times 2}} = \frac{5000}{e^{2}} = \frac{5000}{1.2214} = ₹\ 4{,}093.66$$

1.6.3. Present Value of a series of cash flows

In our previous discussion, we have considered only the present value of a single cash flow at some future time period. However, an investment project often generates a series of cash flows over the life-span of the project. In that case, the business firm may be interested in finding out the present value of that aggregate income stream.

Let us assume that the life-span of an investment project is 'n' years and it generates cash flows of x_0, $x_1, x_2, \ldots, x_n$ during the time period 0, 1, 2, ..., n. In this case, the present value of this future income stream can be estimated as follows :

$$P = x_0 + \frac{x_1}{(1+r)} + \frac{x_2}{(1+r)^2} + \ldots + \frac{x_n}{(1+r)^n}$$

$$P = \sum_{t=0}^{n}\frac{x_t}{(1+r)^t} \text{ .... (1.16)}$$

Here, $\left\{\frac{1}{(1+r)^t}\right\}$ is considered as the Present Value Interest Factor (PVIF).

So, $P = x_0\,(PVIF)_0 + x_1\,(PVIF)_1 + \ldots + x_n\,(PVIF)_n$

[Please note that $(PVIF)_0 = \frac{1}{(1+r)^0} = 1$]

$$\therefore P = \sum_{t=0}^{n} x_t\,(PVIF)_t$$

Example 1.15

Let us consider the following cash flow stream of an investment project with a life-span of 5 years.

Period	0	1	2	3	4	5
Cash flow (₹)	(–) 1,00,000	25,000	30,000	35,000	40,000	40,000

The market interest rate is assumed to be 8% p.a. We have to estimate the present value of the investment project.

In this example, we observe that there has been an cash outflow at the initial period of the investment project (It may represent a loan taken from the bank for this project). When the present value of all cash outflows of an investment project is deducted from the present value of all the future stream of cash inflows arising out of that project, it is called Net Present Value (NPV) of that stream of cash flow. We shall discuss this concept in our next section.

Let us consider our example :

Period	Cash flows (₹)	PVIF	Present Value (₹)
(a)	(b)	(c)	(d) = (b) × (c)
0	(–) 1,00,000	$\frac{1}{(1+0.08)^0}=1$	(–) 1,00,000
1	25,000	$\frac{1}{(1+0.08)^1}=0.9260$	23,150
2	30,000	$\frac{1}{(1+0.08)^2}=0.8573$	25,719
3	35,000	$\frac{1}{(1+0.08)^3}=0.7938$	27,783
4	40,000	$\frac{1}{(1+0.08)^4}=0.7350$	29,400
5	40,000	$\frac{1}{(1+0.08)^5}=0.6806$	27,224
		TOTAL	= ₹ 33,276

Our discussion clearly reveals that the present value of a stream of cash flow can be considered as the present payment amount which is equivalent to the entire cash flow stream arising out of an investment project.

1.8.4. Relation between Present Value and Future Value

The method of estimation of the present value of a cash flow stream shows that it is just the reciprocal of what we have followed in estimating the future value of a cash flow stream. The future value (FV) of a cash flow stream has been estimated as follows :

$$FV = x_0(1+r)^n + x_1(1+r)^{n-1} + x_2(1+r)^{n-2} + \dots + x_n$$

On the other hand, the present value (PV) of the cash flow stream is

$$PV = \frac{FV}{(1+r)^n} = x_0 + \frac{x_1}{(1+r)} + \frac{x_2}{(1+r)^2} + \dots + \frac{x_n}{(1+r)^n}$$

where $\frac{1}{(1+r)^t}$ = PVIF

Thus, PV is just the reciprocal of FV.

1.8.5. Present Value of an Annuity

If the future cash inflows are in equal amount during the life-span of the investment project then it is called as an annuity.

This can be denoted by

$$A_1 = A_2 = A_3 = \dots = A_n = A \text{ (say), for 't' periods } (t = 1, 2, \dots, n)$$

So, the present value of such annuity can be estimated as follows :

$$PV = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \frac{A_3}{(1+r)^3} + \dots + \frac{A_n}{(1+r)^n}$$

$$= A\left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \dots + \frac{1}{(1+r)^n}\right] \qquad [\because A_1 = A_2 = A_3 = \dots = A_n = A]$$

$$= A\left[\sum_{t=1}^{n} \frac{1}{(1+r)^t}\right] \text{ ...... (1.17)}$$

Here, $\sum \frac{1}{(1+r)^t}$ for $t = 1, 2, \dots, n$ can be termed as **Annuity Discount Factor.**

1.8.6. Present Value of deferred annuity

In this case, the annuity arises at the end of each period.

Let this deferred annuity be

$$A_1 = A_2 = A_3 = \dots = A_n = A$$

Present value of this annuity will be

$$P = A\left\{\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \dots + \frac{1}{(1+r)^n}\right\}$$

$$= A\left\{\sum_{t=1}^{n} \frac{1}{(1+r)^t}\right\} \text{ ...... (1.18)}$$

Here, $\sum_{t=1}^{n} \frac{1}{(1+r)^t}$ = Present Value Annuity Factor at the discount rate r% for 'n' years.

It is a G.P. series with subsequent value decreased by a factor $\frac{1}{(1+r)}$. The sum of this G.P. series is

$$S_n = \frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \dots + \frac{1}{(1+r)^n}$$

$$= (1+r)^{-1} + (1+r)^{-2} + \dots + (1+r)^{-n} \text{ .... (1)}$$

Now multiplying both the sides of (1) by $(1 + r)$, we get

$$(1+r)S_n = 1 + (1+r)^{-1} + (1+r)^{-2} + ... + (1+r)^{-n+1} \text{ .... (2)}$$

Now, subtracting (1) from (2) we get

$$r\,S_n = 1 - (1+r)^{-n}$$

or, $$S_n = \frac{1-(1+r)^{-n}}{r} = \frac{1}{r} - \frac{1}{r(1+r)^n}$$

$$= \frac{(1+r)^n - 1}{r(1+r)^n}$$

$$\therefore \quad P = A\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\} \text{ .... (1.19)}$$

Example 1.14

Consider the flow of a deferred annuity of ₹ 2,000 for 5 years. Calculate the present value of this cash flow stream considering the market interest rate of 8% p.a.

Solution :

Period	Annuity (₹)	Present Value Annuity Factor ($r = 0.08$)	PV (₹)
(a)	(b)	(c)	(d) = (b) × (c)
1	2,000	0·9260	1852·00
2	2,000	0·8573	1714·60
3	2,000	0·7938	1587·60
4	2,000	0·7350	1470·00
5	2,000	0·6806	1361·20
		TOTAL =	₹ 7985·40

Here, PV = 2,000 (0·9260 + 0·8573 + 0·7938 + 0·7350 + 0·6806)

= 2000 × 3·9927

= ₹ 7985·40

We can also calculate the present value by using the formula (1.19) :

$$P = A\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$= 2000\left\{\frac{(1+0.08)^5 - 1}{0.08(1+0.08)^5}\right\}$$

$$= 2000\left\{\frac{1.469328 - 1}{0.08 \times 1.469328}\right\}$$

$$= 2000\left\{\frac{0.469328}{0.117546}\right\}$$

$$= 2000 \times 3.9927$$

$$= ₹\ 7985.40$$

1.6.7. Present Value of a perpetual annuity

Perpetual annuity means an annuity for ever, i.e., $A_1, A_2, A_3, ... = A$ (a constant annuity for ever). Now, if the market rate of interest (r) is used for discounting this cash flow stream then the present value of the perpetual annuity will be

$$P = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \frac{A_3}{(1+r)^3} + ...$$

$$= A\left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + ...\right] \quad [\because A_1 = A_2 = A_3 ... = A]$$

$$= A\,[1 + (\text{PVAF}) + (\text{PVAF})^2 + ...] - A$$

Here, $\text{PVAF} = \frac{1}{(1+r)}$ = Present Value Annuity Factor.

Since, we have added '1' in the series, so that extra (A × 1) has to be deducted again.

Further, in this case,

$0 < \text{PVAF} < 1$ [Say, $r = 10\% = 0.1$]

$\Rightarrow \text{PVAF} = \frac{1}{1+0.1} = 0.91$]

$$\therefore \quad P = A\left[\frac{1-(PVAF)^{\infty}}{1-PVAF}\right] - A$$

or, $$P = A\left[\frac{1}{1-PVAF}\right] - A \quad [\because (PVAF)^{\infty} = 0]$$

However, $1 - PVAF$

$$= 1 - \left(\frac{1}{1+r}\right) = \frac{1+r-1}{1+r} = \frac{r}{1+r}$$

or, $$P = \left(\frac{A}{\frac{r}{(1+r)}}\right) - A$$

$$= \frac{A(1+r)}{r} - A$$

$$= \frac{A + Ar - Ar}{r}$$

$$P = \frac{A}{r} \text{ .... (1.20)}$$

Example 1.15

If an investment yields a constant sum of ₹ 800 p.a. in perpetuity then the present value of that perpetual annuity will as follows (considering a market rate of interest of 8% p.a.)

$$P = \frac{A}{r} = \frac{800}{0.08} = ₹\ 10{,}000$$

1.6.8. Present Value of Annuity due

The present value of annuity due formula is similar to that of present value of deferred annuity with only one additional component of $(1 + r)$ as shown below :

$$P = A\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}(1+r) \quad \ldots\ldots (1.21)$$

Example 1.16

Consider a flow of annuity due of ₹ 2,000 for 5 years. Calculate the present value of this cash flow stream considering the market interest rate of 8% p.a.

Solution :

Here we can calculate the present value using the formula (1.21) :

$$P = A\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}(1+r)$$

where A = ₹ 2,000, $n = 5$, $r = 0{\cdot}08$

$$\therefore \quad P = 2000\left\{\frac{(1+0{\cdot}08)^5 - 1}{0{\cdot}08(1+0{\cdot}08)^5}\right\}(1+0{\cdot}08)$$

$$= 2000\left\{\frac{1{\cdot}469328 - 1}{0{\cdot}08 \times 1{\cdot}469328}\right\}(1{\cdot}08)$$

$$= 2000\left\{\frac{0{\cdot}469328}{0{\cdot}117546}\right\} \times (1{\cdot}08)$$

$$= 2000 \times 3{\cdot}9927 \times 1{\cdot}08$$

$$= 7985{\cdot}40 \times 1{\cdot}08$$

$$= ₹\ 8624{\cdot}23$$

Thus, other things remaining same, the present value of annuity due becomes higher than of a deferred annuity.

1.6.9. Present value with continuous compounding

Now, we shall take into consideration the present value of a cash flow stream where 'r' is compounded continuously at 'm' equally spaced periods (say, bi-annual, quarterly, monthly etc.) per year.

Let us consider that the nominal interest rate r is compounded continuously and the cash flows occur in time periods $t_0, t_1, \ldots, t_n$ and the cash flow at the end of each period is being denoted by $x_0, x_1, x_2, \ldots, x_n$.

So, in this case,

$$PV = \sum_{t=0}^{n} \frac{x_t}{\left[1+\left(\frac{r}{m}\right)\right]^{mt}}$$

$$= \sum_{t=0}^{n} \frac{x_t}{\left[1+\left(\frac{r}{m}\right)^{\frac{m}{r}}\right]^{rt}}$$

$$\text{and } PV = \sum_{t=0}^{n} \lim_{m\to\infty} \frac{x_t}{\left[1+\left(\frac{r}{m}\right)^{\frac{m}{r}}\right]^{rt}}$$

$$\text{or, } PV = \sum_{t=0}^{n} \frac{x_t}{e^{rt}} = \sum_{t=0}^{n} x_t \cdot e^{-rt} \quad \ldots\ldots\ldots (1.22)$$

1.7. Net Present Value (NPV)

When the present value of all cash outflows of an investment project is deducted from the present value of all the future stream of cash inflows arising out of that project, it is called the Net Present Value or NPV of that stream of cash flow. The NPV method is generally considered as the most important method for evaluating the capital investment proposals. The following steps are involved in estimating the NPV of an investment project :

(a) At first all cash inflows and cash outflows associated with the investment project are to be worked out.

(b) Appropriate discount rate has to be identified to determine the present value of all the cash flows during the life-span of the project. This discount rate reflects the *cost of capital* or the minimum rate of return which must be earned to undertake such risky venture and a return that would keep the market value of the firm (viz., the market value of the shares and debentures of the firm) intact.

(c) Calculate the present value (PV) of all cash outflows, and calculate the PV of all cash-inflows associated with the investment project using an appropriate discount rate (sometimes the market interest rate is used as the close substitute of the cost of capital).

(d) The Net Present Value (NPV) has to be estimated by subtracting the aggregate PV of all cash outflows from the aggregate PV of all cash inflows.

1.7.1. Evaluation of an Investment Project

An investment project or proposal can be evaluated on the basis of the NPV of the possible cash flow stream arising out of that project.

If NPV > 0 then the project should be accepted. However, the project should be rejected if NPV < 0. If NPV = 0, the firm remains indifferent between its acceptance and rejection.

- **Conventional cash flows :**

When cash outflow is expected to occur only at the start of the investment project then such pattern of cash flow is considered as the conventional cash flows.

Let x_0 = Cash outflow at the initial period

x_t = Cash inflow at the t-th time period (t = 1, 2, ..., n)

r = market rate of interest representing the discount factor or the cost of capital.

Please note that the terminal cash inflows from a project may also include the salvage value (i.e., the scrap value of the discarded machineries and other physical assets of the project) of the project and the recovery of working capital (say, recovery of some bills receivable).

So, in this case

$$NPV = \sum_{t=1}^{n} \frac{x_t}{(1+r)^t} - x_0 \text{ ...... (1.23)}$$

- **Non-Conventional cash flows :**

When the cash outflows occur not only at the initial period but also at subsequent periods, that cash flow pattern is considered as non-conventional cash flow.

Let x_t^* = Cash outflow at period t (where t = 0, 1, 2, ..., n)

In this case

$$NPV = \sum_{t=1}^{n} \frac{x_t}{(1+r)^t} - \sum_{t=0}^{n} \frac{x_t^*}{(1+r)^t} \text{ ...... (1.24)}$$

Example 1.17

The management of SPUTNIK HEALTH Co. Ltd. proposes to purchase a new ventilation machine. Two alternative ventilation machines are available — each having an initial investment of ₹ 1,00,000. Based on the following information, state which of the alternatives you consider financially preferable :

		Machine X	Machine Y
(i) Initial investment (₹)		1,00,000	1,00,000
(ii) Estimated life (Year)		5	5
(iii) Estimated cash inflows (after tax) (₹)	Year-1	25,000	10,000
	Year-2	30,000	15,000
	Year-3	35,000	25,000
	Year-4	40,000	25,000
	Year-5	40,000	25,000

The investor expects a minimum rate of return of 15% p.a. on the investment.

Solution :

Here, the minimum expected return of the investor should be considered as the discounting factor in estimating the NPV of the alternative investment proposals.

For Machine X :

$$NPV = \left[\frac{25,000}{(1+0.15)} + \frac{30,000}{(1+0.15)^2} + \frac{35,000}{(1+0.15)^3} + \frac{40,000}{(1+0.15)^4} + \frac{40,000}{(1+0.15)^5}\right] - 1,00,000$$

$= [25,000\,(0.8696) + 30,000\,(0.7561) + 35,000\,(0.6575) + 40,000\,(0.5718) + 40,000\,(0.4972)] - 1,00,000$

$= [21,740 + 22,683 + 23,012 + 22,872 + 19,888] - 1,00,000$

$= 1,10,195 - 1,00,000 =$ ₹ 10,195

For Machine Y :

$$NPV = \left[\frac{10,000}{(1+0.15)} + \frac{15,000}{(1+0.15)^2} + \frac{25,000}{(1+0.15)^3} + \frac{25,000}{(1+0.15)^4} + \frac{25,000}{(1+0.15)^5}\right] - 1,00,000$$

$= [10,000\,(0.8696) + 15,000\,(0.7561) + 25,000\,(0.6575) + 25,000\,(0.5718) + 25,000\,(0.4972)] - 1,00,000$

$= [8,696 + 11,342 + 16,437 + 14,295 + 12,430] - 1,00,000$

$= 63,200 - 1,00,000$

$=$ (–) ₹ 36,800.

Here NPV > 0 for the purchase decision of Machine X, i.e., in this case, the present value of the cash inflows is more than the initial cost of the project or the initial cash outflow.

On the other hand, NPV < 0 for the purchase decision of Machine Y, i.e., in this case, the present value the cash inflows is not sufficient to cover the initial cash outflow.

Hence, the firm should accept the investment proposal related to the purchase of Machine X.

Tabular Presentation

Year	Machine-X			Machine-Y		
	Cash inflow (₹)	PVDF (15%)	PV (₹)	Cash inflow (₹)	PVDF (15%)	PV (₹)
1	25,000	0·8696	21740	10,000	0·8696	8696
2	30,000	0·7561	22683	15,000	0·7561	11342
3	35,000	0·6575	23012	25,000	0·6575	16437
4	40,000	0·5718	22872	25,000	0·5718	14295
5	40,000	0·4972	19888	25,000	0·4972	12430
	Total		1,10,195			= 63,200
	Less : Cash outflow		– 1,00,000			(–) 1,00,000
	NPV		₹ 10,195			(–) ₹ 36,800

∴ NPV < 0 for Machine - Y

and NPV > 0 for Machine - X.

Example 1.18

Calculate the PV of the following income stream of a project considering the discount rate as 10%. If initial cost of the project is ₹ 40,000 and the salvage value of the project is ₹ 10,000 then calculate NPV and comment.

Year	Net cash flow (₹)	Present value discount factor	Present Value (₹)
1	7,000	0.9091	6,364
2	9,800	0.8264	8,182
3	10,800	0.7513	8,114
4	11,100	0.6830	7,582
5	9,400	0.6209	5,836
6	7,600	0.5645	4,290
7	5,700	0.5132	2,926
8	4,000	0.4665	1,866
9	2,000	0.4241	848
10	2,000	0.3855	772
Total			= 46,780
10	Salvage : 10,000	0.3855	3,855

Please note that here the present value discount factors are to be calculated. Thus,

$\frac{1}{(1+0.10)} = 0.9091$; $\frac{1}{(1+0.10)^2} = 0.8264$; $\frac{1}{(1+0.10)^3} = 0.7513$; and so on.

Now, the initial cost of the project is ₹ 40,000 and it is denoted by C_0. However, the present value of the scrap value or the salvage value of the project has to be deducted from the initial project cost to get the net project cost. So, Net project cost = x_0 – Present value of the salvage value = 40,000 – 10,000 (0.3855) = 40,000 – 3,855 = ₹ 36,145

Thus, the NPV is calculated as NPV = $\sum_{t=1}^{n} \frac{x_t}{(1+r)^t} - \left[x_0 - \frac{S_n}{(1+r)^n}\right]$ where x_0 = Cash outflow at the initial period or the initial project cost, S_n = Salvage value of the project at n-th period.

$$\text{or, NPV} = \sum_{t=1}^{n} \frac{x_t}{(1+r)^t} + \frac{S_n}{(1+r)^n} - x_0$$

$$= (46,780 + 3,855) - 40,000$$

$$= ₹\ 10,635$$

Decision : If NPV > 0 the project should be accepted. Here, NPV > 0. So this project is accepted.

Example 1.19

An investment proposal of ABC Ltd. requires an initial outlay of ₹ 4,00,000 with an expected uniform annual cash flow after tax of ₹ 1,00,000 for 5 years. Should the proposal be accepted if the rate of discount is (a) 15% or (b) 6% ?

Solution :

Statement showing the calculation of present value of cash flows with the discount factor of 15% and 6%

Year	Cash inflow after tax (CIAT) (₹)	Present Value of ₹ 1 @ 15%		Present Value of ₹ 1 @ 6%	
		Discount factor (₹)	Present Value (₹)	Discount factor (₹)	Present Value (₹)
1	1,00,000	0.8696	86,960	0.9434	94,340
2	1,00,000	0.7561	75,610	0.8900	89,000
3	1,00,000	0.6575	65,750	0.8396	83,960
4	1,00,000	0.5718	57,180	0.7921	79210
5	1,00,000	0.4972	49,720	0.7473	74,730
		3.3522	3,35,220	4.2124	4,21,240
Less : Initial Outlay			4,00,000		4,00,000
NPV			(–) 64,780		21,240

It is evident that the proposal cannot be accepted at the discount rate of 15% because of negative NPV i.e., NPV < 0. But at the discount rate of 6%, the proposal gives a positive NPV, suggesting that it may be acceptable. It is important to note that the discount rate is one of the most important factors used in the calculation of the present value because different discount rates will give different present values suggesting different decisions.

Another important factor should be noted in respect of calculation of NPV. If annual cash inflows accrue at uniform or even rate at the end of each year, then the above procedure may be simplified by multiplying sum of the present value factors with the amount of investment.

For example,

at discount rate of 15%, total present value = ₹ 1,00,000 × 3.3522 = ₹ 3,35,220 and,

at discount rate of 6%, total present value = ₹ 1,00,000 × 4.2124 = ₹ 4,21,240

Therefore, under this situation, sum of the present value factors can be applied directly with the amount of initial cash outlay.

- **Mutually exclusive Projects :**

For mutually exclusive projects, the firm computes the NPV for all possible investment projects, and then ranks these projects in order of their NPVs. Thus, it becomes easier for the firm to appraise any investment project in terms of the NPV.

Example 1.20

Rank the following Projects on the basis of NPV criterion assuming 10% discount rate.

Project	Initial Capital Investment (₹)	Annual Cash Flow (₹)	Project Life (Yrs)	Present Value factor for Annuity at 10%
A	25,000	3,000	10	6·1446
B	3,000	1,000	5	3·7908
C	12,000	2,000	8	5·3349
D	20,000	4,000	10	6·1446
E	40,000	8,000	12	6·8137

Solution :

Here, it is observed that for each project the annual cash flow remains given. So, each cash inflow is to be considered as an annuity.

Project	Initial Capital Investment (₹)	Annual Cash Flow (₹)	Project Life	Present Value factor for Annuity at 10%	NPV (₹)	Profitability Index	Rank (based on Profitability index)
(a)	(b)	(c)	(d)	(e)	(f)=[(c) × (e)] − (b)	(g) = 1+(f)/(b)	(h)
A	25,000	3,000	10	6·1446	(−) 6566·2	Rejected	Rejected
B	3,000	1,000	5	3·7908	790·8	1·26	2
C	12,000	2,000	8	5·3349	(−) 1330·2	Rejected	Rejected
D	20,000	4,000	10	6·1446	4578·4	1·23	3
E	40,000	8,000	12	6·8137	14,509·6	1·36	1

For NPV < 0, the projects A & C have been rejected. Project E ranks first in terms of profitability index.

It is important to note that the present value factor for annuity at 10% interest rate is calculated, say for 5 years, as follows :

$$\frac{1}{(1+0\cdot10)} = 0\cdot9091;\ \frac{1}{(1+0\cdot10)^2} = 0\cdot8264;\ \frac{1}{(1+0\cdot10)^3} = 0\cdot7513;\ \frac{1}{(1+0\cdot10)^4} = 0\cdot6830;\ \frac{1}{(1+0\cdot10)^5} = 0\cdot6209;$$

and we get $(0\cdot9091 + 0\cdot8264 + 0\cdot7513 + 0\cdot6830 + 0\cdot6209) = 3\cdot7907 \approx 3\cdot7908$

Profitability Index (PI) : The Profitability Index (PI) is a variant of NPV method for project evaluation. It is defined as the ratio of present value (PV) of income stream of a project over its life-span to the initial project cost. So, $PI = \dfrac{\sum_{t=1}^{n}\frac{x_t}{(1+r)^t}}{x_0}$ and we know that $NPV = \sum_{t=1}^{n}\frac{x_t}{(1+r)^t} - x_0$

or, $NPV + x_0 = \sum_{t=1}^{n}\frac{x_t}{(1+r)^t}$ So, $PI = \dfrac{NPV + x_0}{x_0} = 1 + \dfrac{NPV}{x_0}$

This formula has been used in this estimation process.

Example 1.21

Let us consider that the initial cost of undertaking the Project-X is ₹ 1 lakh and Project-Y also requires the same initial cost. The life of the Project-X is 2 years and that of Project-Y is 3 years ; the cash inflows in these two projects during their life-span are as follows :

Year	Project-X Cash inflow (₹ in lakh)	Project-Y Cash inflow (₹ in lakh)
1	2	0
2	3	3
3	–	5

Make an evaluation regarding which of these projects is to be accepted by the investor based on NPV method of project evaluation considering the market interest rate of 10% as discounting factor.

Solution :

In case of Project-X, $NPV = \dfrac{x_1}{(1+r)} + \dfrac{x_2}{(1+r)^2} - x_0 = \sum_{t=1}^{2}\dfrac{x_t}{(1+r)^t} - x_0$

or, $NPV = \dfrac{2}{(1+0\cdot1)} + \dfrac{3}{(1+0\cdot1)^2} - 1 = (1\cdot818 + 2\cdot479) - 1 =$ ₹ 3·297 lakh

In case of Project-Y, $NPV = \dfrac{x_1}{(1+r)} + \dfrac{x_2}{(1+r)^2} + \dfrac{x_3}{(1+r)^3} - x_0 = \sum_{t=1}^{3}\dfrac{x_t}{(1+r)^t} - x_0$

or, $NPV = \dfrac{0}{(1+0\cdot1)} + \dfrac{3}{(1+0\cdot1)^2} + \dfrac{5}{(1+0\cdot1)^3} - 1 = (0 + 2\cdot479 + 3\cdot756) - 1 =$ ₹ 6·235 − 1 = ₹ 5·235 lakh

So, in this case, NPV > 0 for both the projects but NPV of Project-Y is greater than that of Project-X. So, Project-Y should be accepted.

1.7.2. Merits of NPV method

Now we can indicate some of the merits and demerits of the NPV method.

Merits of the NPV method :

The NPV method of appraising any investment project has several merits :

(i) **This method explicitly recognises the time value of money,** i.e., this method takes into consideration the present value of the future income stream of any investment project while evaluating the project.

(ii) **Total benefits** arising out of any investment project over the life-span of the project are also **taken into account** in the NPV method.

(iii) This method is also **capable of accommodating** changes in the cost of capital by changing the magnitude of the discounting factor (*i.e.*, *r*).

(iv) This method is also instrumental in fulfilling the financial objective of a firm to maximise the wealth of the shareholders. In this method, the discounting factor can be considered as the minimum rate of return required by the investors. If NPV = 0, it implies that the expected rate of return is just equal to the actual rate of return (at which the present value of the future income stream is just equal to the initial outlay of the project). However, if NPV > 0, the rate of return would be higher than the expected rate. This would lead to an increase in the share prices of the firm, and hence, the wealth of the shareholders. Thus, this method is logically consistent with the financial objective of a firm.

1.7.3. Demerits of the NPV method

The NPV method has some shortcomings also. These are noted below :

(a) **Its calculation is difficult** compared to some other project evalution methods (ARR or Pay Back method).

(b) **Sometimes it becomes difficult for the firm to calculate the required rate of return (or the cost of capital) to discount the cash-flows.**

(c) Generally, **this method favours the project with higher NPV.** However, such a project may also involve a larger amount of initial outlay. Hence, for investment projects involving different amounts of initial outlay, the NPV method may not give dependable results for appraising any investment project. However, this difficulty can be avoided by comparing the *NPV/Co* ratio or by comparing the profitability index or the benefit-cost ratio of different projects.

(d) This method **may not also give satisfactory results when two investment projects have different life-spans.** The present value of the future income stream of a project having a longer life-span may be higher compared to a project having shorter life-span. However, the firm may prefer the latter if it does not want its capital to be locked in for a longer period, and wants a quick return on capital invested.

1.8. Internal Rate of Return (IRR)

The Internal Rate of Return of a project is defined as that discount rate at which the present value of the net cash inflow from the project is just equal to the initial cost of the project. It is important to note that the IRR is defined without any reference to the prevailing interest rate. This IRR is determined entirely by the cash flow structure of the project. Thus, this discount rate is determined internally without reference to the external financial world, and therefore, it is called as the internal rate of return.

So, this is a process of trial and error and continues until we find that discount rate at which the present value of the net cash inflow from the project is just equal to the initial cost of the project (or the supply price of capital). So, here, $\sum_{t=1}^{n} \frac{x_t}{(1+i)^t} = C_0$ where i = IRR

- **Decision \ Evaluation :**

If the IRR > Opportunity cost of capital investment (say, the market interest rate) then the project is accepted.

If the IRR < Opportunity cost of capital investment (say, the market interest rate) then the project is rejected.

The detailed procedure for the calculation of IRR can be explained under two different situations ;

(a) **When the annual cash inflows (after-tax) are equal over the life of the project :** The procedure can be explained with the help of following illustration.

Example 1.22

A machine requires an initial investment of ₹ 60,000. The annual cash inflow (after-tax) is estimated at ₹ 20,000 for 5 years. There is no salvage value. Calculate the internal rate of return.

Solution :

Following *Steps* are involved in the process of calculation of IRR, when cash inflows are equal :

(1) Calculate the pay back period to obtain a *quotient* for making an approximation of the IRR.

Pay Back Period \ Quotient \ Present value factor/P-Ratio

$PB = \frac{I}{C}$, where PB = Pay Back period, I = Initial Investment, and C = Constant Annual Cash Inflow.

Here, $PB = \frac{60,000}{20,000} = 3$

(2) Go across the 5-year (life of the machine) row of the **present value of annuity table-2** (given at the end of this book) and search a value nearest to 3 (*quotient*). For better understanding **a portion of present value of annuity table** is given below :

Present Value of Annuity Table

	Present Value of an Annuity of ₹ 1					
Year	17%	18%	19%	20%	21%	22%
1	0·8547	0·8475	0·8403	0·8333	0·8264	0·8197
2	1·5852	1·5656	1·5465	1·5278	1·5095	1·4915
3	2·2096	2·1743	2·1399	2·1065	2·0739	2·0422
4	2·7432	2·6901	2·6386	2·5887	2·5404	2·4936
5	3·1993	3·1272	$\boxed{3·0576}$	$\boxed{2·9906}$	2·9260	2·8636
6	3·5892	3·4976	3·4098	3·3255	3·2446	3·1669

(3) The nearest figures are given in rate 19% (i.e., 3·0576) and the rate 20% (i.e., 2·9906). It indicates that the IRR of the machine is expected to lie between 19% and 20%.

(4) Apply simple interpolation technique as follows :

Discount Rate (%)	Present Value (PV) Factor
19	3·0576
IRR ⇒	⇐ 3
20	2·9906

$$\frac{IRR-19}{20-19} = \frac{3-3·0576}{2·9906-3·0576}$$

$$\text{or, } \frac{IRR-19}{1} = \frac{-0·0576}{-0·0670}$$

$$\therefore IRR = 19 + 0·8597 = 19·86\%$$

If the market rate of interest is assumed to be 8% then this project is accepted since IRR > 8%.

(b) **When annual cash inflows (after-tax) are not equal over the life of the project :** In this situation, the procedure of estimating IRR can also be explained with the help of following illustration.

Example 1.23

An investment Project requires an initial investment of ₹ 86,000, and the cash inflow during the life span of 4 years of this project is as follows :

Year	Cash inflow (₹)
1	20,000
2	30,000
3	35,000
4	40,000

Calculate the internal rate of return.

Solution :

When the cash inflows are not equal or uniform, the calculation of IRR is not easy because it depends on the frustrating trial and error method and hence complicated. To minimise hazards, following Steps may be recommended :

1. Calculate the payback period to obtain a *quotient* for making an approximation of the IRR with the help of following *modified payback period formula* (*often called as fake payback period*).
2. PB = (Initial Investment)/Average Cash Flow
3. $$\frac{86{,}000}{\frac{20{,}000+30{,}000+35{,}000+40{,}000}{4}} = \frac{86{,}000}{31{,}250} = 2{\cdot}752$$
4. *Go across the 4-year (i.e., life of the equipment) row of the present value of annuity table (given at the end of the book) and search a value nearest to 2·7520 (quotient). This may be considered as the first trial rate for IRR.*
5. *The nearest figures are given in rate 16% (2·7982) and the rate 17% (2·7432). It is important to note that these results are very much rough approximation of IRR.*
6. *Find out the NPV for both of these approximate rates. If the NPV is positive, apply the higher rate of discount and if the higher discount rate still provides a positive NPV, increase the discount rate further until the NPV becomes negative and vice versa.*

Year	Annual Cash Inflow (₹)	Present Value of ₹ 1 i.e., Discount Factor							
		16% PV factor	16% PV (₹)	17% PV factor	17% PV (₹)	15% PV factor	15% PV (₹)	14% PV factor	14% PV (₹)
1	20,000	0·8621	17,242	0·8547	17,094	0·8696	17,392	0·8772	17,544
2	30,000	0·7432	22,296	0·7305	21,915	0·7561	22,683	0·7695	23,085
3	35,000	0·6407	22,424	0·6244	21,854	0·6574	23,012	0·6750	23,625
4	40,000	0·5523	22,092	0·5337	21,348	0·5718	22,872	0·5921	23,684
Total PV			84,054		82,211		85,959		87,938
Less : Initial Investment			86,000		86,000		86,000		86,000
NPV			(–) 1946		(–) 3789		(–) 41		1938

The above calculation shows that NPV is negative for both the discount rates 16% and 17% and therefore the rate should be lowered to make NPV positive. At 15% rate, it is (–) ₹ 41 and at 14%, the NPV is positive. Therefore, IRR should lie between 14% and 15% rate.

In other words, initial investment of ₹ 86,000 falls between total present value of ₹ 85,959 and ₹ 87,938. and the IRR must lie within corresponding discount factor of 15% and 14% respectively. We can apply simple interpolation technique as follows :

Discount Rate (%)	Present Value (PV) Factor
14	87,938
IRR ⇒	⇐ 86,000
15	85,959

$$\frac{IRR-14}{15-14} = \frac{86{,}000-87{,}938}{85{,}959-87{,}938}$$

$$\text{or, } \frac{IRR-14}{1} = \frac{-1938}{-1979} \quad \text{or, } IRR = 14 + 0{\cdot}9793 = 14{\cdot}98\%$$

It indicates that at 14·98% discount rate (i.e., present value factor), the NPV is zero. This may be verified as follows :

Year	*Annual Cash Inflow (₹)*	*P.V. factor at 14·98% rate*	*Present value (₹)*
1	20,000	0·8697	17,394
2	30,000	0·7564	22,692
3	35,000	0·6579	23,026
4	40,000	0·5722	22,888
			86,000
Less : Initial Investment			86,000
NPV			0

Therefore, IRR = 14·98%

If the market rate of interest is assumed to be 8% then this project is accepted since IRR > 8%.

Example 1.24

Let us consider that the initial cost of undertaking the Project-X is ₹ 1 lakh and Project-Y also requires same initial cost. The life of Project-X is 1 year and that of Project-Y is 2 years; the cash inflows in these two projects during their life-span are as follows :

Year	*Project-X Cash inflow (₹ in lakh)*	*Project-Y Cash inflow (₹ in lakh)*
1	2	0
2	–	3

Make an evaluation regarding which of these projects is to be accepted by the investor based on IRR method of project evaluation considering the market interest rate of 10%.

Solution :

The IRR method suggests that in case of Project-X, $\frac{x_1}{(1+i)} = x_0$ where x_0 = ₹ 1 lakh and x_1 = ₹ 2 lakhs and i = IRR

or, $\frac{2}{(1+i)} = 1$ or, $2 = 1(1+i)$ or, $i = 2 - 1 = 1$ (i.e. 100%)

The IRR method also suggests that in case of Project-Y, $\frac{x_1}{(1+i)} + \frac{x_2}{(1+i)^2} = x_0$ where x_0 = ₹ 1 lakh and x_1 = ₹ 0, and x_2 = ₹ 3 lakhs and i = IRR

or, $\frac{3}{(1+i)^2} = 1$ or, $3 = 1(1+i)^2$ or, $\sqrt{3} = \sqrt{(1+i)^2}$ or, $\sqrt{3} = 1 + i$ or, $\sqrt{3} - 1 = i$

or, $i = 1{\cdot}732 - 1 = 0{\cdot}732$ (i.e. 73·2%)

Here, IRR for Project-X is higher than that of Project-Y. So, Project-X has to be accepted.

Now we can indicate some of the merits and demerits of the IRR method.

1.8.1. Merits of the IRR method

Some of the advantages of the IRR method in appraising any investment decision are as follows :

(a) It takes into account the time value of money.

(b) This method also considers the net cash flow stream of an investment project in its entirety.

(c) Business executives and non-technical persons prefer this method because the notion of IRR is easy to understand.

(d) It does not use the concept of the required rate of return (or the cost of capital) which is difficult to determine with accuracy.

(e) While appraising independent investment projects, this method is consistent with the overall financial objective of a firm to maximise the value of the firm (and thereby, maximising the wealth of its shareholders).

1.8.2. Demerits of the IRR method

Despite some of its virtues, this method also suffers from serious limitations.

(a) The iterative procedure involves tedious calculations and often leads to complicated computational problems due to application of trial and error method.

(b) This method may lead to multiple internal rates of return for any investment project.

(c) This method also fails to provide adequate guidance for evaluating mutually exclusive projects because selection of projects based on higher IRR may not be profitable.

(d) It is assumed under this method that the future cash inflows are reinvested at a rate equal to IRR for the remaining life of the proposal. This is not a justified assumption. In actual practice it is noticed that only in rare cases, a reinvestment rate represents reinvestment rate of intermediate cash flows. On contrary, in the case of the NPV method, the implied reinvestment rate i.e., the required rate of return or the cost of capital is the same for all investment proposals and therefore, considered as conservative rate and seems to be more logical than IRR. It indicates that the NPV method has virtue of having a single reinvestment rate for all investment proposals. From this point of view, NPV has less error-free assumption than IRR.

(e) This method does not use the concept of required rate of return, whereas it provides the rate of return which is indicative of the profitability of investment proposal.

(f) The results under this method may be inconsistent compared to NPV method if the projects differ in their expected lives, investment or timing of cash inflow.

1.8.3. Relationship between NPV and IRR

In our discussion, we have presented two variants of the discounted cash flow techniques in evaluating the capital investment decisions. We have realised that three important properties must be satisfied by an investment-appraisal method, and these are :

(a) The method must consider all cash flows throughout the entire life of an investment project ;

(b) The method must consider the time value of money ; and

(c) If the method is used to select from a set of mutually exclusive projects (i.e., these projects are capable of performing the same task for the firm ; and hence, if one of them is accepted, the others will be rejected), it must choose that project which will maximise the current value of the firm.

1.8.4. Similarities between NPV and IRR

Both NPV and IRR methods satisfy properties (a) and (b) and both lead to identical decisions regarding the appraisal of *independent or single investment project* (i.e., accept/reject decisions). Such independent projects can be accepted or rejected simultaneously. In fact, the IRR criterion for acceptance of any project requires that the project's cost of capital (r) be less than the IRR (i). We know that NPV = 0 when the discounting factor (r) equals to 'i'. Thus when $i > r$, it implies that NPV > 0 and the project is accepted following the NPV criterion. Similarly, when $i < r$, it implies that NPV < 0, and the project is rejected.

Proof : If all the annual net cash flows from a project are either positive or zero, NPV or IRR criterion will give same direction of project evaluation.

If NPV > 0 then a project is accepted. So we get

$$NPV = \left[\frac{x_1}{(1+r)} + \frac{x_2}{(1+r)^2} + \cdots + \frac{x_n}{(1+r)^n}\right] - x_0 = \sum_{t=1}^{n} \frac{x_t}{(1+r)^t} - x_0 > 0 \text{ .... (1)}$$

As per IRR, we have

$$\frac{x_1}{(1+i)} + \frac{x_2}{(1+i)^2} + \cdots\cdots + \frac{x_n}{(1+i)^n} = x_0 \text{ .... (2) where } i = \text{IRR}$$

Now, substituting the value of x_0 shown in (2) in (1) we get

$$\sum_{t=1}^{n} \frac{x_t}{(1+r)^t} - \sum_{t=1}^{n} \frac{x_t}{(1+i)^t} > 0$$

$$\text{or, } \sum_{t=1}^{n} x_t \left[\sum_{t=1}^{n} \frac{1}{(1+r)^t} - \sum_{t=1}^{n} \frac{1}{(1+i)^t}\right] > 0$$

Here,

$$\sum_{t=1}^{n} x_t > 0$$

So, for NPV > 0, we must have $\left[\sum_{t=1}^{n}\frac{1}{(1+r)^t}-\sum_{t=1}^{n}\frac{1}{(1+i)^t}\right]>0$

In period $t = 1$, we have $\left[\frac{1}{(1+r)}-\frac{1}{(1+i)}\right]>0$

or, $\left[\frac{1}{(1+r)}>\frac{1}{(1+i)}\right]$

or, $(1+i)>(1+r)$

or, $i>r$

It implies that IRR > opportunity cost of capital investment (i.e., market rate of interest). Therefore, the project is accepted both in terms of IRR and NPV criteria.

Similarly, it can be proved that when NPV < 0 and a project is rejected then that project will also be rejected as per IRR criterion.

Here NPV $=\left[\frac{x_1}{(1+r)}+\frac{x_2}{(1+r)^2}+\cdots\cdots+\frac{x_n}{(1+r)^n}\right]-x_0=\sum_{t=1}^{n}\frac{x_t}{(1+r)^t}-x_0<0$ .... (3)

As per IRR, we have

$$\frac{x_1}{(1+i)}+\frac{x_2}{(1+i)^2}+\cdots\cdots+\frac{x_n}{(1+i)^n}=x_0 \text{ .... (4) where } i=\text{IRR}$$

Now, substituting the value of x_0 shown in (4) in (3) we get

$$\sum_{t=1}^{n}\frac{x_t}{(1+r)^t}-\sum_{t=1}^{n}\frac{x_t}{(1+i)^t}<0$$

or, $\sum_{t=1}^{n}x_t\left[\sum_{t=1}^{n}\frac{1}{(1+r)^t}-\sum_{t=1}^{n}\frac{1}{(1+i)^t}\right]<0$

Here, let us assume (normally it happens) that

$$\sum_{t=1}^{n}x_t>0$$

So, for NPV < 0, we must have $\left[\sum_{t=1}^{n}\frac{1}{(1+r)^t}-\sum_{t=1}^{n}\frac{1}{(1+i)^t}\right]<0$

In period $t = 1$, we have $\left[\frac{1}{(1+r)}-\frac{1}{(1+i)}\right]<0$

or, $\left[\frac{1}{(1+r)}<\frac{1}{(1+i)}\right]$

or, $(1+i)<(1+r)$

or, $i<r$

It implies that IRR < opportunity cost of capital investment (i.e., market rate of interest). Therefore, the project is rejected both in terms of IRR and NPV criteria.

Example 1.25

The NPV and IRR methods lead to identical result with regard to the accept/reject decision as far as **an independent or a single project is concerned.** This can be illustrated graphically with the help of the figures used in previous illustration (1.23). For better understanding, the data may be re-written as follows :

Initial investment for equipment :		₹ 86,000
Estimated life span :		4 years
Cash inflow :		
Year	1	₹ 20,000
	2	₹ 30,000
	3	₹ 35,000
	4	₹ 40,000
IRR (as computed earlier)		14.98% or 15% (say)

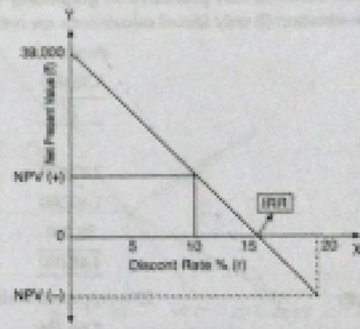

Fig.-1.3

Let us see how NPVs of the said proposal behave with the different discount rates. NPV profiles of the proposal (detail calculations are not shown) are shown below :

Discount Rate % (r)	NPV of Equipment (₹)
0	39,000
5	23,400
10	10,590
14.98 or 15 (say)	0
20	(–) 10,305

Fig.-1.3 shows the relationship between the NPVs of the proposal and the discount rate. It is observed that when discount rate is assumed to be zero (though it is not a real situation), the NPV is maximum of ₹ 39,000. NPV of the equipment falls rapidly as the discount rate increases. At 15% rate, the NPV is zero. This is of course the IRR by definition. If the cost of capital (r) of the firm is assumed to be 10%, we find that NPV is ₹ 10,590 which is positive and hence the proposal is acceptable and so, it is less than IRR as IRR > r, i.e., 15% > 10%. If we assume the cost of capital (r) to be 20%, the proposal is not acceptable as NPV is negative [(–) ₹ 10,305] and so, it is more than IRR as IRR < r, i.e., 15% < 20%. Thus, the two methods i.e., IRR and NPV lead to the same acceptance or rejection decision so far as independent or a single project is concerned.

1.8.5. Conflict between NPV and IRR

However, in case of mutually exclusive projects, the NPV and IRR method will give conflicting ranking to the proposals. This may happen under the following situations :

1. Difference in cash flow patterns or timings among different alternative proposals.
2. Difference in scale or size (i.e., amount) of cash outflow among different alternative proposals.
3. Difference in expected life (i.e., unequal lives) among different alternative proposals.

In the present section we can illustrate this phenomenon graphically with the help of following example by considering the situation (i) only (detail calculations are not shown).

	Project-X	*Project-Y*
Initial investment	3,36,000	3,36,000
Estimated life (in years)	3	3
Estimated Cash Inflow (₹) :		
Year 1	2,80,000	28,000
Year 2	1,40,000	1,68,000
Year 3	28,000	3,02,000
	4,48,000	4,98,000
Internal Rate of Return (IRR)	23% (approximately)	17% (approximately)
NPV at 9% Discount Rate	₹ 60,256	₹ 64,276
Ranking :		
Using IRR	1st	2nd
Using NPV	2nd	1st

NPV Profiles of Project-X and Y
(Detail calculations are not shown)

	Discount Rate % (r)	Net Present Value (NPV) Project-X ₹	Net Present Value (NPV) Project-Y ₹
$NPV_Y > NPV_X$ but $IRR_X > IRR_Y$ ∴ There is a conflict	0	1,12,000	1,62,000
	5	81,732	1,03,960
	9	60,256	64,276
Intersection Rate	10	55,200 (approximately)	55,200 (approximately)
$NPV_X > NPV_Y$ and also $IRR_X > IRR_Y$ ∴ no conflict	15	31,864	14,084
	20	10,612	(–) 21,226
	25	(–) 9064	(–) 51,456
	30	(–) 25,060	(–) 77,602

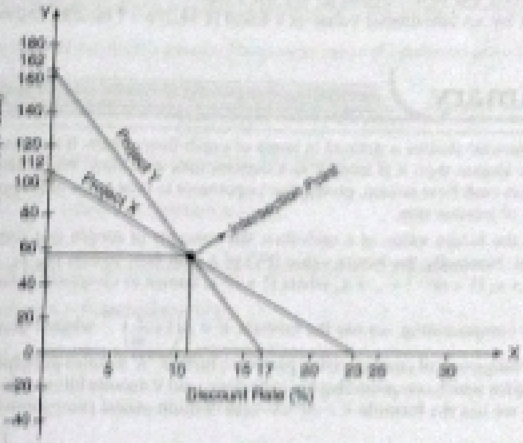

Fig.-1.4

The NPV profiles of Project-X and Y are plotted in Fig.-1.4. It is evident from NPV profile as well as from Fig.-1.4 that the NPV of both the projects declines as the discount rate increases. The IRR of Projects X and Y are 23% and 17% respectively. The NPV profiles of two projects intersect at 10% discount rate.

It is the discount rate where the NPVs of the two projects are equal (₹ 55,200, approximately). In case of Project-X, NPV is maximum of ₹ 1,12,000, if discount rate is assumed to be zero (unreal situation) ; and NPV is zero, corresponding to IRR of 23%. Similarly, in case of Project-Y, NPV is maximum of ₹ 1,62,000, when discount rate is zero ; and NPV is zero, corresponding to IRR of 17%. Now, the points representing maximum NPV (along Y axis) and IRR (along X axis) for both the projects are joined to

form two straight lines. These two lines are called NPV profile curve for the investment projects. The two lines intersect at 10% rate known as intersection rate where NPVs of both the projects are equal. Now it is evident from Fig.-1.4 as well as from the NPV profile that at the discount rates less than the intersection rate i.e. 10%, Project-Y has the higher NPV but lower IRR (17%). On the other hand, at the discount rates greater than the intersection rate (10%), Project-X has both higher NPV as well as higher IRR (23%). Thus, if the required rate of return is greater than the intersection rate, both NPV and IRR methods will give consistent result. It means, the project with higher IRR will also have higher NPV. However, for any required rate of return below the intersection rate (10%), ranking of both the projects under the two methods will give contradictory results. It means, the project with higher IRR will have lower NPV and vice versa.

In the present case, the cost of capital is assumed to be 9% and at this rate both the projects give contradictory results. At 9% rate, both the Projects-X and Y generate positive NPV of ₹ 60,256 and ₹ 64,276 respectively and therefore NPVy > NPVx. But at the same time, Project-X has a higher IRR (23%) than Project-Y (17%). Now the question is which project should we choose? The answer should be in line with the effect of the decision on the maximisation of shareholders' wealth (or current market value of the firm). The IRR method is not compatible with the objective of wealth maximisation. It is concerned with the rate of return on investment or yield rather than the total yield on the investment. Therefore, the firm should go for the Project-Y by following NPV rule and thereby the firm shall be richer by an additional value of ₹ 4,020 [₹ 64,276 − ₹ 60,256] rather than earning a higher rate of return.

Summary

Investment in financial studies is defined in terms of a cash-flow stream. If there remains no uncertainty in this cash-flow stream, then it is termed as a deterministic cash flow. While determining the present value of the future cash flow stream, giving due importance to time value of money, we must first know the basic theory of interest rate.

In determining the future value of a cash-flow, the concepts of simple and compound interest rates become relevant. Normally the future value (FV) of a cash flow stream $(x_0, x_1, ..., x_n)$ is expressed as $FV = x_0(1+r)^n + x_1(1+r)^{n-1} + ... + x_n$ where $(1+r)^n$ is known as Compound Value interest factor. In case of multi-period compounding, we use the formula $V = A\left(1+\frac{r}{m}\right)^{mt}$ where r denotes interest rate p.a.; and m denotes frequency of compounding per year; further, 'A' denotes principal amount, t shows the number of years for which compounding has to be done, and V denotes future value. In case of continuous compounding, we use the formula $V = Ae^{rt}$. In case of multi-period compounding, the effective rate of interest (r_E) becomes more than the nominal interest rate (r), and we have $r_E = \left[1+\left(\frac{r}{m}\right)\right]^{m} - 1$. In case of continuous compounding $r_E = e^r - 1$.

The future compounded value of a deferred annuity is estimated by a formula: $V = A\left\{\frac{(1+r)^n-1}{r}\right\}$, where A = annuity received at the end of every year (viz., deferred annuity). The future value of an 'annuity due' is estimated with the formula:

$$V = A\left\{\frac{(1+r)^n-1}{r}\right\}(1+r), \text{ where } A = \text{Annuity received at the beginning of each year.}$$

The present value of a single cash flow is estimated by using the formula:

$$P = V\left\{\frac{1}{(1+r)^n}\right\}, \text{ where } P = \text{Present value,}$$

V = Given amount of cash flow at some future time period, r = given interest rate (or discount rate). Further, the multi-period discounting of a single cash flow is done by using the formula:

$P = V\left\{\frac{1}{\left(1+\frac{r}{m}\right)^{mt}}\right\}$. Similarly, in case of continuous discounting, we use the formula:

$$P = V\left(\frac{1}{e^{rt}}\right) = V.e^{-rt}.$$

In case of a series of cash flow stream, the present value is estimated by using the formula: $P = \sum_{t=0}^{n}\frac{x_t}{(1+r)^t}$.

The present value and the future value of a cash flow stream are related in the following way: $PV = \frac{FV}{(1+r)^n}$

where FV = Future Value of a cash flow stream. The present value of a deferred annuity is estimated with the help of the following formula: $P = A\left\{\sum_{t=1}^{n}\frac{1}{(1+r)^t}\right\}$ where $\sum\frac{1}{(1+r)^t}$ = Present Value Annuity Factor.

The present value of a deferred annuity is estimated by the formula:

$$P = A\left\{\frac{(1+r)^n-1}{r(1+r)^n}\right\}, \text{ where } A = \text{Deferred annuity.}$$

Similarly, the present value of a perpetual annuity is estimated by the formula:

$$P = A\left(\frac{1}{r}\right) \text{ where A = Perpetual annuity.}$$

The present value of annuity due is estimated by the formula:

$$P = A\left\{\frac{(1+r)^n-1}{r(1+r)^n}\right\}(1+r) \text{ where A = Annuity due.}$$

In case of continuous discounting the present value (PV) of a future cash flow is estimated as

$$PV = \sum_{t=0}^{n}\frac{x_t}{e^{rt}} = \sum_{t=0}^{n}x_t \cdot e^{-rt}$$

An investment project is accepted if NPV > 0, and the investment project is rejected when NPV < 0, where the Net Present Value (NPV) is estimated as $NPV = \sum_{t=1}^{n}\frac{x_t}{(1+r)^t} - x_0$

where x_0 = Cash outflow at the initial period.

It is also possible to rank different investment projects on the basis of their NPV.

The internal rate of return (IRR) of an investment project is defined as that discount rate at which the present value of the future cash flow stream is just equal to the initial cost of the project, i.e., $\sum_{t=1}^{n} \frac{x_t}{(1+i)^t} = x_0$

where x_0 denotes the initial cost of starting the project, and i = IRR. A project is accepted if IRR is greater than the opportunity cost of capital (say, the market rate of interest). However, estimation of IRR entails a long trial-and-error process. In case of some mutually exclusive projects, IRR and NPV may give conflicting results. However, financial economists give more importance to NPV criterion in evaluating an investment project.

Assignment

Short answer-type questions

1. What do you mean by deterministic cash flow stream ? (See Section 1.2)
2. What is time value of money ? (See Section 1.2)
3. Mention two reasons for differences in the value of cash flow at different points of time. (See Section 1.2)
4. If an amount of ₹ 1,000 is invested at a simple interest rate of 8% p.a for 5 years then estimate the future value of this cash flow. (See Section 1.3)
5. If an amount of ₹ 1,000 is invested at a compound interest rate of 8% p.a for 6 years then calculate the value received after 6 years. (See Section 1.3)
6. State the *four* parts of the value received in the process of compounding interest. (See Section 1.3)
7. What is 7-10 rule ? (See Section 1.3)
8. What is the implication of a negative cash flow at the initial period of an investment project ? (See Section 1.4)
9. What is meant by an ideal bank ? (See Section 1.4)
10. What is compound value interest factor ? (See Section 1.4)
11. State the formula for estimating a future value of a given principal amount with multi-period compounding of interest rate. (See Subsection 1.4.1)
12. How do you estimate the future value of single cash flow for continuous compounding of interest rate ? (See Subsection 1.4.4)
13. What is an effective interest rate ? (See Subsection 1.4.5)
14. If the nominal interest rate is 10% p.a. and if it is compounded quarterly than find the effective interest rate. (See Subsection 1.4.5)
15. If the nominal interest rate is 10% p.a. and it is compounded continuously during a year, then estimate the effective interest rate. (See Subsection 1.4.5)
16. What is a deferred annuity ? (See Section 1.5)
17. What is the difference between a deferred annuity and an annuity due ? (See Section 1.5)
18. State the formula for calculating the future value of a deferred annuity. (See Subsection 1.5.1)
19. Estimate the future value of a deferred annuity of ₹ 1,000 at an interest rate of 8% for 5 years. (See Subsection 1.5.1)
20. State the formula for estimating the future value of an annuity due. (See Subsection 1.5.2)
21. Estimate the future value of an annuity due of ₹ 1,000 received at the beginning of each year for 10 years at an interest rate of 8% p.a. (See Subsection 1.5.2)
22. State the formula for estimating the present value of a single cash flow received after a certain period. (See Section 1.6)

23. Estimate the present value of ₹ 10,000 to be received after 5 years assuming a given interest rate of 8% p.a. (See Subsection 1.6.1)
24. How can you estimate the present value of a single cash flow with multi-period discounting ? (See Subsection 1.6.2)
25. Find out the present value of ₹ 5,000 at an interest rate of 10% p.a. receivable at the end of 5 years where discounting is done bi-annually. (See Subsection 1.6.2)
26. State the formula for estimating the present value of a given cash flow where discounting is done continuously. (See Subsection 1.6.2)
27. What is present value interest factor ? (See Subsection 1.6.3)
28. State the formula for estimating the present value of a cash flow stream. (See Subsection 1.6.3)
29. What is present value annuity factor ? (See Subsection 1.6.5)
30. State the formula for estimating the present value of a deferred annuity. (See Subsection 1.6.6)
31. Calculate the present value of a deferred annuity of ₹ 1,000 for 5 years considering the market interest rate of 8% p.a. (See Subsection 1.6.6)
32. What is perpetual annuity ? (See Subsection 1.6.7)

    or, Determine the present Value of a perpetuity that pays ₹ 7,200 per year with 15% interest rate. (See Subsection 1.6.7)

    [C.U., B.Sc. (H), Sem-V, 2020]
33. Calculate the present value of a perpetual annuity of ₹ 1,000 at a market interest rate of 6% p.a. (See Subsection 1.6.7)
34. State the formula for estimating the present value of an annuity due. (See Subsection 1.6.8)
35. Estimate the present value of an annuity due of ₹ 1,000 for 10 years considering a market interest rate of 10% p.a. (See Subsection 1.6.8)
36. State the formula for estimating the present value of a cash flow stream with continuous discounting. (See Subsection 1.6.9)
37. What is Net Present Value (NPV) of an investment project ? (See Section 1.7)
38. State the criterion for the evaluation of an investment project based on NPV method. (See Subsection 1.7.1)
39. Mention any *two* merits of NPV method. (See Subsection 1.7.2)
40. State any *two* demerits of NPV method. (See Subsection 1.7.3)
41. What is IRR ? (See Section 1.8)
42. Mention the process of evaluating any investment project on the basis of IRR. (See Section 1.8)
43. Mention any *two* demerits of IRR method. (See Subsection 1.8.2)
44. Mention any *two* merits of IRR method. (See Subsection 1.8.1)

Long-answer type question

1. (a) Explain the concept of deterministic cash flow. (See Section 1.2)

   (b) Discuss the concept of time value of money. (See Subsection 1.2.1)
2. Distinguish between simple and compound interest rate in connection with the estimation of future value of a single cash flow. (See Section 1.3)
3. Explain the process of estimatimating the future value of a single cash flow when (i) interest rate is compounded bi-annually, quarterly and monthly, (ii) interest is compounded continuously. (See Subsection 1.4.1-1.4.4)
4. Illustrate the notion of effective interest rate. (See Subsection 1.4.5)
5. (a) Distinguish between the notions of deferred annuity and annuity due.

   (b) Prove that the future value of a deferred annuity can be stated as $V = A\left\{\frac{(1+r)^n - 1}{r}\right\}$

   (See Section 1.5 and Subsection 1.5.1)

6. (a) Prove that the future value of an annuity due can be estimated by a formula : $V = A\left[\frac{(1+r)^n-1}{r}\right](1+r)$ (See Subsection 1.6.2)

(b) An individual invests a fixed sum of ₹ 2,000 at the beginning of each year for 8 years at an interest rate of 10% p.a. Calculate the future value of this cash flow.

7. Explain the concept of multi-period discounting for estimating the present value of a single cash flow. (See Subsection 1.6.3)

8. How can you estimate the present value of a cash flow stream ? Find out the present value of the following cash flow :

Period	0	1	2	3	4	5	6
Cash flow (₹)	(–) 10,000	1,200	1,250	1,350	1,300	1,200	1,180

(See Subsection 1.6.3)

9. Explain the relation between present value and future value of a cash flow stream. (See Subsection 1.6.4)

10. Prove that the present value of a deferred annuity can be estimated by the formula : $P = A\left[\frac{(1+r)^n-1}{r(1+r)^n}\right]$ (See Subsections 1.6.6)

11. (a) How can you estimate the present value of a perpetual annuity ?

(b) Prove that the present value of annuity due can be estimated by the formula : $P = A\left[\frac{(1+r)^n-1}{r(1+r)^n}\right](1+r)$

(See Subsections 1.6.7-1.6.8)

12. (a) Distinguish between conventional and non-conventional cash flow in estimating the NPV.

(b) The financial manager of Star Pvt. Ltd. Co. proposes to purchase a new machine for a particular job. Two alternative machines are available, each having an initial investment requirement of ₹ 2,00,000. State which of the alternatives the financial manager will consider based on the following information :

		Machine X	Machine Y
(i) Initial investment (₹)		2,00,000	2,00,000
(ii) Estimated life (yrs.)		6 Yrs.	6 Yrs.
(iii) Estimated cash flow (after tax) (₹)	Year-1	40,000	25,000
	Year-2	30,000	30,000
	Year-3	35,000	30,000
	Year-4	36,000	40,000
	Year-5	40,000	50,000
	Year-6	50,000	55,000

The investor expects a minimum rate of return of 12% p.a. on investment.

(See Section 1.7 and Subsection 1.7.1)

13. An investment proposal of TCKK Pvt. Ltd. requires an initial outlay of ₹ 6,00,000 with an expected annual cash flow (after tax) of ₹ 1,20,000 for 6 years. Should this proposal be accepted when the discount rate is 16% and when it is 8% ? (See Subsection 1.7.1)

14. Rank the following projects on the basis of NPV criterion assuming 10% discount rate.

Project	Initial cash outlay (₹)	Annual cash inflow (₹)	Project life (years)	Present Value Factor for Annuity at 10%
A	25,000	3,000	10	6·1446
B	3,000	1,000	5	3·7908
C	12,000	2,000	8	5·3349
D	30,000	5,000	10	6·1446
E	50,000	6,000	12	6·8137

(See Subsection 1.7.1)

15. Discuss the merits and demerits of NPV methods. (See Subsections 1.7.2-1.7.3)

16. What is IRR ? Consider the following information regarding two alternative investment projects, and select any one of these projects based on IRR method.

Year	Project - X Cash inflow (₹ in lakh)	Project - Y Cash inflow (₹ in lakh)
1	1·2	0
2	—	2

[Project life : X - 1 Year ; Y - 2 Years] (See Section 1.8)

17. Discuss the merits and demerits of IRR method. (See Subsections 1.8.1-1.8.2)

18. Prove that if all annual cash flows from a project are either positive or zero then NPV or IRR criterion of project evaluation will give same result. (See Subsection 1.8.4)

19. (i) A person keeps ₹ 4,500 in each of investment options, I_1 and I_2, for 5 years. I_1 provides 8% simple interest rate p.a. Whereas I_2 provides 6% interest rate compounded yearly. What will be the maturity values of these two investments ? [C.U., B.Sc. (H), Sem-V, 2020]

[Hint : In case of I_1, $V = 4{,}500\ (1 + 0{\cdot}08 \times 5) =$ ₹ 6,300 ;
and in case of I_2, $V = 4{,}500\ (1 + 0{\cdot}06)^5 =$ ₹ 6,022]

(ii) What is the difference between simple and compound interest ? [C.U., B.Sc. (H), Sem-V, 2020] (See Section 1.3)

(iii) Suppose, you got ₹ 1,070 on maturity of a deposit of ₹ 1,000 for one year. If the inflation rate for that year was 5% then what was the rate of interest that you actually received on your deposit ?

[C.U., B.Sc. (H), Sem-V, 2020] (See Subsection 1.3.1)

2

Bond Price, Yield Rate & Term Structure

2.1. Introduction

Several financial instruments are traded in the capital market and money market of an economy. While capital market denotes an institutional arrangement for the transaction of long-term financial assets, the money market implies an institutional arrangement that facilitates purchase and sales of short-term securities in an economy. For example, the money market instruments are certificate of deposit (CD), Commercial Paper (CP), Government Treasury Bills (TB) etc. On the other hand, the fixed interest bearing securities which are transacted in the capital market include dated government securities (or long-term government bonds), municipal bonds, corporate bonds etc.

It should be noted that these financial assets may not have intrinsic value like physical assets (say, gold, silver etc.) but possess some extrinsic value and are traded in financial markets. Hence, these assets are termed as **financial instruments**.

The values of these financial assets are derived from the promises given by the issuers of these assets (say, the promise to pay a given rate of interest). So, they are **promissory notes**. If there remain well developed institutional arrangements for the transactions of these financial instruments on easy terms, then these financial instruments can be treated as a security. The liquidity of any financial security depends on the ease with which they can be converted into cash.

So, when we talk about **fixed-income securities**, then it implies the trading of such securities which offer a given rate of interest per annum to the investor.

2.2. Bonds : Basic Concept

Normally, the bonds carry a given rate of interest. Whenever the corporate houses and the government need to borrow money, they do it by issuing a promissory note known as 'bond'. It creates an obligation on the part of the bond issuer to pay a given rate of interest on the face value of the bond to the bond holders. The purchasers of these bonds are the lenders to either the government or to corporate houses. Different variants of such bonds have evolved over time keeping in view the changing needs of corporate houses and the government.

2.2.1. Main features of a bond

There are five principal features of a bond, viz., the face value of a bond, its coupon rate, periodicity of coupon payment, maturity and redemption value. These features along with other terms and conditions are contained in a document called '*indenture*'.

(a) **Face Value or par Value of a bond :** It implies the amount of money which is stated on the face of the instrument. Usually the bonds are issued with a face value of ₹ 100 or ₹ 1,000 or ₹ 10,000 etc. However, there is no such definite rule that prescribes the amount of face value or par value of a bond. The issue price of the bond, however, might be different from its face value. When the bonds are issued at a price higher than its face value, the additional amount is called a '*premium*'.

However, when it is issued at a price lower than its face value then the difference between the par value and the issue price is referred to as 'discount'.

(b) **Coupon rate :** The rate of interest offered by the issuer of the bond to its subscriber or purchaser is referred to as the coupon rate. Most of the bonds offer periodic coupon payments. In fact, the issuer of the bonds used to attach such 'coupon' with the bond certificate showing the given rate of interest per time period. Such physical coupons are rare at present. However, the terminology still prevails. This coupon rate is governed by the factors such as the current economic conditions, the risk associated with the bond, credit worthiness of the issuer and so on.

(c) **Periodicity of coupon payments :** Normally the coupon rate of a bond is specified as an interest rate per annum. The issuer of the bond may, however, decide to pay this interest at regular intervals, say, quarterly or half-yearly. However, from the view point of time value of money, the value of a bond offering quarterly interest would be higher than that offering semi-annual interest.

(d) **Maturity Period of a bond :** The duration from the date of issuance of a bond till its redemption date is referred to as the maturity period of the bond. When the maturity period of a bond is within 1 year (say, 91-days or 182-days or 364-days Treasury Bills issued by the government) then they are considered as money market instruments. However, when the maturity period is more than 1 year (say, 5 years or 10 years) then those bonds are referred to as capital market instruments.

(e) **Redemption value of a bond :** At the end of the maturity period, the issuer of the bond, viz., the borrower must refund the borrowed amount to the purchaser of the bond or the lender. The amount of money paid to the lender at the time of maturity of the bond is referred to as the redemption value of the bond. Normally the bonds are redeemed at its par value or face value. Sometimes, however, they are redeemed at premium or discount with reference to their par values.

(f) **Ask and bid price of a bond :** The ask price of a bond signifies the minimum price expected by the seller of the bond, while the bid price implies the maximum price the buyer is ready to pay for that bond. The difference between the bid price and the ask price is known as the bid-ask spread. The most liquid and widely traded bonds have narrow spreads.

2.2.2. Bonds : Different Types

There are different types of bond in the bond market. One particular classification of these bonds may be based on the nature of issuer, viz., the government or the private corporate houses. We can have a brief discussion on the types of such bonds.

(a) **Dated government securities :** The long-term government bonds are generally termed as dated government securities (or in short 'G-sec'). These securities normally carry a fixed coupon rate. However, in some cases, there may be floating interest rates on the face value of these bonds. The maturity period becomes more than 1 year.

(b) **Government Treasury Bills :** These treasury bills are short-term government bonds issued either by the Union Government or by the State Government with a maturity period of 1 year or less, e.g., 91-days Treasury Bills, 182-days Treasury Bills, 364-days Treasury Bills etc. Generally these treasury bills are issued at a discount. For instance, if the face value of a 364-days treasury bill is ₹ 100 and it is issued at ₹ 95 (i.e., at a discount), then the difference between the maturity value and the issue price would mean the return (assured) from this treasury bill.

These government securities practically carry no 'default risk' and, hence, they are called risk-free **'gilt-edged'** instruments.

(c) **Fixed rate and floating rate bonds :** Some bonds offer fixed interest rate payable at a given time interval (say, quarterly, half-yearly etc.). In some other cases, bonds offer floating interest rates. Interest rates on these bonds are not fixed. Rather they are linked to some benchmark rate (say, the London Inter-Bank Offer Rate or LIBOR, i.e. call money rate for inter-bank lending/borrowing). It gives a cushion to the bond issuer against fluctuations in market rates of interest. For instance, the State Bank of India (SBI) was the first bank to introduce such bonds in India where the interest

rate was linked to the bank's term deposit rate that served as an anchor rate. These bonds are issued with a 'cap' and a 'floor'. The 'cap' is the maximum interest rate that the issuer can pay, whereas the 'floor' refers to the minimum interest rate that a subscriber (lender) should receive.

(d) **Indexed bond** : These bonds also do not specify any fixed interest rate. Rather they are linked to some price index with a view to protect the bondholder from price inflation, and hence, against a loss in the purchasing power of the bondholders.

(e) **Zero coupon bonds** : These bonds do not carry any periodic interest payments. They are sold at a discount on their face values. In order to provide adequate return to the bondholders, these bonds are issued at substantial discount on their respective face values.

(f) **Deep discount bonds** : These are also zero coupon bonds with long maturity period (say, 15 years), and these bonds are also issued at a discount on their face values. In India, the Industrial Development Bank of India (IDBI) first issued this type of bond in 1992.

(g) **Callable/Puttable bonds** : Generally the bonds are issued with a given maturity period. However, in case of callable bonds, the issuer keeps an option for early redemption of these bonds. Here the issuer may reserve a right (but no obligation) to call the bond prior to the prescribed maturity. However, in case of such callable bonds, the issuer announces a repurchase price of the bond known as the 'call price'. If there is a substantial fall in the market rate of interest after the issuance of these bonds at higher coupon rate, the issuer calls back the bond to safeguard its interest.

On the other hand, in case of puttable bonds, an option is given to the bondholder to redeem the bond prior to the maturity period. If the market interest rate becomes higher than the coupon rate after the purchase of the bond, then the bondholder may want to reclaim the principal amount from the issuer to invest the same at higher interest rate. So, the puttable bonds give a scope of exit from the present investment and reinvest the principal amount in more profitable ventures.

(h) **Convertible bonds** : Sometimes the issuer of a bond merges some features of an equity share with that bond so that some portion of the face value of the bond becomes convertible into a predetermined number of equity shares of the issuer company. Number of equity shares to be received per convertible bond is referred to as '**conversion ratio**'. The price per equity share at which a convertible bond can be converted into a common stock is called as '**conversion price**'. For instance, if a holder of a fully convertible bond with a face value or par value of ₹1,000 can convert it into 20 equity shares of the issuer company at any time before the maturity date, then the conversion ratio is 20 : 1 (i.e., 20 equity shares per bond), and the conversion price would be ₹ 50 (i.e., $\frac{₹\,1000}{20} = ₹\,50$). A convertible bond is more attractive to an investor compared to non-convertible bonds because it gives an opportunity to the bondholders to gain from the potential increase in market value of equity shares. It also reduces the cost of debt servicing for the issuer firm. These bonds also help in conveying a positive signal in the financial market about the expected performance of the issuer company.

- **Some money market instruments :**

(1) Commercial Paper (CP)

A commercial paper is an unsecured short term promissory note. It is negotiable and transferable by endorsement. It is issued by big corporate houses at a discount to face value to meet their working capital requirements. It is also known as finance paper, industrial paper or corporate paper. This paper is subscribed by individuals, banks and corporate houses. Presently commercial papers can be held in a dematerialised form.

In 1987, the Working Group on Money Market suggested the introduction of Commercial Papers (CP) in India. Accordingly, the RBI introduced the CP in January 1990. Initially only large and highly rated corporate houses could issue CP in India. However, later on the CPs could be issued by the All-India financial institutions (say, IFCI, SIDBI, etc.) and the Primary Dealers. The CP can be

issued to individuals, commercial banks, joint stock companies and other registered corporate bodies in India. The CPs can also be issued to NRIs only on a non-transferable and non-repatriable basis. The FIIs are also eligible to invest in CPs as per the norms fixed by the SEBI.

(2) Certificate of Deposit (CD) :

The Certificate of Depos.. is issued by commercial banks and development banks for raising short-term funds. It is an unsecured and negotiable short term instrument issued in bearer form. Scheduled commercial banks and co-operative banks can issue certificate of deposit for a period of not less than three months and upto a period of not more than one year.

The Certificates of Deposit (CDs) were introduced in India in June 1989. Only scheduled commercial banks excluding Regional Rural Banks (RRBs) were allowed to issue CDs. However, in 1992, the development finance institutions were also allowed by the RBI to issue CDs (e.g., six financial institutions, viz. IDBI, IFCI, ICICI, SIDBI, IRBI & EXIM Bank were permitted to issue CDs).

2.2.3. Annuity

An 'annuity' is a contract between the annuitant (the holder of the annuity) and the issuer firm. It pays a given sum of money to the annuitant over a period of time maintaining certain time intervals (say, monthly, quarterly or half-yearly). From the view point of an investor, an annuity can be considered as 'fixed-income instrument' but **these annuities cannot be traded in the financial market** (in fact, the issuer would not allow a change in annuitant when the annuity payments are tied to the life of the holder of the annuity.) Hence, **annuities cannot be considered as securities.** In our previous chapter/unit, we have already discussed the concepts of deferred annuity, annuity due, and perpetual annuity. We have also discussed about the process of estimating the present value of the cash flow stream arising out of such annuity payments.

2.2.4. Bond price and yield

The bond price or the value of the bond implies the amount one would pay at present in exchange of the future cash flow that accrues over the remaining life of the bond to the bond-holder. So, the price one pays today for that bond must be equal to the present value of the future cash-flows (viz., the coupon payments) from the bond at a specific discount rate. While calculating the present value of the future cash-flow stream from the bond, the discount rate would reflect the rate of return expected by the investor. So, this discount rate should take into account the following factors :

1. **Risks involved in the cash-flow :** Normally the bonds offer fixed coupon payments, and therefore the risks involved in the future cash-flows are minimum. In case of government bonds, the risk component is zero. However, in case of corporate bonds issued by the private corporate houses, there might be some risks depending upon the risk profile of the issuer.

   If the risk element is higher then the discount rate used for deriving the present value of the future cash-flow stream from the bond would also be higher.

2. **Economic environment and market condition :** If the economic environment is characterised by a recessionary trend then the market interest rates would be lower. In that case a lower discount is used to derive the present value of the future cash-flows. However, if there remains an inflationary trend in the economy and the market interest rates remain higher then the discount rate used in estimating the present value of the future cash-flows from the bond would also be higher.

3. **Periodicity of cash-flows :** More distant cash-flows must be discounted at higher rate simply because such distant time period adds uncertainty to cash-flow stream.

   Thus, the discount rate at which the present value of the cash-flow stream generated from the bond is just equal to the current bond price, can be considered as the **yield rate** of the bond.

2.5.5. Cash-Flow from a bond

Normally the cash-flows from a conventional bond consist of two parts : (i) A periodic coupon payment (a fixed amount), and (ii) The payment of final redemption value.

Let us consider a bond with a face value of ₹ 100 with a coupon rate of 10% payable half-yearly, and it is redeemable at 5% premium after 3 years. In this case, the cash-flows to the bond holder who holds it till maturity will be as follows :

Table - 2.1

Time (months)	0	6	12	18	24	30	36
(i) Coupon received (₹)	0	5	5	5	5	5	5
(ii) Principal paid (₹)	(–) 100	–	–	–	–	–	–
(iii) Principal redeemed (₹)	–	–	–	–	–	–	+ 105
Total Cash-Flow (₹)	(–) 100	5	5	5	5	5	110

Here, the cash outflow from the bond holder (at the initial period when he purchases the bond) has been shown with a 'minus' (–) sign.

Now, the bond price or the value of the bond based on our previous example) can be estimated with the help of the following formula :

$$\text{Value of a bond } (B_p) = P = \sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{r}{m}\right)^{mt}} + \frac{R_n}{\left(1+\frac{r}{m}\right)^{mt}}$$

$$= \sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{r}{m}\right)^{k}} + \frac{R_n}{\left(1+\frac{r}{m}\right)^{n}} \quad \text{...... (2.1)}$$

Where, m = Number of times the coupon rate or the interest rate is paid in a year.

r = Discount rate.

t = Maturity period of the bond.

k = $m.t$. $(k = 1, 2, \ldots, n)$

[If t = 3 years and m = 2, then k = 6]

C_k = Coupon payment at the time period 'k'

R_n = Redemption Value at the time period 'n'.

B_p = Bond price = P = Value of the Bond

In our previous example, we have

C_k = ₹ 5 at the interval of 6 months

R_n = ₹ 105 ;

m = 2, t = 3, and therefore $m.t$. = 6, and k = 1, 2, ..., 6; Face Value of the bond = ₹ 100.

Since coupon rate is 10% payable half-yearly, so the bond holder receives ₹ 5 as coupon payment at a regular interval of 6 months, and the bond matures at 36th month (i.e., after 3 years) with a premium of 5% on its face value. So, the redemption Value (R_n) is ₹105.

Now, that coupon rate may be justified from the view point of economic environment and expectations of the investors at the time of the issuance of the bond. However, the economic environment is expected to change and an inflationary pressure may develop in the economy. As a result, in determining the

present value of the cash-flows from the bond, it is justified to raise the discount rate above the coupon rate. Let the discount rate be 12% (r = 0·12).

$$B_p = P = \sum_{k=1}^{6} \frac{C_k}{\left(1+\frac{r}{m}\right)^{k}} + \frac{R_n}{\left(1+\frac{r}{m}\right)^{n}}$$

$$= \frac{5}{\left(1+\frac{0\cdot12}{2}\right)} + \frac{5}{\left(1+\frac{0\cdot12}{2}\right)^2} + \cdots + \frac{5}{\left(1+\frac{0\cdot12}{2}\right)^6} + \frac{105}{\left(1+\frac{0\cdot12}{2}\right)^6}$$

$= 5(0·943) + 5(0·889) + 5(0·839) + 5(0·792) + 5(0·747) + 5(0·705) + 105(0·705)$

$= 4·715 + 4·449 + 4·195 + 3·96 + 3·735 + 3·525 + 74·025$

$=$ ₹ 98·60

It becomes obvious that there remains an inverse relationship between the discount rate (r) and the bond price (B_p), i.e., as the discount rate becomes higher the bond value or the bond price (B_p) becomes lower and vice versa (Fig. - 2.1).

This inverse relationship between the discount rate (r) and the bond price (Bp) can be shown with the help of the following example.

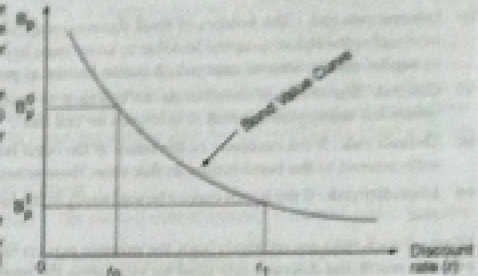

Fig.-2.1

Example 2.1

Let us consider a bond with a face value of ₹ 100, bearing a coupon rate of 10% payable yearly, and the maturity period of this bond is 10 years, the redemption value being ₹ 100. Here, we can show that as the discount rate increases gradually from 5% to 15%, the value of the bond (i.e., the present value of the future cash-flows from the bond) or the bond price will fall gradually (Table - 2.2).

Table - 2.2

Year	Cash flow (₹)	Discount rate (r) 5%	8%	10%	12%	15%
1	10	9·52	9·26	9·09	8·93	8·70
2	10	9·07	8·57	8·26	7·97	7·56
3	10	8·64	7·94	7·51	7·12	6·58
4	10	8·23	7·35	6·83	6·36	5·72
5	10	7·84	6·81	6·21	5·67	4·97
6	10	7·46	6·30	5·64	5·07	4·32
7	10	7·11	5·83	5·13	4·52	3·76
8	10	6·77	5·40	4·67	4·04	3·27
9	10	6·45	5·00	4·24	3·61	2·84
10	110	67·53	50·95	42·41	35·42	27·19
	TOTAL	138·62	113·41	100·00	88·70	74·91

The example shows that:

(i) When discount rate = coupon rate, then bond price (B_0) = Face Value of the bond.

(ii) When discount rate > coupon rate, then B_0 < Face Value of the bond.

(iii) When discount rate < coupon rate, then B_0 > Face Value of the bond.

Thus, when the discount rate = 10% = coupon rate then B_0 = ₹ 100 = Face Value of the bond ...

When the discount rate = 5% < coupon rate = 10% then B_0 = ₹ [illegible] > Face Value of the bond ...

When the discount rate = 15% > coupon rate = 10% then B_0 = ₹ [illegible] < Face Value of the bond ...

2.2.6. Discount rate and risk

An investor may face different types of risks as mentioned below:

(a) Inflation risk: There is always a chance of reduced real rate of return on any financial asset due to inflationary pressure.

(b) Interest rate risk: The holders of fixed income securities (viz. bonds carrying a given coupon rate) suffer the risk of a capital loss due to an increase in the interest rate in future. This is known as capital risk or interest rate risk. It is also known as price risk.

(c) Call risk: The issuers of some bonds can 'call back' or purchase those bonds from the bondholders before the redemption period. It is known as 'call risk'.

(d) Default risk: If the borrower or the issuer of the bond fails to repay the principal amount along with interest to the bond holder in due time, there arises a default risk.

(e) Liquidity risk: If the bonds cannot be sold easily in the financial markets, it gives rise to liquidity risk.

When all such risks are relatively high, the investors expects higher rate of return from the investment and as a result, the discount rate used for estimating the present value of the future cash flows from a bond must also be higher. Hence, the discount rate (r) becomes an increasing function of financial risks (FR), i.e., r = r(FR), r' > 0.

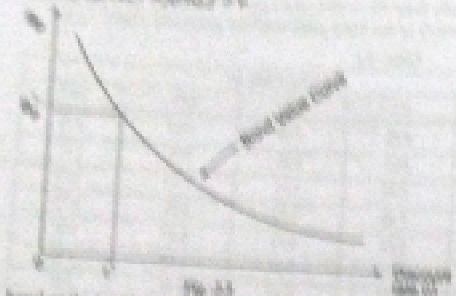

Fig. 2.3

As discount rate rises, the bond price falls. Hence, the risky bonds are traded at lower prices. Hypothetically, as the discount rate tends to zero, the bond price tends to infinity (∞). However, even for any risk-free bond (say, the government bonds), the investors expect a minimum rate of return on investment. This may be termed as the 'risk-free' return. This minimum expected rate of return results in a positive (minimum) discount rate (say, r*) which places a limit to the value of the bond or the bond price (Fig. 2.3). In Fig. 2.3 at r = r* (minimum discount rate reflecting the minimum rate of return expected by the investors), the bond price is $B_0 = B_0^*$.

2.2.7. Yield on a bond

We have already shown that the value of a bond can be estimated with the help of the following formula:

$$B_0 = P = \sum_{t=1}^{k} \frac{C_t}{\left(1+\frac{r}{m}\right)^t} + \frac{R_k}{\left(1+\frac{r}{m}\right)^k}$$

where k = m.n (m = Number of times the coupon rate is paid in a year; n = maturity period of the bond); C_t = Coupon payment at the time period t; R_k = Redemption value at the time period k; P = bond price. If the discount rate (r) is known then this formula can be used to determine the bond price (B_0).

In real life, however, it is observed that at the time of issue of a bond, its coupon payments and the maturity period are known. Then with the listing of such bonds in the capital market (viz. with stock exchanges), their prices keep on changing with changes in buying and selling pressures of these bonds in the stock market. Now, for any given cash flow stream from the bond and given its prevailing market price (which is supposed to be different from its face value), what return this bond would give to the investor becomes an important question. This expected rate of return from a bond given its prevailing market price and the coupon rate, is known as the yield on a bond. Thus, when any investor purchases a bond from the secondary market (i.e., where there are second-hand transactions of bonds, viz., in the stock exchanges) at its prevailing market price then he/she is concerned with the yield of the bond.

2.2.8. Different Variants of Yield on a bond

The 'yield' on a bond has different variants. The most common variant is current yield. It is the simplest estimate of return from a bond.

(i) Current Yield: The current yield of a bond is estimated as follows:

$$\text{Current Yield} = \frac{\text{Coupon rate} \times \text{Face value}}{\text{Current price of Bond}}$$

If a bond with a face value of ₹ 100 has a current price of ₹ 90, and if the coupon rate is 10% then current yield $= \frac{0.1 \times 100}{90} = \frac{10}{90} = 0.111$ or 11.1%.

(ii) Yield to Maturity (YTM): The YTM refers to that yield which an investor would earn by holding the bond till maturity. It is the interest rate at which the present value of the stream of coupon payments and the final redemption value would just be equal to the prevailing market price of the bond.

If the prevailing market price of a bond is denoted by P_0 then our previous formula used for the estimation of the present value of the cash flows from a bond, can also be used to estimate the yield to maturity (YTM).

Thus in the following formula:

$$P_0 = \sum_{t=1}^{k} \frac{C_t}{\left(1+\frac{\lambda}{m}\right)^t} + \frac{R_k}{\left(1+\frac{\lambda}{m}\right)^k} \quad \text{......(2.4)}$$

Here λ = YTM.

Notice, in this formula, P_0, C_t, R_k, m and k (= mn) values are known. The only unknown value is λ, viz., the yield to maturity. It is just like the internal rate of return (IRR) which is used to evaluate any investment proposal.

Example 2.2

(a) Let us first consider a pure-discount bond which does not pay any coupon payment before maturity. Let Bond 'A' with a face value of ₹ 1,000 matures after 1 year and is redeemed at par. Its current price is ₹ 934.58. The yield to maturity (YTM) can be calculated as follows :

So, here $P_0 = \dfrac{R_k}{\left(1+\frac{\lambda}{m}\right)^k}$

Where $m = 1$, $k = mt = t = 1$; $P_0 = 934.58$, $R = ₹\ 1000$ and $\lambda = \text{YTM}$.

$\therefore\ ₹\ 934.58 = \dfrac{1000}{(1+\lambda)}$

or, $1 + \lambda = \dfrac{1000}{934.58} = 1.0699$

or, $\lambda = 1.0699 - 1 = 0.07$ or 7%

$\therefore$ YTM = 7%

Example 2.3

Let us consider another pure-discount bond with a face value of ₹ 1000 and it can be redeemed at par after a maturity period of 2 years. This bond is purchased at present at a price of ₹ 857.34. Here YTM can be measured as follows :

$$P_0 = \sum_{k=1}^{2} \frac{R_k}{\left(1+\frac{\lambda}{m}\right)^k} = \frac{1000}{(1+\lambda)^2}\ (\because R_1 = 0)$$

Here, $P_0 = 857.34$, $m = 1$, $k = m.t = t$; $k = 1, 2$; $R_1 = 0$, $R_2 = 1000$; and $\lambda = \text{YTM}$.

$\therefore\ 857.34 = \dfrac{1000}{(1+\lambda)^2}$

or, $(1+\lambda)^2 = \dfrac{1000}{857.34} = 1.166$

or, $\sqrt{(1+\lambda)^2} = \sqrt{1.166} = 1.0798$

or, $1 + \lambda = 1.0798$

or, $\lambda = 1.0798 - 1 = 0.0798 \approx 8\%$.

Example 2.4

Let us consider another bond with a face value of ₹ 1000 with a coupon rate of 5% payable annually having a maturity period of 2 years, and the bond is redeemable at par. If it is sold at present at a price of ₹ 946.93 then the YTM can be estimated as follows :

$$P_0 = \sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{\lambda}{m}\right)^k} + \frac{R_n}{\left(1+\frac{\lambda}{m}\right)^n}$$

Here, $P_0 = ₹\ 946.93$; $m = 1$, $k = m.t = t$ $(k = 1, 2)$; $C_k = 50$, $R_2 = ₹\ 1{,}000$.

$$946.93 = \sum_{k=1}^{2} \frac{50}{(1+\lambda)^k} + \frac{1000}{(1+\lambda)^2}$$

$$= \frac{50}{(1+\lambda)} + \frac{50}{(1+\lambda)^2} + \frac{1000}{(1+\lambda)^2}$$

$$946.93 = \frac{50}{(1+\lambda)} + \frac{1050}{(1+\lambda)^2}$$

In this case, a trial and error process has to be followed to find out the YTM (i.e., the value of λ).

However, one can follow the following approximation towards the value of λ.

1st step : Divide the purchase price of the bond (i.e., the initial investment) by the average annual cash flow.

Here, it is $\dfrac{946.93}{\frac{50+1050}{2}} = \dfrac{946.93}{550} = 1.72$

2nd step : Now move along 2 year (maturity period of the bond) row of the present value of annuity table (given at the end of this book) and search a value nearest to 1.72 (quotient) and you get an approximation to the discount rate. For better understanding of this procedure, we can present a portion of the present value annuity table as given below :

The Present Value of an Annuity of ₹ 1

Year	5%	6%	7%	8%	9%	10%	11%
1	0.952	0.943	0.935	0.926	0.917	0.909	0.901
2	1.859	1.833	1.808	1.783	1.759	$\boxed{1.736}$	$\boxed{1.713}$

The present value annuity table shows that nearest figures are given in discount rates 10% (1.736) and 11% (1.713).

Hence, we can start with a discount rate of 11% to determine the present value of cash flows of the bond. If we find that prevailing bond price is still higher then we must reduce that discount rate step by step to find out the desired result. This is shown below :

(a) $\dfrac{50}{(1+0.11)} + \dfrac{1050}{(1+0.11)^2} = 897.27 < 946.93$

(b) $\dfrac{50}{(1+0.10)} + \dfrac{1050}{(1+0.10)^2} = 913.21 < 946.93$

(c) $\dfrac{50}{(1+0.09)} + \dfrac{1050}{(1+0.09)^2} = 929.71 < 946.93$

(d) $\dfrac{50}{(1+0.08)} + \dfrac{1050}{(1+0.08)^2} = 946.49 < 946.93$

(e) $\dfrac{50}{(1+0.07975)} + \dfrac{1050}{(1+0.07975)^2} = 946.93$

$\therefore$ YTM = 7.975%

It is important to note that **if a bond is sold at its face value or par value then YTM = coupon rate.**

Example 2.5

A bond with a face value of ₹ 100 carrying an annual coupon rate of 10%, a maturity period of 2 y... and redeemable at par, is sold at par value.

$$\therefore \quad 100 = \frac{10}{(1+\lambda)} + \frac{110}{(1+\lambda)^2}$$

Here, YTM = λ = 0·10 = 10% = coupon rate.

Further, if the bond is sold at a discount then its prevailing price is less than its par value. In... case, YTM > Coupon rate.

Example 2.6

Let us consider our previous example. In that case, if the bond is transacted at a discount and... investor purchases it at a price of ₹ 90 then YTM can be estimated as follows :

$$90 = \frac{10}{(1+\lambda)} + \frac{110}{(1+\lambda)^2}$$

In this case, an approximate value of λ can be estimated as follows :

(a) $\frac{90}{\frac{10+110}{2}} = \frac{90}{60} = 1 \cdot 5$

(b) Move along the 2 year (maturity period of the bond) row of the present value of annuity table... search a value nearest to 1·5. This approximate discount rate is found to be 21% as shown be...

Present Value of Annuity of ₹ 1

Year	15%	16%	17%	18%	19%	20%	21%
2	1·626	1·605	1·585	1·566	1·547	1·528	$\boxed{1·509}$

So, we make a trial and error process starting from, say, 20% discount rate.

If λ = 20%, then $\frac{10}{(1\cdot2)} + \frac{110}{(1\cdot2)^2} = 84\cdot71 < 90$

If λ = 18%, then $\frac{10}{(1\cdot18)} + \frac{110}{(1\cdot18)^2} = 87\cdot47 < 90$

If λ = 16%, then $\frac{10}{(1\cdot16)} + \frac{110}{(1\cdot16)^2} = 90\cdot36 > 90$

If λ = 16·2%, then $\frac{10}{(1\cdot162)} + \frac{110}{(1\cdot162)^2} = 90\cdot06 \simeq 90$

$\therefore$ YTM = 16·2% > coupon rate = 10%

Again, if the bond is sold at a premium then its prevailing market price is more than its face value. In that case, YTM < coupon rate.

Example 2.7

Let us again consider our previous example and let us assume that the bond is sold at a premium, say at ₹ 110. In this case the YTM is calculated as follows :

$$110 = \frac{10}{(1+\lambda)} + \frac{110}{(1+\lambda)^2}$$

Hence, the approximate value of λ can be estimated as follows :

(a) $\frac{110}{\frac{10+110}{2}} = \frac{110}{60} = 1 \cdot 83$

(b) Move along the 2 year row of the present value annuity table and search a value nearest to 1·83, and that discount rate is found to be 6% as shown below :

Present Value Annuity table for ₹ 1

Year	3%	4%	5%	6%
2	1·913	1·886	1·859	$\boxed{1·833}$

Hence, we can begin our trial and error considering λ = 6%

If λ = 6% then $\frac{10}{(1\cdot06)} + \frac{110}{(1\cdot06)^2} = 107\cdot32 < 110$

If λ = 5% then $\frac{10}{(1\cdot05)} + \frac{110}{(1\cdot05)^2} = 109\cdot29 < 110$

If λ = 4·6% then $\frac{10}{(1\cdot046)} + \frac{110}{(1\cdot046)^2} = 110\cdot09 \simeq 110$

$\therefore \quad \lambda$ = YTM = 4·6%

Thus, in this case, YTM = 4·6% < coupon rate = 10%.

From our previous analysis, we can say that the notion of YTM takes into account the time value of money, as well as the capital gain or loss upon maturity of the bond. However, the current yield of the bond does not consider these aspects of bond valuation. If the prevailing price of the bond is less than its redemption value then YTM will be higher than the current yield since the investor would have a capital gain upon the maturity of the bond. In our previous example [Example-2.4]

current yield = $\frac{50}{946\cdot93}$ = 0·0528, i.e., 5·28%

However, YTM = 7·795% $\simeq$ 8%

Example 2.8

Let us take another example where the bond is traded at a price more than its face value (in our previous example-2.4 the face of the bond was ₹ 1000 but it was traded at a price lower than its face value).

Let the face value of a bond be ₹ 100 and let us assume that it carries an annual coupon payment of ₹ 10 with a maturity period of 3 years, and it is redeemable at par upon maturity. The prevailing price of the bond is, say, ₹ 113·60.

In this case,

Current yield = $\frac{10}{113\cdot60}$ = 0·088, i.e., 8·8%

But the YTM will be lower than current yield in this case.

Here, ₹ 113·60 = $\frac{10}{(1+\lambda)} + \frac{10}{(1+\lambda)^2} + \frac{110}{(1+\lambda)^3}$

The trial and error process would suggest that λ = 5% because

$$\frac{10}{(1+0\cdot05)} + \frac{10}{(1+0\cdot05)^2} + \frac{110}{(1+0\cdot05)^3} = 9\cdot52 + 9\cdot07 + 95\cdot02 = ₹\ 113\cdot61$$

So, in this case YTM = 5% < current yield = 8·8%.

(iii) **Realised Yield :** While estimating the yield to maturity (YTM) it is assumed that the bondholder holds the bond till its maturity. However, in real life situation, the bondholder may sell it before maturity at the prevailing market price. If the bondholder sells the bond before its maturity, the current yield may not be affected but the discount rate at which the cash-flows from the bond would be equal to its current price might be different from the yield to maturity. This yield is known as **realised yield**.

Example 2.9

Let us consider our previous example where a bond with a face value of ₹ 100 carries an annual coupon rate of 10% with a maturity period of 3 years, and it is redeemable at par upon maturity. The prevailing market price of this bond is ₹ 113·60.

We have already shown that its current yield $= \frac{10}{113 \cdot 60} = 0 \cdot 088$, i.e., 8·8%

and YTM = 5%

Now, if the bondholder sells this bond after 2 years (before the maturity) at a prevailing price of ₹ 113·60 then the realised yield can be estimated by the following formula (same as before) :

$$P_0 = \sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{\lambda}{m}\right)^k} + \frac{F_k}{\left(1+\frac{\lambda}{m}\right)^k} \quad \text{......} \quad (2.3)$$

Here, $m = 1$; $k = m.t. = t$ ($k = 1, 2$) ; $P_0 =$ ₹ 113·60, F_k = Face value of the bond at k-th period (when the bond is sold) = ₹ 100. So, here λ = Realised Yield rate.

Therefore, we have

$$113 \cdot 60 = \frac{10}{(1+\lambda)} + \frac{10}{(1+\lambda)^2} + \frac{100}{(1+\lambda)^2}$$

$$113 \cdot 60 = \frac{10}{(1+\lambda)} + \frac{110}{(1+\lambda)^2}$$

Here also, through the process of trial and error, we find that $\lambda = 2 \cdot 9\%$

[Here, it is assumed that the coupon payments received at the interim period have been reinvested at 5% rate of interest].

The approximate value of λ, as we have discussed earlier, can be determined as follows :

Step 1 : We devide the sales price of the bond by the average annual cash flow, i.e.,

$$\frac{113 \cdot 60}{\frac{10+110}{2}} = \frac{113 \cdot 60}{60} = 1 \cdot 89$$

Step 2 : Now we move along 2 year row of the present value of annuity table and search a value nearest to 1·89 and we find that it is 4% (as shown below for convenience).

The Present Value of Annuity of ₹ 1

Year	1%	2%	3%	4%
2	1·970	1·942	1·913	$\boxed{1 \cdot 886}$

Hence, the approximate discount rate is 4%.

Hence, we follow the trial and error process as shown below to determine the appropriate realised yield rate :

(a) $\frac{10}{(1+0 \cdot 04)} + \frac{110}{(1+0 \cdot 04)^2} = 111 \cdot 31 < 113 \cdot 60$

(b) $\frac{10}{(1+0 \cdot 03)} + \frac{110}{(1+0 \cdot 03)^2} = 113 \cdot 38 < 113 \cdot 60$

(c) $\frac{10}{(1+0 \cdot 029)} + \frac{110}{(1+0 \cdot 029)^2} = 113 \cdot 60$

Therefore, $\lambda = 2 \cdot 9\%$.

2.3. Bond Price Theorem

In our previous discussion we have been acquainted with the terminologies such as 'coupon rate', 'redemption period' (it is also referred to as 'term-to-maturity' of a bond, i.e., the amount of time left until the last promised payment is made), 'yield to maturity', 'realised yield' etc. We have also seen that

(i) When a bond is sold at par, the prevailing market price of the bond is equal to its face value (or *par value*), and in that case YTM = coupon rate;

(ii) When a bond is sold at a discount then its prevailing market price is less than its face value, and in that case, YTM > coupon rate; and

(iii) When a bond is sold at a premium then its prevailing market price is more than its face value, and in that case YTM < coupon rate.

On the basis of these relations, **five theorems related to bond pricing have been derived by the** financial analysts. For simplicity, we assume that coupon payments are made annually or at an interval of every 12 months.

1st Theorem

The first theorem on bond pricing suggests that if the market price of a bond rises then its 'yield' must decrease; conversely, if the market price of a bond declines then its yield will increase.

Based on our previous examples, we can prepare the following table in support of this theorem :

Coupon rate (r)	Face Value of the bond	Market price of the bond	YTM	Remarks
10%	₹ 100	₹ 100	10%	YTM = r
10%	₹ 100	₹ 90	16·2%	YTM > r
10%	₹ 100	₹ 110	4·6%	YTM < r

2nd Theorem

The second theorem on bond pricing suggests that if the 'yield' of a bond does not change over its life then the size of its discount or premium will fall as its life gets shorter.

Example 2.10

Let us consider a bond having a face value of ₹ 100, and carrying an annual coupon rate of 10%, can be redeemable at par upon its maturity; the maturity period is assumed to be 5 years. Let the current market price of this bond be ₹ 92·80, i.e., it is sold at a discount. Hence, the YTM of this bond is estimated as follows :

$$₹\,92{\cdot}80 = \sum_{t=1}^{4} \frac{10}{(1+k)^t} + \frac{110}{(1+k)^5}$$

Through a trial and error process the YTM (k) of this bond is estimated to 12%.

Thus, $\frac{10}{(1+0{\cdot}12)} + \frac{10}{(1+0{\cdot}12)^2} + \frac{10}{(1+0{\cdot}12)^3} + \frac{10}{(1+0{\cdot}12)^4} + \frac{110}{(1+0{\cdot}12)^5} = ₹\,92{\cdot}80$

Now, after 1 year, if the yield still remains 12% then its present market price would be ₹ 93·93 as

$$₹\,93{\cdot}93 = \frac{10}{(1+0{\cdot}12)} + \frac{10}{(1+0{\cdot}12)^2} + \frac{10}{(1+0{\cdot}12)^3} + \frac{110}{(1+0{\cdot}12)^4}$$

Hence, the discount on the bond declines from ₹ 7·20 (= ₹ 100 – 92·80) to ₹ 6·07 (= ₹ 100 – 93·93).

This theorem can also be interpreted in an alternative manner :

If two bonds have the same coupon rate, face value and yield rate then one bond with shorter life will be sold at small discount or premium as shown below :

Bond : 1	Bond : 2
Face value ₹ 100 Coupon Payment (p.a.) : ₹ 10 Life : 5 years Yield : 12% Price Discount : ₹ 7·20	Face value ₹ 100 Coupon Payment (p.a.) : ₹ 10 Life : 4 years Yield : 12% Price Discount : ₹ 6·07

This relationship can also be shown with the help of a diagram (Fig.-2.3)

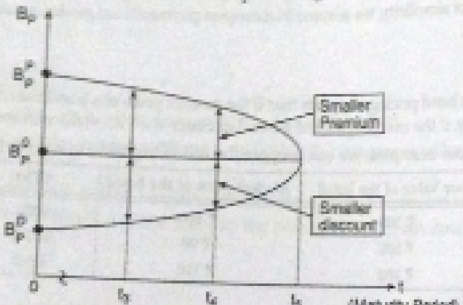

Fig.-2.3

In Fig.-2.3, B_P^0 denotes the par value or face value of the bond ; B_P^P denotes the price of the premium bond (i.e., the bond which is sold at a premium) and B_P^D denotes the bond price when it is sold at a discount.

When the period left for the maturity of the bond declines from $t_5 t_4$ to $t_4 t_3$ then the amount of discount or premium becomes smaller (Fig.-2.3).

3rd Theorem

The third theorem on bond pricing suggests that if the yield of a bond does not change over its life then the size of its discount or premium will decrease at an increasing rate as the life of a bond gets shorter.

Example 2.11

Let us consider our example given in Theorem-2. We have seen that when the yield (YTM) = 12% then the bond with a maturity period of 5 years (having a par value of ₹ 100 and redeemable at par, and with a coupon rate of 10% p.a.) is sold at a discount ₹ 92·80. After 1 year, if the yield (YTM) remains same, the current market price of the bond becomes ₹ 93·93 (already shown the estimation).

Now, after 2 years, if the yield (YTM) still remains same then the present market price of the said bond will be as follows :

$$\frac{10}{(1+0{\cdot}12)} + \frac{10}{(1+0{\cdot}12)^2} + \frac{110}{(1+0{\cdot}12)^3} = ₹\,95{\cdot}19$$

and therefore, the discount on the bond becomes ₹ 100 - 95·19 = ₹ 4·81.

Hence, we observe that —

when the maturity period of the bond declines from 5 years to 4 years then the amount of discount declines from

₹ 7·20 [= ₹ 100 – 92·80]

to ₹ 6·07 [= ₹ 100 – 93·93].

Thus, the discount amount falls by ₹ 1·13 [= ₹ 7·20 – 6·07], i.e., it is $\frac{1{\cdot}13}{₹\,100} \times 100 = 1{\cdot}13\%$ of the par value.

Further, when the maturity period declines from 4 years to 3 years then the amount of discount declines further from

₹ 6·07 [= ₹ 100 – 93·93] to

₹ 4·81 [= ₹ 100 – 95·19].

Thus, the discount amount falls by

₹ 1·26 [= ₹ 6·07 – 4·81], i.e. it is

$\frac{₹\,1{\cdot}26}{₹\,100} \times 100 = 1{\cdot}26\%$ of the par value.

Hence, as the life of the bond gets shorter, the discount in this case declines at an increasing rate. This can also be shown with the help of a diagram (Fig.-2.4)

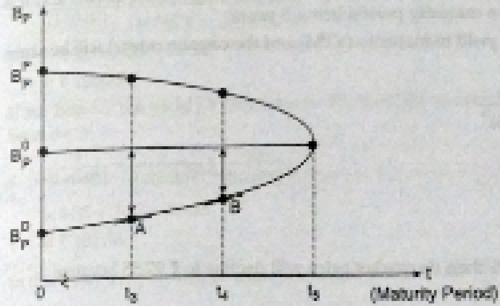

Fig.-2.4

In Fig.-2.4, the graph shows the relation between bond price (B_p) and the maturity period (t) of the bond. When a bond is sold at a discount then the size of that discount is $(B_P^0 - B_P^D)$.

Here, $B_p^O > B_p^D$ and hence, $(B_p^O - B_p^D) > 0$;

B_p^O = Par value or face value of the bond.

B_p^D = discounted price of the bond.

As the maturity period declines from

t_3t_5 to t_4t_5, $(B_p^O - B_p^D)$ falls,

i.e., $d(B_p^O - B_p^D) < 0$ and $d(t) < 0$

$$\therefore \quad \frac{d(B_p^O - B_p^D)}{dt} > 0$$

$$\text{But} \quad \frac{d^2(B_p^O - B_p^D)}{dt^2} > 0$$

Thus, this distance between B_p^O and B_p^D declines at an increasing rate with a fall in maturity period.

4th Theorem

The fourth theorem on bond pricing suggests that a decrease in bond's yield will raise the price of bond by an amount that is greater in size compared to a situation where an equal increase in bond's yield causes a fall in bond's price.

Example 2.12

Let us consider a bond having a face value of ₹ 100 carrying an annual coupon rate of 10% and it is redeemable at par upon its maturity, the maturity period being 3 years.

Now, if this bond is sold at par then the yield to maturity (YTM) and the coupon rate (r) will be same, because

$$\frac{10}{(1+0\cdot1)} + \frac{10}{(1+0\cdot1)^2} + \frac{10}{(1+0\cdot1)^3} + \frac{100}{(1+0\cdot1)^3}$$

$$= \frac{10}{(1\cdot1)} + \frac{10}{(1\cdot1)^2} + \frac{110}{(1\cdot1)^3}$$

$$= 9\cdot09 + 8\cdot26 + 82\cdot64$$

$$= ₹\ 99\cdot99 \simeq ₹\ 100$$

However, if its yield (YTM) rises to 11% then its market price will decline to ₹ 97·55 because

$$\frac{10}{(1+0\cdot11)} + \frac{10}{(1+0\cdot11)^2} + \frac{110}{(1+0\cdot11)^3}$$

$$= 9\cdot0 + 8\cdot12 + 80\cdot43 = ₹\ 97\cdot55$$

Thus, the bond price falls by

₹ 2·45 = ₹ 100 – ₹ 97·55

However, if the yield (YTM) declines to 9% then the market price of the bond would rise to ₹ 102·53 because

$$\frac{10}{(1+0\cdot09)} + \frac{10}{(1+0\cdot09)^2} + \frac{110}{(1+0\cdot09)^3}$$

$$= 9\cdot17 + 8\cdot42 + 84\cdot94$$

$$= ₹\ 102\cdot53$$

In this case, the bond price increases by an amount of ₹ 2·53 = ₹ 102·53 – 100 which is higher than ₹ 2·45 (i.e., the change in bond price when the yield increased from 10% to 11%).

5th Theorem

The fifth theorem on bond pricing suggests that the percentage change in bond's price due to a change in its yield will be smaller if the coupon rate of the bond remains higher.

Example 2.13

Let us consider two bonds, viz., bond-1 and bond-2 with the following features :

Table - 2.3

Features	Bond-1	Bond-2
Face Value	₹ 100	₹ 100
Life	3 years	3 years
Coupon rate (p.a.)	9%	7%
Redemption Value	₹ 100	₹ 100
Yield (YTM)	7%	7%
B_p	₹ 105·25	₹ 99·99 ≃ ₹ 100

For Bond-1, we have

$$\frac{9}{(1+0\cdot07)} + \frac{9}{(1+0\cdot07)^2} + \frac{109}{(1+0\cdot07)^3}$$

$$= 8\cdot41 + 7\cdot86 + 88\cdot98$$

$$= ₹\ 105\cdot25$$

If for Bond-1 the yield (YTM) rises to 8% then the market price of Bond-1 will decline to ₹ 102·58 because

$$\frac{9}{(1+0\cdot08)} + \frac{9}{(1+0\cdot08)^2} + \frac{109}{(1+0\cdot08)^3}$$

$$= 8\cdot33 + 7\cdot72 + 86\cdot53$$

$$= ₹\ 102\cdot58$$

In this case, the percentage change in bond price is $\frac{105\cdot25 - 102\cdot58}{105\cdot25} \times 100$

$$\text{or } \frac{2\cdot67}{105\cdot25} \times 100 = 2\cdot54\%$$

Now, for Bond-2, if the yield (YTM) also rises to 8%. Then the market price of bond-2 will be ₹ 97·42 because

$$\frac{7}{(1+0\cdot08)} + \frac{7}{(1+0\cdot08)^2} + \frac{107}{(1+0\cdot08)^3}$$

$$= 6\cdot48 + 6\cdot00 + 84\cdot94$$

$$= ₹\ 97\cdot42$$

In this case, the percentage change in Bond price will be $\frac{100-97.42}{100} \times 100$

$= 2.58\% > 2.54\%$.

2.3.1. Convexity in bond valuation

Here, the first and fourth theorems of bond pricing can lead to a concept of bond valuation which is known as 'convexity'.

The first theorem, as we have already discussed, shows an inverse relationship between the bond price (B_P) and the yield rate (YTM). On the other hand, the fourth theorem shows that B_P and YTM are not linearly correlated. Rather, $B_P = f(y)$ where

y = YTM and $\frac{d(B_P)}{dy} < 0$ and $\frac{d^2(B_P)}{dy^2} > 0$

Thus, the price-yield curve will be convex to the origin (Fig.-2.5)

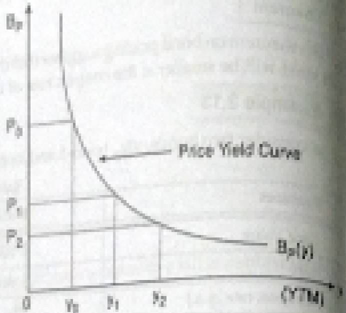

Fig.-2.5

In Fig.-2.5, if y declines by 1%, say, from y_1 to y_3 then B_P rises from P_1 to P_3, i.e., by an amount of P_3P_1. However, when y rises by 1% from y_1 to y_2 then B_P falls by an amount of $P_1P_2 < P_3P_1$. Hence, the price-yield curve becomes negatively sloped and convex to the origin.

2.3.2. Qualitative nature of price-yield curve : More discussion

In our previous discussion, we have already analysed the nature of the price-yield curve that shows the relationship between bond price (B_P) and the yield rate (YTM). However, by studying a price-yield curve we can show the relationship between B_P, YTM, coupon rate and the maturity period of a bond. Hence, a qualitative study of the price-yield curve helps us in identifying the factors which influence the construction of bond portfolio of an investor and understanding the nature of interest rate risk associated with a bond portfolio.

In a free market economy, yields of different bonds have a close linkage with one another and the yields of these bonds along with the interest rates on other fixed income securities normally track one another.

If any fixed-income security offers, say, 9% interest rate then the investors would be reluctant to buy a bond (with almost similar risk component) that offers an yield rate of, say, 7%. Hence, the general interest rate environment in the economy exerts such an influence upon the yield rates of different bonds so that those rates tend to move in tandem with other interest rates.

However, we have already shown that a change in the bond price can also lead to a change in the yield rate (YTM). The changes in bond price, in turn, depend to a large extent upon the structure of a bond, viz., the coupon rate, maturity period etc.

Hence, even when the yield rates of different bonds move in harmony, the prices of those bonds may vary by different amounts.

Let us now consider the following four bonds with same maturity period of , say, 30 years :

Table - 2.4

Bond	Face Value	Annual coupon rate (p.a.)	Redemption
Bond : 1	₹ 100	0	At par
Bond : 2	₹ 100	9%	At par
Bond : 3	₹ 100	10%	At par
Bond : 4	₹ 100	15%	At par

Here, Bond-1 is a zero-coupon bond. Normally, these are pure-discount bonds which are sold at a discount. We know that for this type of bond, we can estimate the yield rate (YTM) using the following formula :

$$P_0 = \sum_{k=1}^{n} \frac{R_k}{\left(1+\frac{\lambda}{m}\right)^k} \quad \text{........ (2.4)}$$

$$= \frac{R_{30}}{(1+\lambda)^{30}}$$

(The notations have their usual meanings)

Since redemption value (R) is obtained in 30th year upon the maturity of the bond, so, $R_1, R_2, ..., R_{29} = 0$. Now, given the current market price of the bond (P_0), we can estimate the yield rate or yield to Maturity (λ).

If the current market price of this pure-discount bond is equal to its par value, i.e., ₹ 100, then $\lambda = 0$

i.e., $100 = \frac{100}{(1+\lambda)^{30}}$, Where $\lambda = 0$

So, here YTM = λ = Coupon rate = 0

Now, if the present market price of this zero-coupon bond is ₹ 90 then

$90 = \frac{100}{(1+\lambda)^{30}}$ or, $(1+\lambda)^{30} = \frac{100}{90} = 1.11$

or, $(1+\lambda) = (1.11)^{\frac{1}{30}}$

or, $(1+\lambda) = 1.00348$

or, $\lambda = 0.00348$

$= 0.35\%$

Similarly, if its current price is ₹ 88

then, $(1+\lambda)^{30} = \frac{100}{88} = 1.136$

or, $\lambda = (1.136)^{\frac{1}{30}} - 1 = 1.0042 - 1 = 0.0042 = 0.42\%$

Thus, we get an inverse relation between the yield to maturity (YTM = λ) and the bond price (B_p), and the price-yield curve becomes negatively sloped. Now, the vertical intercepts of the all these bonds

and some points on each of the price-yield curves representing bonds with different coupon rates (with same maturity period of 30 years) can easily be calculated as follows :

Table : 2.5

Bond	Face Value (₹)	Annual coupon rate (%)	B_p (₹) for YTM = 0	B_p (₹) for YTM = coupon rate
Bond : 1	100	0	100	100
Bond : 2	100	5%	5 × 30 = 150 + Redemption value = 100 = 250	100
Bond : 3	100	10%	10 × 30 = 300 + 100 = 400	100
Bond : 4	100	15%	15 × 30 = 450 + 100 = 550	100

For any bond, if we consider YTM = 0 it implies that the bond is priced as if it offers no interest, and we get a value for B_p that would show the vertical intercept (i.e., Max. value of B_p when YTM = 0) of the respective price-yield curve. For instance, in case of 5% coupon bond, if YTM = 0, then the present value of the bond will be ₹ 5 (coupon payment per year) for 30 years (maturity period) and the redemption value of ₹ 100 (since it is redeemable at par), i.e., [(5 × 30) + 100] = ₹ 250.

Again, for this bond, if YTM = coupon rate = 5%. Then market value of the bond will be its par value, i.e., ₹ 100 [denoted by point *A* on $B_2(5)$ price-yield curve].

Following the similar process, we can locate vertical intercepts for different coupon bearing bonds. In Fig.-2.6, the 10% coupon bearing bond, i.e., $B_3(10)$ has B_p = ₹ 400 for YTM = 0, and B_p = ₹ 100 for YTM = 10% (denoted by point *B*).

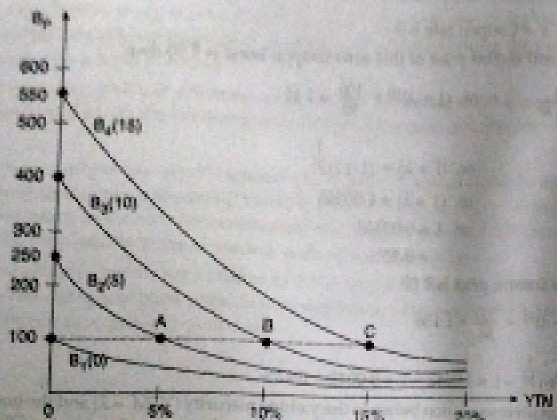

Fig.-2.6

2.3.3. Price-Yield curve & the influence of maturity period

The price-yield curves can also be used to show the influence of the maturity period of the bond on the bond price.

To illustrate this point, we consider the following 3 bonds with similar coupon rates, viz., 10% but with different maturity periods, viz.,

(i) For Bond - A, face value = ₹ 100 ; coupon rate = 10% p.a., redemption value = ₹ 100 (at par), and maturity period = 30 years.

(ii) For Bond - B, face value = ₹ 100 ; coupon rate = 10% p.a., redemption value = ₹ 100 (at par), and maturity period = 10 years.

(iii) For Bond - C, face value = ₹ 100, coupon rate = 10% p.a., redemption value = ₹ 100 (at par), and maturity period = 3 years.

Now, for Bond - A, if YTM = 0, then

B_p = (10 × 30) + 100 = ₹ 400;

for Bond - B, if YTM = 0, then

B_p = (10 × 10) + 100 = ₹ 200 ; and

for Bond - C, if YTM = 0, then

B_p = (10 × 3) + 100 = ₹ 130 ;

These values would determine the vertical intercepts of the respective price-yield curves.

Similarly, when YTM = coupon rate = 10% then B_p = Par Value for each bond = ₹ 100.

As a result, the price-yield curves would intersect at a point (E) as shown in Fig.-2.7.

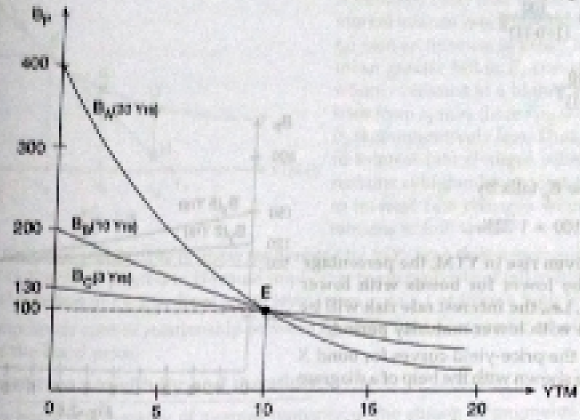

Fig.-2.7

2.3.4. Interest rate risk and price-yield curve

The absolute slope of the price-yield curve also shows the interest rate risk associated with a bond. Fig.-2.7 indicates that other things remaining the same (viz., the face value of a bond, the coupon rate and the redemption value), the price-yield curve becomes steeper for bonds with higher maturity. Hence, for 1% change in the YTM, say, for an 1% increase in YTM, the magnitude of fall in bond price

(B_p) will be more for long maturity bond compared to that for a short-maturity bond. Hence ... rate risk will be higher for long-maturity bond in comparison with short-maturity bonds.

Example 2.14

Let us consider a bond (say, Bond X) with a face value of ₹ 100 and a coupon rate of 10% ... redeemable at par upon maturity, the maturity period being 5 years. Let us consider another (say, Bond Y) with similar features like Bond X but the maturity period is 2 years.

Now, if an investor purchases Bond X at par, and then if the YTM rises to 11% then its market ... will decline to ₹ 96.30 since

$$\sum_{t=1}^{5}\frac{10}{(1+0.11)^t}+\frac{100}{(1+0.11)^5}$$

$$=\frac{10}{(1.11)}+\frac{10}{(1.11)^2}+\frac{10}{(1.11)^3}+\frac{10}{(1.11)^4}+\frac{110}{(1.11)^5}$$

$$= 9 + 8.12 + 7.31 + 6.59 + 65.28$$

$$B_p = ₹\ 96.30$$

So, the bond price declines from ₹ 100 to ₹ 96.30 as yield rate (YTM) rises from 10% to 11%. So, ... declines by $\frac{100-96.30}{100}\times 100 = 3.7\%$.

However, in case of bond Y, if YTM rises from 10% to 11% then the fall in B_p will be as follows

$$\sum_{t=1}^{2}\frac{10}{(1+0.11)^t}+\frac{100}{(1+0.11)^2}$$

$$=\frac{10}{(1.11)}+\frac{110}{(1.11)^2}$$

$$= 9 + 89.28$$

$$B_p = ₹\ 98.28$$

So, in this case the B_p falls by

$$\frac{100-98.28}{100}\times 100 = 1.72\%$$

Hence, for any given rise in YTM, the percentage fall in B_p will be lower for bonds with lower maturity period, i.e., the interest rate risk will be lower for bonds with lower maturity period.

In this example, the price-yield curves for bond X and Y can also be shown with the help of a diagram (Fig.-2.8).

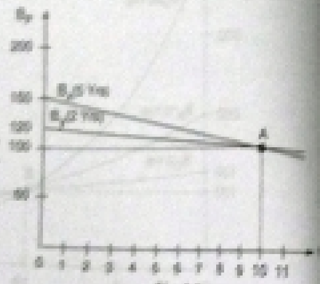

Fig.-2.8

In Fig.-2.8, B_p(5 years) denotes the price-yield curve of bond X with a maturity period of 5 years, and 'B_p (2 yrs)' denotes the price-yield curve of bond Y with a maturity period of 2 years. When YTM = coupon rate = 10%, then B_p = ₹ 100 = par value for both X and Y. Hence, the price-yield curves of bond X and Y intersects at point A (Fig.-2.8). For B_p (5 yrs), if YTM = 0, then B_p = (10 × 5) + Redemption value (100) = ₹ 150, and we get the vertical intercept of B_p (5 yrs) curve.

Similarly, for bond Y, when YTM = 0 then B_p = (10 × 2) + 100 = ₹ 120.

...ce, B_p (5 yrs) curve or the price-yield curve of bond-X becomes steeper than that of B_p (2 yrs) curve ...plying higher interest rate risk for bond-x compared to that of bond-Y.

- Yield risk : Our previous discussion implies that the investors are also subject to yield risk. ...the yield rate changes then the bond price (B_p) will also change. It implies an impact upon the near-term value of the bond, and therefore, an immediate risk. However, the investor can hold the bond ...to its maturity and hence, can receive the periodical coupon payments and the redemption value ...on its maturity. In case of fixed-income securities, this cash-flow (viz., the fixed coupon payments) ... not affected by changes in yield rate. But if the investor plans to sell the bond before maturity then ...e bond price will be governed by the price-yield curve.

2.3.5. Interest rate sensitivity of the bond price

In our previous discussion we have shown that a price-yield curve showing a negative relationship between bond price (B_p) and the yield rate (YTM) becomes convex to the origin. This convexity of the price-yield curve can reflect the sensitivity of the bond price (B_p) to changes in interest rates (the YTM can be considered as the market rate of interest). If the price-yield curve is linear then for every 1% change in the YTM, the B_p will change at a given percentage always. In that case, the B_p becomes equally sensitive to changes in interest rates at all levels (i.e., either at lower level or at higher level of market interest rates).

However, because of the convexity of the price-yield curve, this sensitivity will differ. In Fig.-2.9, YY denotes the price-yield curve, which is convex to the origin. If this price-yield curve is linear then it would be a downward sloping straight line as denoted by Y_1Y_1.

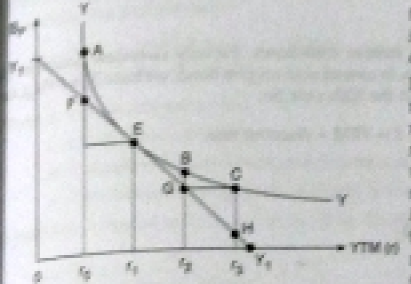

Fig.-2.9

It becomes clear that when the yield rate or the market interest rate (r) remains at a low level (say, r_0) then an increase in r (say, from r_0 to r_1) would mean greater fall in B_p compared to a situation where r remains at a higher level (say, at r_2). If r rises from r_2 to r_3 (here $r_3r_2 = r_1r_0$) then the fall in B_p is comparatively less. Thus, B_p is less sensitive to interest rate changes when the interest rate remains at higher levels, and B_p is more sensitive to interest rate changes when the interest rate remains at low levels.

However, if the price-yield curve is linear (as denoted by Y_1Y_1 line) then a movement along Y_1Y_1 line from point F to E or from point G to H would imply that the magnitude of fall in B_p would be the same for any equal increase in r either from a low level (from r_0 to r_1) or from a high level (from r_2 to r_3). Hence, the non-linear convex relationship between B_p and YTM (r) helps in analysing the interest rate sensitivity of the bond price.

2.3.6. Interest rate sensitivity and duration

Duration of a bond is a measure of average maturity of the stream of payments associated with a bond.

The interest rate sensitivity of a bond can be measured by the 'duration' of a bond. For fixed income securities 'duration' is defined as the weighted average of the maturities of all the individual cash flows from the fixed-income security, the weights being the present values of the individual cash flows as a proportion of current bond price at each period.

The duration of a bond is measured using the following formula :

$$D = \frac{\sum_{k=1}^{n} PV(t_k)t_k}{PV = P_0} \text{ ...... (2.5)}$$

Where $PV(t_k)$ = Present value of cash flow from a bond that takes place at the time period t_k $(k = 1, 2, ..., n)$

$PV = P_0$ = Current bond price which is equal to the present value of future cash-flows from the bond, i.e.,

$$PV = \sum_{k=1}^{n} PV(t_k)$$

[Here, we consider all the non-negative cash-flows, and the cash outflow at the initial period for the purchase of a bond is not taken into account since it is assumed that the investor already owns the bond].

The duration can also be measured by reusing the previous formula as

$$D = \sum_{k=1}^{n}\left[\frac{PV(t_k)}{P_0} \times t_k\right] \text{ ...... (2.6)}$$

In case of a zero coupon bond, there are no interim cash-flows, the only cash-flow occurs at maturity period, viz., the redemption value. So, in case of zero coupon bond, we have $P_0 = PV(t_n)$. If $k = 10$ years, then the cash-flow will occur at the 10th year. So,

$$P_0 = PV(t_n) = \frac{R_n}{(1+\lambda)^n} \text{ ...... (2.7) where } \lambda = \text{YTM} = \text{discount rate.}$$

$\therefore$ For zero coupon bonds,

$$D = \frac{PV(t_n)}{P_0} \times t_n = \frac{P_0}{P_0} \times t_n = t_n \text{ ...... (2.8)}$$

Thus D = Maturity period of the bond.

= Duration of the zero coupon bond.

It should be noted that for any coupon bearing bond duration will always be less than maturity period of the bond. This is because in our formula, the largest value that t_k can have is t_n, and each of the value of t_k is multiplied by a weight (a fraction) equal to $\frac{PV(t_k)}{P_0}$

$$\therefore D < t_n$$

However, D is greater than t_1, i.e., the lowest value of t_k (since D is the weighted average of t_k, $k = 1, 2, ..., n$)

So, $t_1 < D < t_n$

However, for zero-coupon bond,

$D = t_n$

If the zero-coupon matures after one year then $D = t_1$

$\therefore$ We have $t_1 \le D \le t_n$.

...ount formula for the estimation of duration. If the yield rate (YTM) of the bond is used as the discount rate to estimate the present value of the cash-flows then that formula is often called as 'Macaulay formula' or the 'Macaulay duration', and it is expressed as follows :

$$D = \frac{\sum_{k=1}^{n} \frac{C_k}{(1+\frac{y}{m})^k}\left(\frac{k}{m}\right) + \frac{R_n}{(1+\frac{y}{m})^{mn}}\, n}{PV} \text{ ...... (2.9)}$$

Where, $PV = \sum_{k=1}^{n} \frac{C_k}{(1+\frac{y}{m})^k} + \frac{R_n}{(1+\frac{y}{m})^n}$

C_k = Coupon amount at the time period k $(k = m.t)$

m = Number of times the coupon payment is made in any year. (If $m = 1$ then $k = mt = t$)

y = Yield to maturity or the yield rate or the discount rate to determine the present value (PV) of cash-flows.

R_n = Redemption value of the bond upon maturity.

When the coupon payments are identical then the Macaulay's duration formula is normally expressed in the following explicit form :

$$D = \frac{1+y}{my} - \frac{1+y+n(c-y)}{mc[(1+y)^n - 1] + my} \text{ ...... (2.10)}$$

Where y = yield rate per period

c = coupon rate per period

m = Number of times the coupon payment is made per year.

n = Periods remaining for the maturity of the bond.

If yield rate = coupon rate (i.e., $c = y$), i.e., if the bond is sold at par then the above formula becomes

$$D = \frac{1+y}{my} - \frac{1+y}{my[(1+y)^n - 1] + my} \quad [\because y = c]$$

$$= \frac{1+y}{my} - \frac{1+y}{my(1+y)^n - my + my}$$

$$D = \frac{1+y}{my}\left[1 - \frac{1}{(1+y)^n}\right] \text{ ...... (2.11)}$$

Example 2.15

(i) Consider a bond with a face value of ₹ 100, a coupon rate of 10% payable after every 6 months, and it is redeemable at par after a maturity period of 30 years. It is sold at par at present. Calculate the duration of this bond.

(ii) Consider the same type of bond with the only difference that the 10% coupon rate is paid annually. Calculate the duration of this bond.

Solution :

(i) In this case, the bond is sold at par. So, its coupon rate = yield rate per year. However, since coupon payment is made bi-annually, so 5% on face value is paid after every 6 months. Here we can estimate the duration of the bond using Macaulay's duration formula (where $c = y$) :

$$D = \frac{1+y}{m \cdot y}\left[1 - \frac{1}{(1+y)^n}\right]$$

$$D = \frac{1+0.05}{2 \times 0.05}\left[1 - \frac{1}{(1+0.05)^{60}}\right]$$

$= 10.5\,(1 - 0.05353) = 9.9379$ years.

Here, $c = y = 0.05$, $m = 2$

$n = m \times t$

$= 2 \times 30$

$= 60$

$t = 30$ years.

(ii) In the second case, the duration of the bond will be as follows :

$$D = \frac{1+0.1}{0.1}\left[1 - \frac{1}{(1+0.1)^{30}}\right]$$

Here, $m = 1$, $y = 0.1$, $n = m.t = t = 30$ years

$\therefore D = \frac{1.1}{0.1}[1 - 0.0573]$

$= 11 \times 0.9427$

$= 10.36$ years.

So, the duration of a bond (having same coupon and yield rate) increases with a fall in the value of *m*, i.e., the frequency of coupon payments per year.

Example 2.16

Calculate the duration of a bond based on the following information :

(i) Face value of the bond : ₹ 100 redeemable at par

(ii) Coupon rate : 8% payable annually

(iii) Time left for maturity : 8 years

(iv) Discount rate : 10% p.a.

Solution :

The duration of this bond can be estimated as follows :

Table : 2.6

Time (Yrs) (a)	Cash flow (₹) (b)	PVDF (₹) (c)	PV (₹) (d) = (b) × (c)	Weight (e) = (d) ÷ P_0	Duration (f) = (a) × (e)
1	8	0.9091	7.27	0.0814	0.0814
2	8	0.8264	6.61	0.0740	0.1480
3	8	0.7513	6.01	0.0673	0.2019
4	8	0.6830	5.46	0.0611	0.2444
5	8	0.6209	4.97	0.0556	0.2780
6	8	0.5644	4.51	0.0505	0.3030
7	8	0.5131	4.10	0.0490	0.3430
8	108	0.4665	50.38	0.5641	4.5128
			P_0 = 89.31	1.0000 = 1.0	5.8095

≅ 5.81 years.

[Here, Present Value Discount Factor (PVDF) = $\frac{1}{1+r}$]

In this case, we have used the following formula for the estimation of duration of the bond [viz., formula (2.9)] :

$$D = \frac{\sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{\lambda}{m}\right)^k}\left(\frac{k}{m}\right) + \frac{R_0}{\left(1+\frac{\lambda}{m}\right)^n} n}{PV}$$

Here $PV = \sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{\lambda}{m}\right)^k} + \frac{R_0}{\left(1+\frac{\lambda}{m}\right)^n}$

In our example, C_k = ₹ 8 per year (i.e., 8% p.a.)

$m = 1$

$k = m.t = t\ (= 1, 2, \ldots, 8)$

$\lambda = 0.1$ and R_0 = ₹ 100

$$\therefore D = \frac{\sum_{k=1}^{8} \frac{8}{(1+0.1)^k} \times k + \frac{100}{(1+0.1)^8} \times 8}{PV} \text{ where } PV = P_0 = \sum_{k=1}^{8} \frac{8}{(1+0.1)^k} = ₹\ 89.31$$

$$= \frac{\frac{8}{(1+0.1)} \times 1 + \frac{8}{(1+0.1)^2} \times 2 + \ldots + \frac{8}{(1+0.1)^8} \times 8 + \frac{100}{(1+0.1)^8} \times 8}{89.31}$$

$$= \frac{\frac{8}{(1.1)} \times 1}{89.31} + \frac{\frac{8}{(1.1)^2} \times 2}{89.31} + \frac{\frac{8}{(1.1)^3} \times 3}{89.31} + \ldots + \frac{\frac{108}{(1.1)^8} \times 8}{89.31}$$

$= 0.0814 + 0.1480 + 0.2019 + 0.2444 + 0.2780 + 0.3030 + 0.3430 + 4.5128$

$= 5.81$ years.

2.3.7. Volatility of a bond

The interest rate sensitivity of a bond or the volatility of a bond can be measured by the 'duration' of a bond. The volatility of a bond implies percentage change in the bond price with 1% change in yield rate (YTM). It is expressed as

$$\frac{\text{Percentage change in bond price}}{\text{Percentage change in (1+YTM)}} \approx -D$$

where D = duration.

Here, the symbol '≈' signifies 'approximately equal to'. This formula implies that when the yield rates of two bonds having same duration change by same percentage, then the prices of those bonds will change by approximately equal percentages.

If percentage change in

Bond Price = $\frac{\Delta B_p}{B_p}$, and

percentage change in yield rate = $\frac{\Delta(1+YTM)}{1+YTM}$

$$= \frac{\Delta YTM}{(1+YTM)}$$

Then, we can write

$$\frac{\Delta B_p}{B_p} \approx -D \cdot \frac{\Delta YTM}{(1+YTM)}$$

$$= -\left(\frac{D}{1+YTM}\right) \cdot \Delta(YTM) \quad \text{......... (2.12)}$$

Example 2.17

Let us consider a bond which is currently selling at a price of ₹ 1000 with a yield rate = YTM = 8%. Now, given that the duration of this bond is 10 years, what would be magnitude of the rise in the price of this bond if the yield rate (YTM) rises to 9%?

Solution :

Here, $\Delta(YTM) = 9\% - 8\% = 1\% = 0.01$

$D = 10$ years.

$YTM = 8\% = 0.08$

$$\therefore \frac{\Delta B_p}{B_p} = (-)\frac{10}{(1+0.08)} \times (0.01) = (-)\frac{0.1}{1.08}$$

$$= (-)\, 0.09259$$

$$= (-)\, 0.0926$$

$$= (-)\, 9.26\%$$

It implies that 1% percentage point rise in yield rate will result in about 9.26% decline in the bond's price (B_p).

In this formula, the expression $\left(\frac{D}{1+YTM}\right)$ is often referred to as the **'modified duration'** of a bond. It reflects the percentage change in bond's price for 1% change in the yield rate (YTM) of the bond. Sometimes the **Volatility of the bond** (V_B) is estimated as follows :

$$V_B = (-)\frac{D}{\left[1+\frac{YTM}{m}\right]} = (-)\frac{D}{\left[1+\frac{\lambda}{m}\right]} \quad \text{......... (2.13)}$$

Where D = Duration of the bond

m = Number of times the coupon payment is made during a year.

λ = yield to maturity

Now, if $m = 1$, $\lambda = 8\%$ and $D = 10$ years

$$\text{Thus } V_B = (-)\frac{10}{\left[1+\frac{0.08}{1}\right]} = (-)\frac{10}{1+0.08} = (-)\,9.26$$

Hence, we get the same result as before.

Proof :

We can now show how the formula (2.13) for the estimation of the volatility of the bond price (B_p) or the sensitivity of the B_p due to a change in the yield rate has been derived.

Let us first consider the **Macaulay duration formula** of a bond as given below :

$$D = \frac{\sum_{k=1}^{n} \frac{C_k}{(1+\frac{\lambda}{m})^k}\left(\frac{k}{m}\right)}{PV} \quad \text{......... (2.14)}$$

where, $PV = \sum_{k=1}^{n} \frac{C_k}{(1+\frac{\lambda}{m})^k}$

C_k = Cash flow from the bond at the time period k ; $[k = m.t]$

m = Number of times the coupon payment is made per year (here it is assumed that $m = 1$, and therefore $k = m.t = t$)

λ = yield rate or the yield to maturity of the bond.

Hence, at the time period k,

$$PV_k = \frac{C_k}{(1+\frac{\lambda}{m})^k} = C_k \cdot (1+\frac{\lambda}{m})^{-k}$$

Now, differentiating PV_k with respect to λ, we get

$$\frac{d(PV_k)}{d\lambda} = C_k \cdot \left(\frac{1}{m}\right) \cdot -k\left[1+\frac{\lambda}{m}\right]^{-k-1}$$

$$= -C_k\left(\frac{k}{m}\right)\left[1+\frac{\lambda}{m}\right]^{-(k+1)}$$

$$= -\frac{\left(\frac{k}{m}\right)C_k}{\left[1+\frac{\lambda}{m}\right]^{k+1}}$$

$$= \frac{\left(\frac{t}{m}\right)}{\left(1+\frac{\lambda}{m}\right)} \frac{C_t}{\left(1+\frac{\lambda}{m}\right)^t}$$

$$= \frac{\left(\frac{t}{m}\right)}{\left(1+\frac{\lambda}{m}\right)} PV_t \text{ —— (2.15)}$$

Now, since the bond price at present is $P = \sum_{t=1}^{n} PV_t$, so the above result (2.15) can be expressed as

$$\frac{dP}{d\lambda} = \sum_{t=1}^{n} \frac{d(PV_t)}{d\lambda}$$

$$= -\sum_{t=1}^{n} \frac{\left(\frac{t}{m}\right)}{\left(1+\frac{\lambda}{m}\right)} \frac{C_t}{\left(1+\frac{\lambda}{m}\right)^t}$$

$$= -\frac{1}{\left(1+\frac{\lambda}{m}\right)} \frac{\sum_{t=1}^{n} \frac{C_t}{\left(1+\frac{\lambda}{m}\right)^t}\left(\frac{t}{m}\right)}{PV} PV$$

$$= -\frac{1}{\left(1+\frac{\lambda}{m}\right)} D.P \text{ —— (2.16)}$$

Since, $PV = \sum \frac{C_t}{\left(1+\frac{\lambda}{m}\right)^t} = P$

and $D = \dfrac{\sum_{t=1}^{n} \frac{C_t}{\left(1+\frac{\lambda}{m}\right)^t}\left(\frac{t}{m}\right)}{PV}$

$$\therefore \frac{dP}{d\lambda}\cdot\frac{1}{P} = (-)\frac{D}{\left(1+\frac{\lambda}{m}\right)}$$

or, $\dfrac{dP/P}{d\lambda} = -\dfrac{D}{\left(1+\frac{\lambda}{m}\right)} = -D_M$

or, $\dfrac{dP}{d\lambda}\cdot\dfrac{1}{P} = (-)D_M$ —— (2.17)

Where $\frac{dP/P}{d\lambda}$ = Percentage change in bond price due to a change in yield rate (YTM) = volatility or sensitivity of bond price and D_M = Modified duration

$$= \frac{D}{\left(1+\frac{\lambda}{m}\right)}$$

This modified duration helps us in estimating the volatility or the sensitivity of bond price to a change in yield rate.

However, this estimation of volatility or the sensitivity of bond price assumes a linear relationship between the change in bond price and the change in yield rate. But in reality the price-yield curve is not linear, rather it is convex to the origin. Hence, the estimation of sensitivity of bond price based on the notion of 'duration' would provide us with an approximation to the actual situation [Hence, we used the sign '≈' in our formula (2.12) signifying 'approximately equal to']. This problem has already been analysed before with the help of a diagram (Fig-2.9).

We can also illustrate it with the help of an example.

Example 2.18

Let us consider the case as shown in our previous example No. 2.16, where we have shown that a bond with a face value of ₹ 100 carrying a coupon rate of 8% p.a. (payable annually), and a maturity period of 8 years when the bond is redeemable at par. At a discount rate of 10%, the current price of this bond is found to ₹ 89·31, and the duration of this bond was estimated to be 5·81 years.

Now, based on this duration, we can show the volatility of this bond, i.e., the percentage change in bond price due to 1% change in the yield rate or the discount rate. Let us use the formula (2.13) or (2.17) to estimate this sensitivity or volatility of bond price :

$$V_P = (-)\frac{D}{\left(1+\frac{\lambda}{m}\right)} = (-)\frac{5 \cdot 81}{(1 \cdot 1)} = (-)5 \cdot 28$$

Here D = duration = 5·81 years

$m = 1$, λ = YTM = 10%

This result implies that 1% increase in the yield rate (YTM) would lead to 5·28% fall in the bond price, and 1% fall in the yield rate (YTM) would cause an increase in bond price by 5·28%.

However, we can show that the actual change in bond price due to 1% change in YTM might be different from that estimated by the 'duration' based volatility formula. This is shown in the following table :

Table - 2.7

Time	Cash-flow (₹)	PV of the Cash-flow		
		at λ = 11%	at λ = 10%	at λ = 9%
1	8	7·21	7·27	7·34
2	8	6·49	6·61	6·73
3	8	5·85	6·01	6·18
4	8	5·27	5·46	5·67
5	8	4·75	4·97	5·20
6	8	4·28	4·51	4·77
7	8	3·25	4·10	4·38
8	108	46·86	50·38	54·20
Current Bond price		= ₹ 84·56	= ₹ 89·31	= ₹ 94·47

In this case, we observe that as the yield rate rises from 10% to 11%, the bond price falls by 5·32%.

Thus, $\dfrac{84 \cdot 56 - 89 \cdot 31}{89 \cdot 31} = -\dfrac{4 \cdot 75}{89 \cdot 31} = (-)0 \cdot 05318$

$= (-)5 \cdot 32\%$

Similarly, we observe that as the yield rate falls from 10% to 9%, the bond price rises by 5.78%.

Thus, $\frac{94.47-89.31}{89.31} = \frac{5.16}{89.31} = 0.05777$

$\simeq 5.78\%$

Hence, we find that while the duration-based volatility is – 5.28%, the actual sensitivity or volatility of the actual price-yield curve as opposed of the bond is different. This is because of the convexity of the actual price-yield curve as opposed to the linear relationship between bond price and yield rate assumed in 'duration-based' sensitivity measure. This can be shown with the help of a price-yield curve (Fig.-2.10).

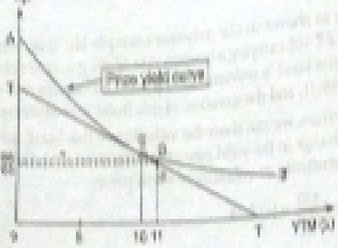

Fig. - 2.10

In Fig. - 2.10, AB denotes the price-yield curve of a bond with a face value of ₹ 100, carrying 8% coupon rate (payable annually) and a maturity period of 4 years (redeemable at par). The duration (D) of this bond is estimated to be 3.81 years [Example : 2.18]. So, its modified duration (D_M) at a yield rate of 10% becomes $D_M = \frac{3.81}{(1+0.1)} = 3.28$.

Now, from (2.17), we get $\frac{dP}{d\lambda} = -D_M.P$

Here, $\frac{dP}{d\lambda}$ denotes the slope of the price-yield curve at λ = 10% (at point E). Thus, the tangent TT' touching point E of the price-yield curve determines the duration-based sensitivity or volatility of the bond [point E corresponds to bond price at λ = 10%].

Now, if yield rate (λ) rises to 11% then the approximate change in bond price (B_p) will be

$dP = -D_M.P.d\lambda$

$\Delta P = (-)\, D_M.P.\Delta\lambda$ [Assuming that $\frac{dP}{d\lambda} = \frac{\Delta P}{\Delta\lambda}$]

$= (-)\, 3.28 \times 100 \times 0.01$

$= (-)\, 3.28$

Hence, example : 2.18 suggests that the bond price should fall from ₹ 89.31 to ₹ 89.31 – 3.28 = ₹ 86.03 (determined by point F on the tangent TT'). But as per actual price-yield curve, the bond price falls to ₹ 84.86 (denoted by point G on the price-yield curve). Thus, the duration-based volatility measure can only provide us with an approximation to the actual change.

2.3.8. Properties of duration

The property of convexity of the price-yield curve and the definition of duration together reveal some particular characteristics of 'duration'.

1. **Duration is less than maturity period of a bond :** The duration of a bond with a given coupon rate is always less than the maturity period of the bond. We know that duration $(D) = \frac{\sum_{k=1}^{n} PV(t_k).t_k}{PV}$

[as already shown in formula (2.5)].

Here, the largest value of t_k is t_n and while estimating D, each value of t_k is multiplied by a fraction $\frac{PV(t_k)}{P_0}$ (Where P_0 = PV = Present price of the bond). Therefore, $D < t_n$.

2. **Duration is higher for bonds with longer maturity :** Since duration is defined as a time weighted average of present values of cash-flows from a bond, the weightage of the redemption value will have the highest multiplier. Hence, with an increase in the value of t_k (k = 1, 2, ..., n), viz., the maturity period of the bond, the duration (D) of the bond will rise (other things remaining the same). Thus, in our formula $D = \frac{\sum_{k=1}^{n} PV(t_k).t_k}{PV}$, D rises with an increase in the value of t_k.

Example 2.19

Consider the following two bonds :

	Bond - 1	Bond - 2
Face Value :	₹ 100	₹ 100
Coupon rate (p.a.) :	10%	10%
Maturity period :	3 years	5 years
Redemption value :	At par	At par

Now, we are to calculate the 'Duration' of these two bonds considering YTM = 8%.

Table - 2.8

Year (t_k)	Bond - 1 Cash Flow (₹)	Bond - 1 PV (t_k) (YTM = 8%)	Bond - 1 $t_k \times \frac{PV(t_k)}{P_0}$	Bond - 2 Cash Flow (₹)	Bond - 2 PV (t_k) (YTM = 8%)	Bond - 2 $t_k \times \frac{PV(t_k)}{P_0}$
1	10	9.26	0.088	10	9.26	0.086
2	10	8.57	0.162	10	8.57	0.159
3	110	87.32	2.490	10	7.94	0.220
4	—	—	—	10	7.35	0.272
5	—	—	—	110	74.86	3.466
		P_0 = PV = ₹ 105.15	D = 2.74		P_0 = ₹ 107.98	D = 4.203

Thus, the duration (D) of the bond with longer maturity period (viz., Bond -2) becomes higher. The duration-based volatility of the bond will be $V_B = -\frac{D}{\left(1+\frac{\lambda}{m}\right)}$

here λ = YTM = 8% and m = 1.

$\therefore$ For Bond -1, $V_B = (-)\frac{2.74}{(1+0.08)} = (-)2.54$

and for Bond -2, $V_B = (-)\frac{4.20}{(1+0.08)} = (-)3.89$

Thus, Bond -2 is more sensitive to changes in yield rate.

In both cases, it is also observed that the duration is less than the maturity period of the bond. In case of Bond -1, maturity period is 3 years but the duration is 2.74 years. In case of Bond -2, the maturity period is 5 years but the duration is 4.20 years.

3. **Duration of bonds with low coupon rate becomes higher :** when the coupon rate falls then other things remaining the same, the present value of the bond becomes lower. In this case, the proportion of redemption value to the present value of the bond becomes higher than that in case of high-coupon bond. As a result, the duration of low-coupon bond becomes more than that of a high-coupon bond. This can be shown with the help of an example.

Example 2.20

Year (t_k)	Low Coupon Bond (8%)			High Coupon Bond (15%)		
	Cash Flow (₹)	PV (t_k) (YTM = 5%)	$t_k \times \frac{PV(t_k)}{P_0}$	Cash Flow (₹)	PV (t_k) (YTM = 5%)	$t_k \times \frac{PV(t_k)}{P_0}$
1	8	7.62	0.069	15	14.28	0.105
2	8	7.26	0.131	15	13.60	0.201
3	8	6.91	0.187	15	12.96	0.287
4	108	88.85	3.212	115	94.61	2.784
		P_0 = PV = ₹ 110.65	D = 3.599		P_0 = ₹ 135.45	D = 3.387

So, here it is observed that the duration of the low-coupon bond (i.e., bond with an annual coupon rate of 8%) is higher (D = 3.599 ≃ 3.60 years) in comparison with that of the high-coupon bond (D = 3.387 ≃ 3.39 years for bond with a coupon rate of 15% p.a.)

Thus, other things remaining the same, the volatility of a low-coupon bond will be higher because

(i) for low-coupon bond, $V_B = (-)\frac{D}{\left(1+\frac{\lambda}{m}\right)}$ [Here λ = 5%, m = 1]

$= (-)\frac{3.60}{(1+0.05)}$

$= (-)3.43$

(ii) for high-coupon bond, $V_B = (-)\frac{3.39}{(1+0.05)}$

$= (-)3.23$

4. **Duration of a bond with lower yield to maturity will be higher :**

The price-yield curve shows that at lower yield rate, the absolute slope of the curve becomes steeper (see Fig.-2.10) implying higher sensitivity of bond price to changes in yield rate. So the duration based sensitivity of bond price also becomes higher at lower yield rate.

This can be shown with the help of an example.

Example 2.21

Year (t_k)	Low Yield Bond			High Yield Bond		
	Cash Flow (₹)	PV (t_k) (YTM = 5%)	$t_k \times \frac{PV(t_k)}{P_0}$	Cash Flow (₹)	PV (t_k) (YTM = 15%)	$t_k \times \frac{PV(t_k)}{P_0}$
1	10	9.52	0.081	10	8.69	0.101
2	10	9.07	0.154	10	7.56	0.176
3	10	8.64	0.220	10	6.57	0.230
4	110	90.50	3.075	110	62.89	2.935
		P_0 = ₹117.73	D = 3.53		P_0 = ₹ 85.71	D = 3.442

This example shows that low-yield bond (with YTM = 5%) has a higher 'duration' (D) [D = 3.53] compared to that of the high-yield bond [Where D = 3.44].

5. **The duration of a zero-coupon bond is equal to the time remaining for its maturity :** We have already discussed this special feature. However, we can now illustrate it with an example. In case of a zero-coupon bond cash-flow occurs only at the maturity period of the bond without any interim cash-flows. So, in that case, the present value of the cash-flow or the bond price would be

$P_0 = PV(t_n) = \frac{R_n}{(1+\lambda)^n}$; Where λ = Yield to maturity and t_n = maturity period.

Since $D = \frac{\sum PV(t_k)t_k}{P_0}$

Therefore, for a zero-coupon bond we have

$D = \frac{PV(t_n)}{P_0} \times t_n = \frac{P_0}{P_0} \times t_n = t_n$

Example 2.22

Period (days)	Cash flow (₹)	$PV(t_n)$ (YTM = 5%)	$t_n \times \frac{PV(t_n)}{P_0}$
0	—	—	—
364	100	95.25	D = 364
		P_0 = ₹ 95.25	

In our example, the maturity period of the zero-coupon bond is 364 days, i.e., $\frac{364}{365}$ = 0.9973 year. The present value of this bond at a yield rate (YTM) of 5% p.a. will be $\frac{100}{1+\left(\frac{0.05}{365}\times 364\right)} = \frac{100}{1.049863}$ = ₹ 95.25.

Thus, the duration of this zero-coupon bond is $364 \times \frac{99.75}{99.75} = 364$ days $= 0.9973$ year.

Therefore, in this case, the volatility or sensitivity of the bond price to change in yield rate w

$V_B = (-)\frac{D}{(1+r)} = (-)\frac{0.9973}{(1+0.05)} = (-)\,0.9498\% = (-)\,0.95\%$.

Thus, the volatility of such short-maturity bonds with low YTM will be close to 1%, i.e. if YTM by 1%, the bond price would fall by almost equal to 1%.

However, if the zero-coupon bond has a long maturity period, say, 25 years then the volatility w be very high. For example, a 25 year zero-coupon bond with a face value of ₹ 100 and a redemp value of ₹ 100 (at par), the present value at 5% yield rate would be $\frac{100}{(1+0.05)^{25}}$ = ₹ 29.53. In this c

the duration of this bond is also 25 years. So the volatility (V_B) of the bond will be $V_B = (-)\frac{25}{(1+0.05)}$ $= (-)\,23.81\%$. Thus, for 1% rise in yield rate, the bond price will fall by about 23.8%. Hence, for lo duration zero-coupon bond, the interest rate risk becomes higher.

2.3.8. Duration of a portfolio of bonds

There can be a portfolio of several bonds of different maturities. Hence, in this case, the portfolio a like a single big fixed-income security. However, the periodic cash-flow from the portfolio may no of equal magnitude due to different maturity periods of bonds within that portfolio.

The duration of such a portfolio of bonds having equal yields can be estimated by the weight average of the durations of individual bonds.

Let us consider only two bonds : X and Y in the portfolio.

Here, the duration of bond X is

$$D^x = \frac{\sum_{k=1}^{n} PV_k^x t_k}{P_0^x} \quad \text{where } P_0^x = PV^x$$

and duration of bond Y is

$$D^y = \frac{\sum_{k=1}^{n} PV_k^y t_k}{P_0^y} \quad \text{where } P_0^y = PV^y$$

$$\therefore P_0^x D^x + P_0^y D^y = \sum_{k=1}^{n} PV_k^x \cdot t_k + \sum_{k=1}^{n} PV_k^y \cdot t_k$$

$$= t_k\left[\sum PV_k^x + \sum PV_k^y\right]$$

Now, the duration of the portfolio (D) is estimated as $D = \frac{P_0^x}{P} \cdot D^x + \frac{P_0^y}{P} \cdot D^y$ ——— (2.18)

where $P = P_0^x + P_0^y$.

If there are 'm' number of bonds in the portfolio then the duration of the portfolio will be

$$D = W_1 D_1 + W_2 D_2 + \dots + W_m D_m = \sum_{i=1}^{m} W_i D_i \quad \text{———— (2.19)}$$

where $W_i = \frac{P_i}{P}$ $(i = 1, 2, \dots, m)$

and $P = \sum_{i=1}^{m} P_i$

Example 2.23

Let us consider our example No. 2.19 where $D_1 = 2.74$ (for Bond : 1) and $D_2 = 4.203$ (for Bond : 2); P_0^1 = ₹ 105.15 and P_0^2 = ₹ 107.98. In that example, the maturity period of Bond - 1 is 3 years with yield rate of 8%, and the maturity period of Bond - 2 is 5 years with same yield rate, i.e., 8%.

So, in this case portfolio duration (D) will be $D = \frac{P_0^1 D_1}{P} + \frac{P_0^2 D_2}{P}$

$$D = \frac{105.15}{213.13} \times 2.74 + \frac{107.98}{213.13} \times 4.203$$

where ₹ 213.13 = 105.15 + 107.98.

or, $D = [0.49 \times 2.74] + [0.51 \times 4.203]$

$= 1.34 + 2.14$

$= 3.48$

Financial analysts are also of the opinion that in cases where the bonds in a portfolio have different yields then an average of those yields can be considered for the calculation of the present values of respective bonds, and the composite duration of the portfolio can be estimated.

The duration of the portfolio will provide us with the sensitivity of the overall present value of the bonds in the portfolio to a change in the yield rate.

In example-2.23, the sensitivity or the volatility of the portfolio will be as follows :

Volatility of Portfolio (V_P) $= (-)\frac{D}{1+YTM}$

$= (-)\frac{3.48}{1+0.08}$

$= (-)\,3.22\%$

It implies that if there is 1% increase in the yield rate then average bond price of the portfolio will decline by 3.22% and *vice versa*.

2.4. Immunisation

We have already discussed the concept of duration of a portfolio of bonds. This portfolio is susceptible to interest rate risks. *Immunisation refers to a bond portfolio management technique that aims at protecting the portfolio from any adverse effects associated with future changes in interest rates.* This technique immunises the portfolio value against changes in interest rates.

Naturally, the way in which a bond portfolio has to be managed depends upon the purpose of that portfolio management. This is shown in the following table with examples (Table -2.9).

Table - 2.9

Purpose of creating a portfolio : Examples	Investment Plan	Risk Implications
1. An amount required after 1 year for the repairment of factory building	(a) 1 - year Treasury Bill (b) 10 - year zero-coupon bonds	(a) Low risk : Since exact amount of redemption value is known. (b) High risk : Since after 1 year, the bond price may fall due an increase in market interest rate.
2. An amount required after 10 years for meeting educational expenditure (say, to continue higher education)	(a) 10 year-zero-coupon bonds (b) 1 year Treasury Bill	(a) Low risk : Since the return is predictable (when it is required). (b) High risk : Since there will be reinvestment risk (as rate of interest may fall).
3. A series of cash obligations to be met in future (as happens in case of an insurance company)	(a) A set of zero-coupon bonds with maturity periods and redemption values matching with the cash obligations. (b) a set of bonds with a value = present value of the stream of cash obligations.	(a) Problem of having adequate zero-coupon corporate bonds (with high return). (b) Low risk : If yield rate does not change for these bonds. High risk : If yield rate changes.

In this table, we see that in some cases [in 3(b)] the value of portfolio may not match with the present value of the stream of cash obligations. This problem is solved through immunisation to some extent **by matching the duration of the portfolio of bonds with the duration of the cash obligations.** We know that the duration of portfolio of bonds measures the sensitivity of the present value of the bond portfolio to the changes in yield rate. If there is an increase in yield rate then the present value of the portfolio will fall. However, in that case, the present value of the future cash obligations will also fall by approximately the same amount. As a result, the value of the portfolio would be sufficient to meet the cash obligations, say, of the insurance company.

Example 2.24

Let us consider a simple situation where the portfolio manager has to meet one single cash obligation of ₹ 10,00,000 after 2 years. Since, there is only one cash outflow (without any interim outflow), the 'duration' of this cash outflow is 2 [just like duration of a zero-coupon bond as shown in formula (2.9)]

Now, to meet that obligation, the portfolio manager is considering the following information to prepare the investment plan :

	Bond : 1	Bond : 2
(a) Face Value :	₹ 1000	₹ 1000
(b) Coupon rate :	8%	7%
(c) Maturity :	3 years	1 year
(d) Redemption :	At par	At par
(e) Present Value (at 10% YTM) :	₹ 950·25	972·73
(f) Duration :	2·78	1

Now, in this investment plan, if the portfolio manager allocates all of its fund in purchasing Bond -2 with a plan to reinvest the proceeds after 1 year to purchase another bond of 1 year maturity and in this way meet the cash obligation after 2 years, then that process would entail a reinvestment risk. This is because the interest rate may fall in the mean time and the proceeds, when reinvested, may not fetch same return as before. On the other hand, the portfolio manager may plan to put all of its fund in purchasing Bond -1. But this investment plan also entails an interest rate risk because these bonds (of 3 years' maturity) will have to be sold after 2 years to meet the cash obligations. However, the yield rate may rise in the mean time, i.e., the bond price will fall and, therefore, the sales proceeds may not be sufficient to meet the cash obligations (by selling those bonds at a lower price).

Hence, the portfolio manager, to overcome this problem, may plan to invest a portion of the portfolio-fund in purchasing Bond -1 and another portion in Bond - 2.

Now, **the immunisation technique suggests the following process** to find out what proportion of portfolio fund to be invested in Bond -1 and what proportion in Bond - 2 :

$w_1 + w_2 = 1$ ..... (i)

$(w_1 \times 2.78) + (w_2 \times 1) = 2$ .... (ii)

Here, w_1 and w_2 suggest the proportions (or weights) of the portfolio's fund to be invested in Bond - 1 and Bond - 2 respectively. Equation (i) shows that the sum of these weights (or proportions) should be equal to 1.

Equation (ii) shows that the weighted average of the durations of the bonds in the portfolio must be equal to the duration of the cash obligation (after 2 years). Thus, we have two unknowns (viz., w_1 and w_2) and two equations.

Hence, the required values of w_1 and w_2 can be found out by solving these two equations.

From (i) we get $w_2 = 1 - w_1$

Now, substituting this value of w_1 in equation (ii) we get :

$[w_1 \times 2.78] + [(1 - w_1) \times 1] = 2$

or, $2.78\, w_1 - w_1 + 1 = 2$

or, $w_1 (2.78 - 1) = 1$

or, $w_1 = \frac{1}{1.78} = 0.5618$

$\therefore$ $w_2 = 1 - w_1 = 1 - 0.5618 = 0.4382$

Thus, the portfolio manager should invest 56·18% of the portfolio fund in Bond-1 and 43·82% of the fund in Bond-2.

Now, the present value of the cash obligation at YTM = 10% would be $\frac{10,00,000}{(1+0.10)^2}$ = ₹ 8,26,446. Thus, the portfolio manager will need this sum in order to purchase Bond-1 and Bond-2 at their desired proportions for immunising the portfolio.

Therefore, investment in Bond-1

$= 0.5618 \times ₹8,26,446$

$= ₹4,64,297$

and investment in Bond-2

$= 0.4382 \times ₹8,26,446$

$= ₹3,62,149$

Since, the current price of Bond-1 = ₹950.25

So, the number of Bond-1 to be purchased will $\frac{4,64,297}{950.25} = 488.6 \approx 489$

Further, as the current market price of Bond-2 = ₹972.73, so the number of Bond-2 to be pu[rchased]

will be $\frac{3,62,149}{972.73} = 372.3 \approx 372$

Now, how this immunisation process can help the portfolio manager to meet the future cash obl[igation] of ₹10,00,000 with its bond portfolio can be shown with the help of the following table.

Table - 2.20

An immunised portfolio to meet a cash obligation after 2 years

	YTM at the end of 1 year		
	9%	10%	11%
(1) Reinvestment of proceeds of Bond-2 and its value at time period t = 2 [₹1,070 × 372.3 × (1 + YTM)]	4,34,233	4,38,197	4,42,[illegible]
(2) Reinvestment of coupons received at t = 1 from Bond-1 [₹80 × 488.6 × (1 + YTM)]	42,606	42,997	43,[illegible]
(3) Value of coupons received at t = 2 from Bond-1 [₹80 × 488.6]	39,088	39,088	39,[illegible]
(4) Sales proceeds from Bond-1 at t = 2 $\left[\frac{₹1080 \times 488.6}{(1+YTM)}\right]$	4,84,227	4,79,716	4,75,[illegible]
TOTAL VALUE OF PORTFOLIO at t = 2	₹10,00,024	₹9,99,998	₹10,00,[illegible]

This table shows that if YTM remains unchanged at 10% upto the second year when cash obliga[tion] has to be met, then total value of the bond portfolio would almost be equal to the value of the c[ash] obligation (viz., ₹10,00,000).

It is also observed that if the yield rate falls to 9% or rises to 11% before one year had passed a[nd] remained at that changed level then the value of the portfolio would be slightly more than the ca[sh] obligation.

2.4.1. Some problems with immunisation technique

In many cases the immunisation technique may fail to protect the bond portfolio from the adver[se] effects of interest rate changes. We can point out some of the weaknesses of this portfolio managem[ent] technique.

(i) **Call risk and default risk have been ignored :** The immunisation technique of bond portfolio management assumes that the bonds in a portfolio would not suffer from any default risk or call risk. However, if the issuer of any bond calls back the bond before maturity or fails to pay the coupons and the maturity value in due time then such call risk or default risk would create problems in immunising a portfolio.

(ii) **Yield rate may not remain same for bonds with different maturity :** The immunisation technique also assumes that the yield rate remains same for bonds of different maturity periods and if there is any change in yield rate then it becomes same for all such bonds. But in reality the yield rate may rise with the rise in maturity period. Further, it has been observed that the yield rates become more volatile in case of bonds with short maturities. In such cases, a portfolio may not be immunised.

(iii) **Change in 'duration' over time may make this technique ineffective :** With the passage of time the 'duration' of bonds in the portfolio as well as the 'duration' of the cash obligation are expected to change. Further with the change in yield rates, the duration will change at different rates. As a result, it becomes difficult to immunise a portfolio. In this situation, the portfolio manager may have to rebalance the portfolio, i.e., some of the bonds currently held by the investor may have to be sold and they are to be replaced by other bonds so as to match the duration of the portfolio with the duration of the cash obligation.

2.5. Term Structure of Interest Rates

The term structure of interest rate implies that the interest rate charged for lending money or the interest rate paid for borrowing money would depend on the length of time for which the money is being lent out or borrowed. For example, when an investor is willing to keep a deposit with a bank then the interest income or the return that he expects from the principal amount depends upon the time for which he invests the money, say, 8% interest rate p.a. for 1-year term deposit, 9% interest rate p.a. for 2-year term deposit, 10% interest rate p.a. for 3-year term deposit etc.

When we calculate the present value of a 3-year bond having a face value of ₹1,000, carrying a coupon rate of 8% payable annually, and redeemable at par, we normally use the current market expectation of, say, 10% return p.a. as discount rate. Thus, we get the following present value (PV) of this bond :

$$PV = \frac{80}{(1+0.10)} + \frac{80}{(1+0.10)^2} + \frac{1080}{(1+0.10)^3}$$

$$= 72.73 + 66.12 + 811.42$$

$$= ₹950.27$$

However, instead of using the uniform discount rate, we can use different expected returns as discount rates for cash-flows at different points of time. In that case, we get the following result :

$$PV = \frac{80}{(1+0.08)} + \frac{80}{(1+0.09)^2} + \frac{1180}{(1+0.1)^3}$$

$$= 74.07 + 67.33 + 811.42$$

$$= ₹952.82$$

Thus, when we estimate the present value or the current bond price at ₹950.27 then the single discount rate or yield rate of 10% would reflect an average yield rate or return over a period of 3 years. However, when we emphasise on the term structure of the yield rate, then the present price of the 3-year bond becomes higher.

The yield structure according to the term of investment can be referred to as the term structure of interest rates. The variation of yield rates across bonds can be explained to some extent by the

differences in the quality of the bonds in question. The quality of bond is often judged by the rating for those bonds. A bond with higher credit rating (say, AAA⁺) is likely to be sold at a higher price and hence it will have lower yield rate. On the other hand, a bond with identical promised cash-flow may have a lower credit rating (say, B⁺) and, therefore, this bond may be sold at a lower price at present, and, hence, this bond will have a higher yield rate.

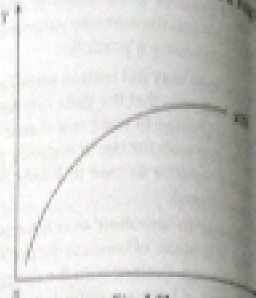
Fig.-2.11

In a similar way, the maturity period of bonds can also explain the differences in their yield rates. Generally a bond with longer maturity period tends to offer higher yield rate compared to a bond with short maturity period. Hence, yield rate (Y) becomes a function of the maturity period (t), i.e., $y = f(t)$ where $f'(t) > 0$ i.e., as t rises, Y also rises and vice versa. This functional relation is depicted by a yield curve.

Thus, we get an upward sloping yield curve (Fig.-2.11). However, the theory related to the term structure of interest rates suggests that the yield curve may take some other shape also (i.e., it can be either downward sloping or horizontal) depending upon the market conditions and the expectations of the investors. If bonds with longer maturity have lower yield rates then we get an inverted yield curve (Fig.-2.12). In this case, the bonds with short maturity give more return and the investors believe that such increase in yield rate during short run is a temporary phenomenon, and in near future the yield rate will fall.

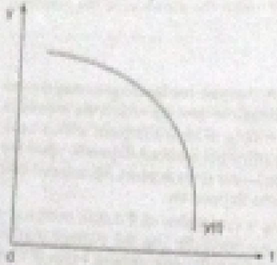
Fig.-2.12

For a proper understanding of the theory of the term structure of interest rates, we are to be acquainted with the notions of spot rate and forward interest rate.

2.5.1. Spot Rate of Interest

A spot rate of interest can be defined as the basic interest rate p.a. charged on any amount of money held during any particular period. It is measured at any given point of time as the yield to maturity (YTM) on a pure discount bond [or a zero coupon bond (ZCB)] and can be considered as the interest rate associated with a spot contract.

In order to eliminate the influence of default risk, it is better to consider a government treasury bill for this purpose. Since the ZCB promises to pay a fixed amount at a fixed date in future, so the ratio of redemption value to the current price of the bond would imply the spot rate for the maturity date. Such a spot contract involves immediate lending of an amount of money from the lender to the borrower. This loan amount along with the agreed interest rate (viz, the spot rate) has to be repaid by the borrower at a specific time in future. Thus, if an amount 'A' is lent out for 1 year, and the spot rate is s_1 then after one year, $A(1 + s_1)$ amount has to be repaid by the borrower. Similarly, s_2 would denote 2-year spot rate (i.e., an interest of s_2 p.a. for 2 years) on lending an amount 'A' for which the borrower would repay $A(1 + s_2)^2$ after 2 years. Hence, for the lender, the money lent out for 2 years grows by a factor of $(1 + s_2)^2$. It is to be noted that the definition of spot rate implicitly assumes a compound interest rate. However, this compounding can be yearly or bi-annually or quarterly etc.

- Yearly Compounding : In this case, the money lent out at a spot rate of s_t grows by a factor $(1 + s_t)^t$.
- *Multi-period compounding in a year* : If the compounding takes place for *m* periods within a year then the money lent out at the spot rate s_t grows by a factor $\left(1+\frac{s_t}{m}\right)^{mt}$. Hence, for bi-annual compounding, $m = 2$; for quarterly compounding, $m = 4$ etc.
- **Continuous compounding in a year** : If the compounding is done continuously within a year then the money lent out at the spot rate s_t grows by a factor $e^{s_t t}$.

However, among all these compounding processes, the yearly compounding appears to be more convenient.

For Zero Coupon Bonds (ZCB), the yield rate and the spot rate of interest are analogous. We can show that for such ZCBs, the spot rate normally rises with the maturity period of the bond. We can also show that the spot rate with multi-period compounding (say, $m = 2$) and yearly compounding ($m = 1$) would be different.

Let us consider a ZCB with a face value of ₹ 100. The current prices of such ZCBs with different maturity periods with associated spot rates are shown below :

Table - 2.11

Current price of the bond (₹)	Maturity period (Year)	Spot rate (%) $m = 2$	Spot rate (%) $m = 1$
98·50	1	1·62	1·62
94·10	3	2·04	2·04
84·00	5	3·52	3·54
58·20	10	5·49	5·56
38·50	15	6·46	6·57
16·25	25	7·40	7·54

Here, given the current price of the ZCB, the spot rate has been estimated using the following formula :

$$P = \frac{F}{\left(1+\frac{YTM}{m}\right)^{mt}} = \frac{F}{\left(1+\frac{S_t}{m}\right)^{mt}} \quad \text{......... (2.20)}$$

Where P = Current bond price

F = Face value of the bond

YTM = Yield to maturity

= S_t = Spot rate at the time period t

m = frequency of compounding in a year

t = maturity period.

For instance, in case 15 year ZCB,

$P = 38·50$, $F = 100$, $t = 15$; if $m = 2$

then the S_1 will be estimated as :

$$38.50 = \frac{100}{\left(1+\frac{S_1}{2}\right)^{2\times 15}}$$

or, $\left(1+\frac{S_1}{2}\right)^{30} = \frac{100}{38.50}$

or, $\left(1+\frac{S_1}{2}\right) = \left(\frac{100}{38.50}\right)^{\frac{1}{30}}$

or, $S_1 = \left[\left(\frac{100}{38.50}\right)^{\frac{1}{30}} - 1\right] \times 2$

$= 0.06459 \approx 6.46\%$

However, other things remaining the same, if $m = 1$ then the spot rate in this case will be as follows :

$$38.50 = \frac{100}{(1+S_1)^{15}}$$

or, $(1+S_1)^{15} = \frac{100}{38.50}$

or, $S_1 = \left(\frac{100}{38.50}\right)^{\frac{1}{15}} - 1$

$= 0.0657$

$\approx 6.57\%$

This example reveals that we must have a discount factor to estimate the spot rate. It is also observed that for ZCB, the yield rate and spot rate schedule will be synonymous. Here, the yield curve or the spot curve will be upward sloping (as already shown in Fig.-2.11).

2.5.2. Discount factor

Our previous discussion clearly indicates that when the spot rate has to be estimated then the corresponding discount factor (d_t) for each time period has to be determined so that the present value of the future cash-flows from a bond can be estimated.

In case yearly compounding, this discount factor would be $d_t = \frac{1}{(1+S_t)^t}$, and for multi-period compounding, it will be $d_t = \frac{1}{\left(1+\frac{S_t}{m}\right)^{mt}}$. However, in case of continuous compounding, $d_t = e^{-s_t t}$.

For any given cash-flow stream $x_0, x_1, x_2, \ldots, x_n$, the present value ($PV$) will be $PV = x_0 + d_1x_1 + d_2x_2 + \ldots + d_nx_n$

Here, $d_0 = \frac{1}{(1+s_0)^0} = 1$.

i.e., $d_t x_t = x_t$

Here, d_t resembles the price for cash received at time period t. Thus, the present value signifies the aggregate value of multiplied by the quantity (of cash-flow) for different points of time.

2.5.3. Method of 'bootstrapping' for estimating spot rates

In deriving the spot curve or yield curve, the problem that we often face is the inadequacy of zero coupon bonds (ZCBs) of different maturity dates. In such case, the spot rate curve can be determined by following a method called **'bootstrapping'**.

It can be explained as follows :

Let us assume that only a zero coupon 1-year government Treasury Bill is available. However, the 2-year bond with a current price 'P' and a face value 'F' is available; it gives a coupon rate 'C' at the end of 1st and 2nd years. So, in this case, we get a relationship :

$$P = \frac{C}{(1+s_1)} + \frac{C+F}{(1+s_2)^2} \quad \ldots\ldots (2.21)$$

where s_1 = 1-year spot rate (which is known)

s_2 = 2-year spot rate

So, we have only one unknown variable, viz., s_2 in this equation. Hence, we can solve this equation for s_2. However, in such cases, the yield rate (YTM) and the spot rate would be different.

Example 2.25

Let us consider the following three coupon bearing bonds :

Table - 2.12

	Type	Current Bond Price (₹)	YTM
Bond : A	1-year 10% bond (Face Value : ₹ 1000)	1018.50	8%
Bond : B	2-year 12% bond (Face Value : ₹ 1000)	1062.50	8.47%
Bond : C	3-year 11% bond (Face Value : ₹ 1000)	1052.40	8.93%

The yield rate (YTM) of the 1-year bond would imply the 1-year spot rate (i.e., s_1). Now, let us consider bond-B (a 2-year bond) and see how s_2 can be estimated given s_1.

Here $P = 1062.50$, $s_1 = 8\%$, $C = ₹\ 120$.

$\therefore\ 1062.50 = \frac{120}{(1+0.08)} + \frac{1120}{(1+s_2)^2}$

or, $1062.50 - 111.11 = \frac{1120}{(1+s_2)^2}$

or, $(1+s_2)^2 = \frac{1120}{951.39} = 1.1772$

or, $1+s_2 = \sqrt{1.1772} = 1.0852$

or, $s_2 = 1.0852 - 1 \quad = 0.0852$

$= 8.52\%$

Then we can consider the bond : C (3-year 11% bond with face value ₹ 1000) to find the 3-year spot rate (i.e., s_3).

Here P = ₹ 1052.40, r_1 = 8%, r_2 = 8.52%, C = ₹ 110.

$$\therefore \quad 1052.40 = \frac{110}{(1+0.08)} + \frac{110}{(1+0.0852)^2} + \frac{1110}{(1+r_3)^3}$$

$$= 101.85 + 93.41 + \frac{1110}{(1+r_3)^3}$$

or, $$1052.40 - 195.26 = \frac{1110}{(1+r_3)^3}$$

or, $$857.14 = \frac{1110}{(1+r_3)^3}$$

or, $$(1+r_3)^3 = \frac{1110}{857.14}$$

or, $$(1+r_3) = \left(\frac{1110}{857.14}\right)^{\frac{1}{3}} = 1.08999$$

$$= 1.09$$

or, $r_3 = 1.09 - 1 = 0.09 = 9\%$

In this way, we can calculate the spot rates even from the information regarding coupon bearing bonds.

Similarly, two coupon bearing bonds with identical maturity dates and different coupon rates can be used to prepare the equivalent of a zero coupon bond.

Example 2.26

Let us consider the following two coupon bearing bounds :

	Current Price
Bond-A : 10% 10-year bond (Face Value : ₹ 100)	₹ 98.72
Bond-B : 8% 10-year bond (Face Value : ₹ 100)	₹ 85.89

Let us consider a bond portfolio with 1 unit of bond-B and (–) 0.8 unit of bond-A. Thus, the face value of the portfolio will be

₹ 100 – (0.8 × 100) = ₹ 20

and the price of the portfolio will be

$$P = P_B - (0.8 \times P_A)$$
$$= 85.89 - (0.8 \times 98.72)$$
$$= 85.89 - 78.976$$
$$= ₹\ 6.91$$

The coupon payment for this portfolio will be (–) 0.8 of 10% of ₹ 100 of bond A = – 8.00

(+) 8% of ₹ 100 of bond B = + 8.00

= 0

So, it becomes a zero-coupon portfolio.

For this portfolio, current price (P) = ₹ 6.91, F = ₹ 20, and therefore 10-year spot rate must satisfy the relationship $(1 + r_{10})^{10}$. P = 20

or, $(1+r_{10})^{10} \times 6.91 = 20$

or, $$r_{10} = \left(\frac{20}{6.91}\right)^{\frac{1}{10}} - 1 = 0.1121$$

$$r_{10} = 11.21\%$$

2.5.4. Forward rates

Forward interest rates denote the interest rates to be paid by the borrower to the lender on the amount borrowed in between two time periods in future based on the terms agreed upon at present.

We can explain it with the help of a simple example. Suppose that the spot interest rates r_1 and r_2 are known, and if ₹ 1 is invested for 2 years then it would grow by a factor $(1 + r_2)^2$ at the end of 2 years. This can be treated as maturity strategy or direct investment strategy. Alternatively one may invest ₹ 1 for 1 year at the spot rate r_1 and then reinvest the maturity value in the next year at the then prevailing rate (denoted by $f_{1,2}$, i.e., forward rate between year 1 and 2 or, the interest rate on money lent out in period 1 and receipt of principal alongwith interest in period-2). This strategy can be considered as the roll-over investment strategy.

In case of direct investment strategy, we have

₹ $1(1 + r_2)^2$ after 2 years.

However, in case of roll-over strategy,

We have ₹ $(1 + r_1)(1 + f_{1,2})$ after 2 years.

Under conditions of perfect certainty and knowledgeable investors, the direct and roll-over strategies must provide same outcome.

$$\therefore \quad (1+r_2)^2 = (1+r_1)(1+f_{1,2})$$

or, $$(1+f_{1,2}) = \frac{(1+r_2)^2}{(1+r_1)}$$

or, $$f_{1,2} = \frac{(1+r_2)^2}{(1+r_1)} - 1 \quad \text{....... (2.22)}$$

Example 2.27

Consider the following spot rates and calculate the forward rate ($f_{1,2}$).

Let r_1 = 9% and r_2 = 10%

So, the direct strategy would result in

$(1 + r_2)^2 = (1 + 0.1)^2 = 1.21$

However, in case of roll-over strategy we get

$(1 + 0.09)(1 + f_{1,2}) = 1.09(1 + f_{1,2})$

So, under conditions of perfect information we must have

$(1 + 0.10)^2 = (1 + 0.09)(1 + f_{1,2})$

or, $1.21 = 1.09(1 + f_{1,2})$

or, $$f_{1,2} = \frac{1.21}{1.09} - 1$$

$= 1.11 - 1 = 0.11$, i.e., 11%

Hence, the forward rate ($f_{1,2}$), in this case, is determined by two spot rates.

■ Generalised rule :

In our previous discussion, we were concerned with only two spot rates [as shown in formula (2.22)]. Now, it can be generalised. The forward rate between times t_1 and t_2 ($t_1 < t_2$) can be denoted by $f_{1,2}$ and it is the interest rate charged for borrowing money at time t_1 which has to be repaid with interest at time period t_2. Normally, such forward rates are expressed on an annualised basis. From theoretical view point, the forward rate is based on the underlying spot rates (here s_1 and s_2). Hence, the calculations of forward rates are often termed as *'implied forward rate'* as opposed to *market forward rate*. This implied forward rate between the time period i and j ($i < j$) with yearly compounding of interest rate is expressed as :

$$(1+s_j)^j = (1+s_i)^i (1+f_{i,j})^{j-i}$$

$$\text{or,}\quad f_{i,j} = \left[\frac{(1+s_j)^j}{(1+s_i)^i}\right]^{\frac{1}{j-i}} - 1 \quad \text{......... (2.23)}$$

If there is multi-period compounding within a year (with frequency of compounding p.a. = m) then (2.23) will be expressed as :

$$\left(1+\frac{s_j}{m}\right)^{mj} = \left(1+\frac{s_i}{m}\right)^{mi}\left(1+\frac{f_{i,j}}{m}\right)^{m(j-i)}$$

$$\text{or,}\quad \left(1+\frac{s_j}{m}\right)^{j} = \left(1+\frac{s_i}{m}\right)^{i}\left(1+\frac{f_{i,j}}{m}\right)^{(j-i)}$$

$$\text{or,}\quad \left(1+\frac{f_{i,j}}{m}\right)^{(j-i)} = \frac{\left(1+\frac{s_j}{m}\right)^{j}}{\left(1+\frac{s_i}{m}\right)^{i}}$$

$$\text{or,}\quad \left(1+\frac{f_{i,j}}{m}\right) = \left[\frac{\left(1+\frac{s_j}{m}\right)^{j}}{\left(1+\frac{s_i}{m}\right)^{i}}\right]^{\frac{1}{(j-i)}}$$

$$\text{or,}\quad \frac{m+f_{i,j}}{m} = \left[\frac{\left(1+\frac{s_j}{m}\right)^{j}}{\left(1+\frac{s_i}{m}\right)^{i}}\right]^{\frac{1}{(j-i)}}$$

$$\text{or,}\quad f_{i,j} = m\left[\frac{\left(1+\frac{s_j}{m}\right)^{j}}{\left(1+\frac{s_i}{m}\right)^{i}}\right]^{\frac{1}{(j-i)}} - m \quad \text{......... (2.24)}$$

• *Role of arbitrage :*

The equality $(1+s_2)^2 = (1+s_1)(1+f_{1,2})$ that we observe in (2.22) can be justified with the help of arbitrage principle.

Here, if $(1+s_1)(1+f_{1,2}) > (1+s_2)^2$

Then the roll-over strategy of investment gives more return compared to direct investment strategy. So, an arbitrageur can borrow for 2 years and lend the same by following the roll-over strategy. Hence, the profit earned by an arbitrageur would be

$$(1+s_1)(1+f_{1,2}) - (1+s_2)^2 > 0$$

The capital investment made by the arbitrageur would be nil since his original borrowed amount would be recovered alongwith some profit. If this process continues then greater borrowing pressure would raise s_2 and gradually the positive gap between $(1+s_1)(1+f_{1,2})$ and $(1+s_2)^2$ will be eliminated. Similarly, if $(1+s_1)(1+f_{1,2}) - (1+s_2)^2 < 0$ then the arbitrageur will reverse his investment strategy and ultimately the equality in return from direct strategy and roll-over strategy will be ensured.

However, this arbitrage process assumes that there *remains no transaction cost* (say, in the form of brokerage fees) in this investment, and the lending and borrowing rates are equal.

2.6. Theories related to Term Structure

Several theories have been developed to explain the term structure of interest rates. However, four primary theories are used to explain the term structure.

These are as follows :

(a) The expectation theory or the unbiased expectation theory;

(b) The liquidity premium theory;

(c) The market segmentation theory; and

(d) The preferred habitat theory.

2.6.1. The expectation theory

This theory explains the differences in interest rates in the financial market on the basis of the expectations or the unbiased expectations of the investors regarding the future spot rate.

Let us consider an investor with ₹ 1 who plans to invest the sum for 2 years. Let us also consider our example No. 2.25

where 1-year spot rate (s_1) = 8%

2-year spot rate (s_2) = 8·52%

Now, the investor can follow a **'maturity strategy'** or **'direct strategy'** and can invest the sum for full two years at 2-year spot rate s_2 = 8·52%. Thus, with this investment strategy, he will have at the end of two years a value equal to ₹ 1·1776 = (1 × 1·0852 × 1·0852)

= $(1 + 0·0852)^2$

Alternatively, he can now invest ₹ 1 for one year at 1-year spot rate s_1 = 8% and receives ₹ 1·08 after one year, and reinvest that sum for another one year. But the investor **does not know what would be the 1-year spot rate after 1 year from now.** The investor has an **'expectation'** about that future 1-year spot rate (which we can be denoted by $s^{e}_{1,2}$).

Let us assume that $s^{e}_{1,2}$ = 10%. As a result, he is expected to receive a sum of ₹ 1·188 = (1 × 1·08 × 1·10) after 2 years from now. This is known as **'roll-over strategy'**.

Here, we find that the roll-over investment strategy generates greater expected value compared to the value generated by following the maturity or direct investment strategy, i.e., ₹ 1·188 > ₹ 1·1779.

But $s^e_{1,2}$ = 10% cannot represent the general view of the market sentiment, because if it is true then most of the investors would opt for roll-over investment strategy. So, demand for 1-year bond would rise, and that of 2-year bond will fall. As a result, to attract the supply of funds to 2-year bonds, 2-year spot rate (s_2) would rise. Similarly, as the supply of funds to 1-year bond becomes more than its demand, 1-year spot rate will fall.

Hence, s_1 = 8%, s_2 = 8·52% and $s^e_{1,2}$ = 10% cannot represent an equilibrium situation.

Now, we can think of another situation where s_1 = 8%, s_2 = 8·52% and $s^e_{1,2}$ = 6%. In this case, the roll-over strategy of investment would generate a value equal to (1 × 1·08 × 1·06) = ₹ 1·1448 < ₹ 1·1779.

Hence, in this case, the investor will prefer maturity strategy.

But this cannot also ensure an equilibrium because greater demand for 2-year bond would lead to a rise in 2-year bond price, i.e., a fall in 2-year yield rate or spot rate. On the other hand, a slack in the demand for 1-year bond would cause a decline in 1-year bond price, i.e., an increase in 1-year yield rate or spot rate.

Now, based on s_1 = 8% and s_2 = 8·52% the forward rate ($f_{1,2}$) would be [based on formula 2.22]:

$$f_{1,2} = \frac{(1+s_2)^2}{(1+s_1)} - 1$$

$$= \frac{(1\cdot0852)^2}{(1\cdot08)} - 1$$

$$= 1\cdot0904 - 1$$

$$= 0\cdot0904 \text{ or } 9\cdot04\%$$

Now, if $f_{1,2} = s^e_{1,2}$, then the roll-over investment strategy would result in a value equal to

(1 × 1·08 × 1·0904) = ₹ 1·1779

Which is just equal to the value received in 'maturity strategy'. Thus, the expectation theory or the **unbiased expectation theory suggests that at equilibrium** $s^e_{1,2} = f_{1,2}$ ...... (2.25)

and it implies that [substituting this value of $f_{1,2}$ in formula 2.22] at equilibrium we have:

$$(1+s_2)^2 = (1+s_1)\left(1+s^e_{1,2}\right) \text{ ...... (2.26)}$$

Following our previous example, it can also be said that an investor who follows a maturity strategy with one-year holding period (i.e., who want to hold a bond for 1 year) can receive a value ₹ 1·08 (= 1 × 1·08) by investing ₹ 1 in 1-year bond.

Alternatively, he can follow a 'naive strategy' (investor having lack of wisdom) where he can purchase a 2-year bond at present and sell it after 1-year. In that case, the expected selling price would be ₹ 1·08 $\left[= \frac{₹\ 1\cdot1779}{(1+0\cdot0904)}\right]$, i.e., the present value of 2-year bond has been derived by discounting it at the expected spot rate $s^e_{1,2}$ = 9·04%. Thus, both maturity and naive strategies result in same expected return. Therefore, the investor would have no incentive to choose one strategy over the other.

Thus, the unbiased expectation theory suggests that the expected interest rates are based on the current spot rates. In general, the current spot rate schedule leads to a set of forward rates such as $f_{1,2}, f_{1,3}$ ...... $f_{1,n}$ and these forward rates define the expected spot rates $s^e_{1,2}, s^e_{1,3}$ ...... $s^e_{1,n-1}$ for the next year.

We know that

$f_{1,2}$ = Forward rate for money borrowed for 1 year, a year from now.

$f_{1,3}$ = Forward rate for money borrowed for 2 years, starting from the next year.

$f_{1,n}$ = Forward rate for money borrowed for 'n' years, starting from the next year.

In a similar way, we can say that the forward rates $f_{1,2}, f_{2,3}$ ...... $f_{n-2,n-1}, f_{n-1,n}$ would define the expected spot rates $s^e_{1,2}, s^e_{2,3}$ ...... $s^e_{n-1,n}$. Thus, the investor can either invest a sum of ₹ P for purchasing bond with longer maturity and receive $P(1+s_n)^n$ after 'n' period, or invest that sum following a roll-over strategy to get

$$P(1+s_1)\left(1+s^e_{1,2}\right)\left(1+s^e_{2,3}\right)\left(1+s^e_{3,4}\right) \text{......} \left(1+s^e_{n-1,n}\right) \text{ after 'n' period.}$$

The unbiased expectation theory suggests that

$$P(1+s_n)^n = P(1+s_1)\left(1+s^e_{1,2}\right)\left(1+s^e_{2,3}\right) \text{......} \left(1+s^e_{n-1,n}\right)$$

or $$(1+s_n)^n = (1+s_1)\left(1+s^e_{1,2}\right)\left(1+s^e_{2,3}\right) \text{......} \left(1+s^e_{n-1,n}\right)$$

or $$(1+s_n) = \left[(1+s_1)\left(1+s^e_{1,2}\right)\left(1+s^e_{2,3}\right) \text{......} \left(1+s^e_{n-1,n}\right)\right]^{\frac{1}{n}}$$

or $$s_n = \sqrt[n]{(1+s_1)\left(1+s^e_{1,2}\right)\left(1+s^e_{2,3}\right) \text{......} \left(1+s^e_{n-1,n}\right)} - 1 \text{ ...... (2.27)}$$

Thus the current interest rate on long-term bonds (s_n) is an unbiased average or geometric mean of the current spot rate (interest rate) and the future expected spot rates of 1-year bonds.

If the investors expect that $s^e_{n-1,n} > s^e_{n-2,n-1} > \text{......} s^e_{2,3} > s^e_{1,2}$ then spot rate schedule or the yield curve must be upward sloping (as shown in Fig.-2.11), and in that case $s_n > s_1$.

Now the question is why do investors often expect the spot rates to increase in future? One possible answer to this question is that during the expansionary phase of business cycle, the investors want to keep their real return intact in an inflationary environment. Hence, if the inflation rate is, say, 4% during a year and the nominal interest rate is 8% p.a., then the real interest rate will be approximately 4% p.a. Now, if inflation rate rises from 4% to 5% then the investors also expect an increase in the nominal interest rate from 8% to 9% to keep their real interest income intact.

Similarly, during the recessionary phase of the business cycle, the investors expect a gradual fall in the future expected spot rates, and that case

$$s^e_{n-1,n} < s^e_{n-2,n-1} < \text{......} < s^e_{2,3} < s^e_{1,2}$$

So, in that case, the spot rate curve or the yield curve must be downward sloping (as shown in Fig.-2.12). However, if the investors expect no change in the future spot rates then

$$s_1 = s_{1,2}^e = s_{2,3}^e = \cdots\cdots = s_{n-1,n}^e$$

and therefore, in this case, we have

$$s_n = \sqrt[n]{(1+s_1)^n} - 1$$

$$= \left[(1+s_1)^n\right]^{\frac{1}{n}} - 1$$

$$= 1 + s_1 - 1$$

or, $s_n = s_1$ .... (2.28)

In this case, the spot curve or the yield curve will be horizontal (Fig.-2.13).

However, the unbiased expectation theory normally suggests an increasing trend in future spot rates with an upward sloping yield curve.

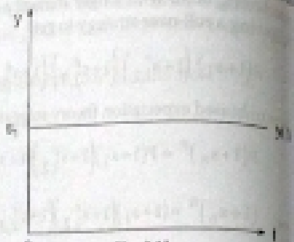

Fig.-2.13

This expectation theory is based on the following assumptions :

(a) The financial markets are perfectly competitive ;

(b) The investors are rational, i.e., they want to maximise the yield on their holding period (i.e., the period for which they want to hold the security) ;

(c) The investors have a perfect foresight and they have uniform expectations regarding future changes in spot rates and security prices ;

(d) The lending and borrowing rates are same ;

(e) There remains no transaction cost in holding or trading in securities ;

(f) The securities of different maturities are perfect substitutes for each other.

- **Weaknesses of the expectation theory :**

The unbiased expectation theory also suffers from some drawbacks.

(a) This theory is based upon some restrictive assumptions most of which have no practical relevance.

(b) This theory suggests that if the spot rate curve is upward sloping then the market expects that the interest rates will rise in future. But in many cases, such expectations have been proved to be wrong.

(c) This theory cannot also explain the term structure of interest rates in the informal or unorganised segment of the financial market.

(d) This theory also ignores the risks and uncertainty involved in investment.

2.6.2. Liquidity Premium Theory

The liquidity premium theory of interest rate structure, as developed by J. R. Hicks, is based upon the preference pattern of the investors regarding short-term and long-term securities. This theory asserts that the investors usually prefer short-term fixed-income securities over long-term securities. It is assumed that the risk of holding long-term securities, which is often measured in terms of the variance of return on capital, is comparatively more than that of short-term securities.

The intuitive logic is that the investors prefer their funds to be liquid rather than tied up or blocked for longer period. It is argued that the investors may require funds (or cash) before the maturity date of the security held. Thus, if any investor holds long-term bonds and needs to sell it before maturity date, then that would entail more 'price risk' or 'interest rate risk' since the bond price may fall substantially at the time the investor wants to sell it.

Hence, the borrower of the long-term fund or the issuer of long-term bonds must offer some premium (in the form of greater expected return) to the investors to induce them for buying long-term securities. Thus, while the unbiased expectation theory suggests the equality between the forward rate ($f_{1,2}$) and expected spot rate in the next period $\left(s_{1,2}^e\right)$ [as shown in formula (2.25)], the liquidity premium theory suggests that $f_{1,2} = s_{1,2}^e + L_{1,2}$......... (2.29)

where $L_{1,2}$ = Liquidity premium for the period starting one year from now and ending two years from now.

$$\therefore L_{1,2} = f_{1,2} - s_{1,2}^e$$

Thus, the difference between forward rate and expected future spot rate is considered as liquidity premium. Therefore, for a 2-year investment period we should have [following the formula (2.26)] :

$$(1+s_2)^2 = (1+s_1)\left(1+s_{1,2}^e + L_{1,2}\right) \text{......... (2.30)}$$

Here, $L_{1,2} > 0$ and therefore, according to liquidity premium theory

$$(1+s_2)^2 > (1+s_1)\left(1+s_{1,2}^e\right) \text{......... (2.31)}$$

This result implies that higher is the maturity period of a bond, higher should be the liquidity premium, and this explains the upward sloping yield curve or spot rate curve. In (2.31), it is assumed that $s_2 > s_1$. However, if we assume that $s_1 > s_2$ then the inequality as shown in (2.31) will hold only if the expected spot rate $s_{1,2}^e$ remains sufficiently lower than the current one-year spot rate (s_1). In this case, the yield curve will be downward sloping.

Example 2.28

Let the current 1-year spot rate be $s_1 = 8\%$, 2-year spot rate be $s_2 = 8{\cdot}52\%$ and the 1-year forward rate be (based on s_1 and s_2) $f_{1,2} = 9{\cdot}04\%$.

The unbiased expectation theory suggests that $f_{1,2} = s_{1,2}^e = 9{\cdot}04\%$ but the liquidity premium theory suggests that $f_{1,2} > s_{1,2}^e$ when $s_2 > s_1$. The gap $\left(f_{1,2} - s_{1,2}^e\right)$ will indicate the liquidity premium.

Hence, $s_{1,2}^e$ should be less then 9·04%, say, $s_{1,2}^e = 8{\cdot}6\%$. It implies that 9·04 – 8·6 = 0·44% = liquidity premium.

So, $(1+s_2)^2 = (1+0{\cdot}0852)^2 =$ ₹ 1·1776

and $(1+s_1)\left(1+s_{1,2}^e\right) = (1+0{\cdot}08)(1+0{\cdot}086) =$ ₹ 1·1729.

That is to say

$$(1+s_2)^2 > (1+s_1)\left(1+s^e_{1,2}\right)$$

or, ₹1·1776 > ₹1·1729

Here, higher return in case of investment in long-term (2-year bond) security can be attributed as reward for taking greater price-risk.

However,

$$(1+s_2)^2 = (1+s_1)\left(1+s^e_{1,2}+L_{1,2}\right)$$

$$(1+0.0852)^2 = (1+0.08)(1+0.0904)$$

$$= ₹1.1776$$

$$[\because s^e_{1,2}+L_{1,2} = 8.6+0.44$$

$$= 9.04\%$$

$$= 0.0904]$$

Thus, the investors should be given a liquidity premium so as to induce them to undertake the long-term investment.

Example 2.29

Let us consider a case where $s_1 = 8\%$, $s_2 = 6\%$ and the 1-year forward rate (based on s_1 and s_2) $f_{1,2} = 4.04\%$.

(You can check that $f_{1,2} = \frac{(1+s_2)^2}{(1+s_1)} - 1$)

If the liquidity premium $L_{1,2} = 0.44\%$ then what should be the expected spot rate $s^e_{1,2}$?

Here, Since $s_1 > s_2$, so the liquidity premium theory suggests that the following inequality must hold [as shown in (2.31)]:

$$(1+s_2)^2 > (1+s_1)\left(1+s^e_{1,2}\right)$$

Thus, $s^e_{1,2}$ should be substantial lower than s_1. Now, based on s_1 and s_2,

$$f_{1,2} = \frac{(1+0.06)^2}{(1+0.08)} - 1$$

$$= \frac{1.1236}{1.08} - 1$$

$$= 0.0404 = 4.04\%$$

As the liquidity premium $L_{1,2} = 0.44\%$

$$\therefore s^e_{1,2} = f_{1,2} - L_{1,2} = 4.04 - 0.44 = 3.6\%$$

Hence, we have $(1+s_1)\left(1+s^e_{1,2}\right) = (1+0.08)(1+0.036) = ₹1.1189$

and $(1+s_2)^2 = (1+0.06)^2 = ₹1.1236$

₹1.1189 < ₹1.1236

Hence, the yield curve will be downward sloping since the 1-year spot rate is expected to decline in future.

Now, if we assume that $s_1 = s_2$ then the term structure becomes flat, i.e., the yield curve becomes horizontal (as shown in Fig. 2.13).

It is to be noted that in case of unbiased expectation theory such horizontal yield curve suggests that expected interest rates in the market remains unaltered. But according to the liquidity premium theory, despite $s_1 = s_2$, the desired inequality as shown in equation (2.31) would be maintained only if $s^e_{1,2}$ remains less than s_1.

Example 2.30

Let $s_1 = s_2 = 8\%$. If the liquidity premium is assumed as 0·44% then what should be expected spot rate $s^e_{1,2}$ according to the liquidity premium theory?

Here, the forward rate $f_{1,2}$ based on the two spot rates s_1 and s_2 would be equal to the spot rate because

$$f_{1,2} = \frac{(1+s_2)^2}{(1+s_1)} - 1 = \frac{(1+0.08)^2}{(1+0.08)} - 1$$

$$= \frac{1.1664}{1.08} - 1$$

$$= 1.08 - 1 = 0.08 \text{ or, } 8\%$$

Hence, to maintain the inequality

$$(1+s_2)^2 > (1+s_1)\left(1+s^e_{1,2}\right)$$

and $s^e_{1,2} < f_{1,2}$

If the liquidity premium is assumed to be 0·44% then

$$s^e_{1,2} = f_{1,2} - L_{1,2} = 0.08 - 0.0044$$

$$= 0.0756$$

$$= 7.56\%$$

$$\therefore (1+s_2)^2 = (1.08)^2 = ₹1.1664$$

$$\text{and } (1+s_1)\left(1+s^e_{1,2}\right) = (1.08)(1.0756)$$

$$= ₹1.1616$$

2.6.3. The Market segmentation theory

One of the important theories in determining the term structure of interest rates is the market segmentation theory. It was developed by **Culbertson.** This theory assumes that the investors are

risk-averters, i.e., they want to minimise the risk involved in purchasing securities of ... maturities.

This theory states that the capital market is divided into a number of segments. Broadly, there are two segments in the capital market:

(a) The short-term security market.

(b) The long-term security market.

Here, it is assumed that the short-term securities are imperfect substitutes of long-term ... The way to minimise risk is to match maturities with holding periods. Let us consider the investment decision of the Life Insurance Corporation of India (LIC). It collects insurance premium ... individuals with an average age of 25-30 years. It is expected that no payment obligation for LIC will arise within next 20 years (depending upon the average life expectancy at birth). So, LIC may invest in long-term securities with a maturity period of, say 20 years. Similarly, some other organisations or individuals can invest in short-term securities to meet short-term obligations.

In each segment of the capital market, we have both demand for and supply of securities. The short-term equilibrium interest rate is determined through the interactions between the supply and demand forces of short-term securities. Similarly, the long-term equilibrium interest rate is determined through the demand and supply forces in the long-term securities market (Fig.-2.14).

In Fig.-2.14, both the demand for and supply of long-term securities are found to be higher than those of short-term securities. However, the demand for long-term securities (D_L) is found to be much higher than the demand for short-term securities (D_S). So, despite a higher supply of the long-term securities the equilibrium rate of interest in long-term securities market (r_L) is found to be higher than that in the short-term securities market (r_S).

Now, joining the equilibrium points E and F, we can determine the yield curve. It is to be noted that the equilibrium interest on long-term securities may even be less than that on short-term securities depending on demand-supply situations.

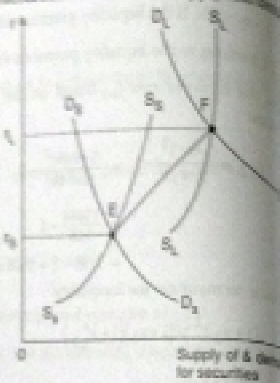

Fig.-2.14

Critical Estimation : This theory emphasises on the risk minimising behaviour of an investor. But in reality, the investors are not only guided by the motivation of risk aversion but also by the motivation of maximisation of return.

2.6.4. The Preferred Habitat Theory

This theory can be treated as the moderate and realistic version of the market segmentation theory. According to this theory the investors can be grouped according to their preferred areas of operation, i.e., a group of investors and borrowers might be risk-averters and hence, they want to deal with only the less risky segment of the securities market. For instance, these investors would be more inclined to invest their fund in either risk-free government bonds or in the corporate bonds issued by the reputed corporate houses. So their preferred habitat would be 'low-risk low-return bonds'. Similarly, there may be another group of investors who are 'risk-lovers' and their preferred habitat would be 'high-risk high-return bonds'. Hence, an investor would be ready to leave his preferred habitat of securities market only if the other segment of the securities market offer significantly high rate of interest on capital invested. Hence, the term structure of interest rates according to this theory would

be determined by higher risk premium with an expectation of rising spot rates in future. However, in this case, the increase in expected spot rate does not necessarily have a positive correlation with the maturity period of the security, rather it is guided by the requirement of extra yield rate to induce the investors and borrowers to shift from one preferred habitat to the other in the securities market.

Summary

A bond is a fixed income security. It creates an obligation on the part of the issuer of the bond to pay a given rate of interest on the face value of the bond to the bond holder. These bonds can be short-term or long-term bonds depending upon their maturity period. These bonds can be issued either at a premium or at a discount. The given interest rate per period offered by the issuer of the bond, viz., the borrower of fund, is called as coupon rate. At the time of maturity, the bond can be redeemed either at par or at a premium or at a discount with reference to its par value.

There can be various types of such bonds, e.g., the government Treasury Bills, dated government securities, floating rate bonds, zero coupon bonds, deep-discount bonds, indexed bonds, callable bonds etc. However, an annuity contract cannot be considered as a security since it cannot be traded in the security market. The discount rate (r) at which the present value (PV) of the future cash-flow stream generated from the bond is just equal to the current bond price (B_p) can be considered as the yield rate of the bond. This cash flow stream includes an initial cash outflow for the purchase of the bond, and the subsequent cash inflows in the form of coupon payments (C_k) and redemption value received at the time of maturity (R_n).

$$\therefore B_p = \sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{r}{m}\right)^k} + \frac{R_n}{\left(1+\frac{r}{m}\right)^n} \quad \text{where } k = m.t, (k = 1, 2, \_\_\_\_, n)$$

m = Number of times the coupon payment or the interest rate is paid in a year (i.e., if it is paid bi-annually then $m = 2$).

t = Maturity period of the bond.

C_k = Coupon payment at time period k.

R_n = Redemption value of the bond at time period 'n'.

r = discount rate.

B_p = Bond price = present value of the cash flow.

As the discount rate (r) rises, the bond price (B_p) falls, and vice versa. Thus, we get a downward sloping bond value curve.

This discount rate accommodates the expected return of the investor for undertaking various risks such as inflation risk, interest rate risk, call risk, default risk, etc. Normally, any investor expects at least a minimum return from investment which is equal to a risk-free rate (normally offered by government bonds).

$$\text{The current yield of a bond} = \frac{\text{Coupon rate} \times \text{Face Value}}{\text{Current bond price}}$$

and the Yield to maturity (YTM) denotes the rate of return which the investor earns by holding the bond till its maturity. It reflects the interest rate or discount rate at which the present value of the future cash-flow stream generated by the bond becomes just equal to the current bond price.

In case of a zero coupon bond, the bond generates only one cash-flow at the time of its redemption period. Such bonds are normally issued at a discount.

If the bondholder sells the bond before its maturity date then the yield rate is called as the **realised yield**.

There are five theorems related to bond pricing.

Theorem-1 : If bond price rises the yield rate will fall, and vice versa.

Theorem-2 : The size of the discount or the premium on the face value of the bond will fall as the bond's life gets shorter provided that the yield does not change during the life of a bond.

Theorem-3 : If yield remains unaltered during the life of a bond then the size of the discount or premium on the face value of the bond decreases at an increasing rate as the life of the bond shortens.

Theorem-4 : A decrease in bond's yield will raise bond price by an amount greater in size compared to the situation where an equal increase in bond's yield causes a fall in bond price.

Theorem-5 : The percentage change in bond's price due to a change in yield rate will be smaller if the coupon rate remains higher.

The price-yield curve of a bond showing an inverse relationship between bond price and yield rate will be convex to the origin because of the first and fourth theorems.

For any pure discount bond (without any coupon payment) the yield to maturity (denoted by λ) can be calculated as follows :

$$B_p = \frac{F_k}{\left(1+\frac{\lambda}{m}\right)^k} \text{ when } k = m.t \ (t = \text{maturity period})$$

If $m = 1$ then $k = t$ ($t = 1, 2, \ldots, n$).

If $\lambda = 0$, then $B_p = F_k$. If the redemption value (F_k) at maturity is at the par value of the bond then current bond price (B_p) will be equal to its par value.

In case of coupon bearing bond, if the yield to maturity (λ) = coupon rate then B_p = Par value of the bond. Other things remaining same, the price-yield curve becomes steeper for bonds with higher maturity period. Thus, interest rate risk will be higher for long-maturity bonds. If the bondholder wants to sell the bond before its maturity then the investor is subject to price-risk or the yield risk.

The interest rate sensitivity of a bond can be measured by 'duration' of a bond. Duration is defined as the weighted average of the maturities of all individual cash-flows from a fixed-income security, the weights being the present values of individual cash-flows as a proportion of the current bond price. The duration (D) of a bond is measured by the formula : $D = \frac{\sum_{k=1}^{n} PV(t_k)t_k}{PV = P_B}$ where

$PV(t_k)$ = Present value of cashflow from a bond that takes place at the time period t_k ($k = 1, 2, \ldots, n$)

$PV = P_B$ = Current price of the bond

In case of zero coupon bond $D = \frac{PV(t_k)}{P_B} \times t_k = \frac{P_B}{PV} \times t_k = t_k$

The duration or 'Macaulay duration' formula is expressed as :

$$D = \frac{\sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{\lambda}{m}\right)^k}\left(\frac{k}{m}\right)}{PV} \text{ where}$$

$$PV = \sum_{k=1}^{n} \frac{C_k}{\left(1+\frac{\lambda}{m}\right)^k}, \text{ and } k = m.t.$$

In its explicit form, this formula is also expressed as :

$$D = \frac{1+y}{my} - \frac{1+y+n(c-y)}{mc[(1+y)^n - 1]+my}$$

Where y = yield rate, c = coupon rate, n = maturity period of the bond, m = frequency of coupon payment per year.

If $c = y$ then $D = \frac{1+y}{my}\left[1 - \frac{1}{(1+y)^n}\right]$

The volatility or interest rate sensitivity of a bond is measured by the formula :

$V_B = (-1)\frac{D}{\left(1+\frac{\lambda}{m}\right)}$ where D = duration of the bond, λ = Yield to maturity. Here, $\frac{D}{\left(1+\frac{\lambda}{m}\right)}$ = modified duration, and $V_B = \frac{\Delta P}{\Delta \lambda} \cdot \frac{1}{P}$. However, the duration-based estimation of the volatility of a bond assumes a linear relationship between the change in bond price and change in yield rate. In reality, however, this relationship is non-linear.

The duration of a portfolio of bonds is estimated as follow : If there are two bonds X and Y, and $P_0^X = PV_X$,

$$P_0^Y = PV_Y, \ P = P_0^X + P_0^Y, \ D^X = \frac{\sum_{k=1}^{n} PV_k^X t_k}{P_0^X}, \ D^Y = \frac{\sum_{k=1}^{n} PV_k^Y t_k}{P_0^Y}$$

then duration of the portfolio :

$$D = \frac{P_0^X}{P} \cdot D^X + \frac{P_0^Y}{P} \cdot D^Y$$

The technique of bond portfolio management is known as 'immunisation'. This technique aims at protecting the bond portfolio from any adverse effects associated with future changes in interest rates. This technique involves the matching of the 'duration' of the portfolio of bonds with the 'duration' of the future cash obligations of the investor. If w_1 and w_2 denote the weights or the proportion of the portfolio fund invested in Bond-1 and Bond-2 respectively and if durations of these two bonds are d_1 and d_2 respectively, and the duration of the cash obligation of the investor is denoted by D_t then we have the following two equations :

$w_1 + w_2 = 1$ .... (i)

$w_1 d_1 + w_2 d_2 = D_t$ .... (ii)

Here, the values of d_1, d_2 and D_t are given. Hence, the required values of w_1 and w_2 can be estimated by solving these two equations. However, there are some particular problems with this immunisation technique. This technique has ignored the problems of call risk and default risk; this technique assumes that the yield rates of bonds in a portfolio remain same and the change, if any, is similar for all bonds. But this assumption is not true in reality.

The term structure of interest rates implies that the interest rate charged for lending money or the interest rate paid for borrowing money would depend upon the length of time for which the money is being lent out or borrowed. The interest rate associated with a spot contract at any given point of time is called as **spot rate**. It involves immediate lending of an amount from the lender to the borrower. For zero coupon bond, the yield rate at the time of its purchase would indicate the spot rate. However, for coupon bearing bonds a method of **'bootstrapping'** can be followed to estimate the spot rates on bonds with different maturity periods. If P = current bond price, C = coupon payment, F = face value of the bond, s_1 = 1-year spot rate, s_2 = 2-year spot rate, and if the values of s_1, P, C and F are known then for a 2-year bond, s_2 can be determined using the formula $P = \frac{C}{(1+s_1)} + \frac{C+F}{(1+s_2)^2}$.

A forward interest rate denotes the interest rate to be paid by the borrower to the lender on the amount borrowed in between two time periods in future based on the terms agreed upon at present (Thus, a forward rate $f_{1,2}$ denotes the interest rate on money lent out in period-1 and the receipt of principal alongwith interest in period-2). Any investor can follow either a maturity (direct strategy) strategy or a

roll-over strategy while investing the fund in purchasing bonds, i.e., $(1+s_2)^2 = (1+s_1)(1+f_{1,2})$ for investment of ₹ 1 for 2 years.

$\therefore f_{1,2} = \frac{(1+s_2)^2}{(1+s_1)} - 1$. Hence, the estimation of the forward rate is based on spot rates. The generalised rule is

$$f_{i,j} = \left[\frac{(1+s_j)^j}{(1+s_i)^i}\right]^{\frac{1}{j-i}} - 1$$

Where $i < j$. In case of multi-period compounding of interest rate, this formula is expressed as

$$f_{i,j} = m\left[\frac{\left(1+\frac{s_j}{m}\right)^j}{\left(1+\frac{s_i}{m}\right)^i}\right]^{\frac{1}{(j-i)}} - m$$

There are four primary theories for the explanation of term structure of interest rates :

1. **The expectation theory :** This theory suggests that the expected future spot rate is determined by the forward rate, i.e., $s^f_{1,2} = f_{1,2}$, and in case of investment in a 2-year bond, an equilibrium is maintained when $(1+s_2)^2 = (1+s_1)(1+s^f_{1,2})$.

   In case of investment for 'n' period, $(1+s_n)^n = (1+s_1)(1+s^f_{1,2})(1+s^f_{2,3})\cdots\cdots(1+s^f_{n-1,n})$

   The left-hand side of this equation suggests the maturity strategy or direct investment strategy (in 2-year bond), and the right-hand side shows the roll-over strategy of investment. Hence, one can invest in long-term bonds (for n years) at the current spot rate for n-year bond(s_n), or he can first invest ₹ 1 for 1 year at the current spot rate for 1-year bond (s_1) and then after 1-year, he can reinvest the maturity value at an expected spot rate in the next year $(s^f_{1,2})$, and then after 2 years, he can again reinvest the maturity value in 1-year bond at an expected spot rate $s^f_{2,3}$, and so on.

   From this relation, we can write $s_n = \sqrt[n]{(1+s_1)(1+s^f_{1,2})(1+s^f_{2,3})\cdots\cdots(1+s^f_{n-1,n})} - 1$

   Hence, according to the unbiased expectation theory, the current long-term spot rate (s_n) is an unbiased average (a geometric mean) of the current 1-year spot rate and the future expected spot rates.

   If the investors expect that $s^f_{n-1,n} > s^f_{n-2,n-1} > \cdots\cdots > s^f_{2,3} > s^f_{1,2}$ then the spot rate schedule or the **yield curve would be upward sloping**, and in that case $s_n > s_1$.

   Similarly, if the investors expect that $s^f_{n-1,n} < s^f_{n-2,n-1} < \cdots\cdots < s^f_{2,3} < s^f_{1,2}$ then the yield curve would **be downward sloping.** However, if the investors expect that $s_1 = s^f_{1,2} = s^f_{2,3} = \cdots\cdots = s^f_{n-1,n}$

   then $s_n = \sqrt[n]{(1+s_1)^n} - 1$
   $= 1 + s_1 - 1 = s_1$

   In this case, the **yield curve will be flat or horizontal.**

2. **The liquidity Premium Theory :** This theory suggests that the investors usually prefer short-term fixed-income securities over long-term securities because they do not like to block their liquid fund for a long time.

   They believe that long-term bonds entail more price risk or interest rate risk because the bond price may fall substantially when the investors want to sell such bonds before maturity dates. Hence, to induce such investors in buying long-term bonds, some liquidity premium has to be offered by the bond issuer so that $f_{1,2} = s^f_{1,2} + L_{1,2}$ where $L_{1,2}$ = liquidity premium for the period starting from next year from now, and ending at two years from now. So, for 2-year bond investment

   $\therefore (1+s_2)^2 = (1+s_1)(1+s^f_{1,2} + L_{1,2})$

   or, $(1+s_2)^2 > (1+s_1)(1+s^f_{1,2})\ [\because L_{1,2} > 0]$

   This result implies that higher is the maturity period, higher should be the liquidity premium. **This explains the upward sloping nature of the yield curve** (here it is assumed that $s_2 > s_1$).

   However, if it is assumed that $s_1 > s_2$ then the above inequality will hold only if the value of $s^f_{1,2}$ is sufficiently low. **In that case, the yield curve will be downward sloping.**

   However, if it is assumed that $s_1 = s_2$ then the term structure becomes flat, i.e., the yield curve becomes horizontal. According to liquidity premium theory, the desired inequality [as shown in equation (2.31)] would be maintained if $s^f_{1,2}$ remains less than s_1 when $s_1 = s_2$. However, the unbiased expectation theory suggests that in case of flat yield curve, the expected spot rates would remain unchanged.

3. **The market segmentation theory :** This theory shows that the securities market can be segmented into short-term securities market and long-term securities market. The term structure of interest rates, according to this theory, depends on the relative strengths of demand and supply in these segmented markets.

4. **The preferred habitat theory :** This theory suggests that the investors, based upon their preference and motivation towards taking risks, have some preferred zones of operation. Hence, if any investors prefers investment in 'low-risk low-return' bonds then he can withdraw himself from his preferred zone to enter into a zone with 'high-risk high-return' bonds only if he gets additional return on his investment. This will explain the term structure of interest rates.

Assignment

Short Answer-type questions

1. Define a financial instrument. *(See Section 2.1)*
2. What is a financial security ? Give an example. *(See Section 2.1)*
3. What is a bond ? *(See Section 2.2)*
4. What is a coupon rate ? *(See Subsection 2.2.1)*
5. What do you mean by the redemption value of a bond ? *(See Subsection 2.2.1)*
6. What is a zero coupon bond ? *(See Subsection 2.2.2)*
   or, What is a certificate of deposit ?
7. What is a Treasury Bill ? *(See Subsection 2.2.2)*
8. Show the difference between a dated government security and the Treasury Bill. *(See Subsection 2.2.2)*
9. Differentiate between fixed rate and floating rate bonds. *(See Subsection 2.2.2)*
10. What is an indexed bond ? *(See Subsection 2.2.2)*
11. What is a convertible bond ? *(See Subsection 2.2.2)*
    or, What is a commercial paper ? *[C.U., B.Sc. (H), Sem-V, 2020]*

12. What is an Annuity ? Can it be considered as a security ? (See Subsection 2.
13. What is inflation risk ? (See Subsection 2.
14. What is an interest rate risk faced by an investor ? (See Subsection 2.
15. Define a call risk. (See Subsection 2.
16. What do you mean by a default risk ? (See Subsection 2.
17. What is a liquidity risk to be faced by an investor ? (See Subsection 2.
18. State the formula to be used for the estimation of the current bond price. (See Subsection 2.
19. What is meant by yield on a bond ? (See Subsection 2.
20. What is current yield of a bond ? (See Subsection 2.
21. What is meant by yield to maturity ? (See Subsection 2.
22. What would be the yield to maturity of a bond when it is sold at its par value ? (See Subsection 2.
23. What is realised yield ? (See Subsection 2.
24. What is the principal proposition of the first theorem regarding bond pricing ? (See Section 2.
25. State the principal proposition of the second theorem of bond pricing. (See Section 2.
26. What is the main proposition of the third theorem on bond pricing ? (See Section 2.
27. State the principal proposition of the fourth theorem of bond pricing. (See Section 2.
28. What is the main proposition of the fifth theorem of bond pricing ? (See Section 2.
29. What is a price-yield curve ? (See Subsection 2.
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30. How the price-yield curve can be used to show the interest rate risk associated with a bond ?
(See Subsection 2.
31. What is meant by yield risk ? (See Subsection 2.
32. What do you mean by 'Duration' of a bond ? (See Subsection 2.
33. State the formula for estimating duration of a bond. (See Subsection 2.
34. A zero coupon bond has a maturity period of 20 year. What will be the 'Duration' (D) of this bond ?
(See Subsection 2.
35. State the formula for estimating 'Macaulay duration'. (See Subsection 2.
36. What is meant by the volatility of a bond ? (See Subsection 2.
37. What is the basic assumption in the process of deriving duration-based estimation of volatility of bond price ?
(See Subsection 2.
38. State the formula for the estimation of duration of the portfolio of bonds. (See Subsection 2.
39. What is 'immunisation' of the portfolio of bonds ? (See Section 2.4
40. What do you mean by term structure of interest rates ? (See Section 2.5
41. What is yield curve ? (See Section 2.5
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42. What is a 'spot rate' in the bond market ? (See Subsection 2.5.1)
43. What is the method of bootstrapping in estimating the spot curve in the bond market ?
44. What is a forward rate in the bond market ? (See Subsection 2.5.
45. Distinguish between maturity or direct strategy of investment and the roll-over strategy of investment in the bond market. (See Subsection 2.5.4)
46. If 1-year spot rate (s_1) = 9% and 2-year spot rate (s_2) = 10%, then estimate the forward rate between year 1 and year-2 (denoted by $f_{1,2}$). (See Subsection 2.5.4)
(ii) If the spot rates for 1 and 2 years are s_1 = 6.3% and s_2 = 6.9%, what is the forward rate $(f_{1,2})$? (See Subsection 2.5.4)
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47. State the formula for the derivation of implied forward rate between the time period i and j $(i < j)$ with yearly compounding based on the spot rates of ith and jth time period respectively. (See Subsection 2.5.4)
48. What is the basic proposition of the expectation theory with regard to the term structure of interest rates ? (See Subsection 2.6.1)
49. When can the yield curve be downward sloping based on the unbiased expectation theory ? (See Subsection 2.6.1)
50. State any two weaknesses of the expectation theory of term structure of interest rates. (See Subsection 2.6.1)
51. Mention the basic proposition of the liquidity premium theory with regard to the term structure of interest rates. (See Subsection 2.6.2)
52. Differentiate between bid price and ask price of a bond. (See Subsection 2.2.1)
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Long Answer-type questions

1. (i) Define a bond.
   (ii) Explain the main features of a bond. (See Section 2.2 & Subsection 2.2.1)
2. Give a short note on different types of bond found in the financial market. (See Subsection 2.2.2)
3. Explain the risks which are to be taken into account while estimating the discount rate for finding out the bond price. (See Subsections 2.2.4 & 2.2.6)
4. Explain the concept of Bond Value Curve. (See Subsections 2.2.5 & 2.2.6)
5. Explain the notion of yield on a bond. (See Subsection 2.2.7)
6. (i) What is current yield ? If a bond with a face value of ₹ 100 has a current price of ₹ 92, and if the coupon rate is 8% p.a. then estimate the current yield of the bond. (See Subsection 2.2.8)
   (ii) What is meant by yield to maturity ? If a pure-discount bond (or a zero coupon bond) has a face value of ₹ 1,000 with a maturity period of 1 year (when it is redeemable at par) and if its current price is ₹ 893·50 then calculate the yield to maturity. (See Subsection 2.2.8)
7. (i) A bond with a face value of ₹ 100 carrying an annual coupon rate of 8%, a maturity period of 2 years and redeemable at par, is sold at par value. What would be its yield to maturity ?
   (ii) What would be the relationship between yield to maturity and the coupon rate of a bond when
   (a) it is sold at a discount ? and
   (b) it is sold at a premium ? (See Subsection 2.2.8)
8. (i) Explain the first and fourth theorems of bond pricing.
   (ii) How these two theorems would help us in explaining the convexity of price yield curve ?
   (See Section 2.3 & Subsection 2.3.1)
9. Discuss the second theorem on bond pricing. (See Subsection 2.3.1)
10. Explain the third theorem on bond pricing. (See Subsection 2.3.1)
11. Explain the fifth theorem on bond pricing. (See Subsection 2.3.1)
12. Draw the price-yield curves of the following bonds (each having a maturity period of 20 years)

Bonds	Face Value (₹)	Annual coupon rate (%)
Bond-A	₹ 100	0
Bond-B	₹ 100	6%
Bond-C	₹ 100	8%
Bond-D	₹ 100	10%

All these bonds are redeemable at par. Comment on the shapes of these price yield curves.
(See Subsection 2.3.2)

13. If there are three bonds with similar face values (₹ 100) and coupon rates (say, 10%) then show that the price-yield curves of these bonds would be different for differences in the maturity periods of these bonds. (See Subsection 2.3.3)

14. How a price-yield curve can be used to analyse the interest rate sensitivity of the bond price ? Explain. (See Subsection 2.3.5)

15. (i) Explain the concept of duration (D) of a bond.
(ii) If the maturity period of bond is denoted by $t_1, t_2, ..., t_n$ then show that $t_1 \leq D \leq t_n$. (See Subsection 2.3.6)

(iii) Calculate the duration of a bond based on the following information :
(a) Face value of the bond (₹) : ₹ 100 (redeemable at par)
(b) Coupon rate : 9% (payable annually)
(c) Time left for maturity : 7 years
(d) Discount rate : 8% p.a. (See Subsection 2.3.6)

16. (i) Use the notion of 'duration' of a bond to estimate the volatility of bond price. (See Subsection 2.3.7)
(ii) A bond is currently sold at a price of ₹ 1,000 with a yield rate of 9%. If the duration of this bond is 8 years then what would be the magnitude of the change in the price of this bond when the yield rate increases to 10% ? (See Subsection 2.3.7)

17. State Macaulay's duration formula. How can you derive the formula for estimating the volatility of bond price (or, interest rate sensitivity of bond price) from this formula ? Explain. (See Subsection 2.3.7)

18. (i) Explain the properties of duration of a bond.
(ii) Calculate the duration of the following two bonds considering the yield rate of 7%.

Description	Bond-1	Bond-2
(a) Face value :	₹ 100	₹ 100
(b) Coupon rate (p.a.) :	9%	9%
(c) Maturity period :	2 years	7 years
(d) Redemption value :	At par	At par

Which property of duration can you infer from this sum ? (See Subsection 2.3.8)

19. Estimate the duration of the following two bonds :

Description	Bond-1	Bond-2
(a) Face value :	₹ 100	₹ 100
(b) Coupon rate (p.a.) :	8%	15%
(c) Maturity period :	4 years	4 years
(d) Redemption value :	At par	At par

The present value schedule for these two bonds with a yield rate of 5% are as follows :

Year (t_k)	PV (Q_k) (YTM : 5%)	
	Bond-1	Bond-2
1	7·62	14·28
2	7·26	13·60
3	6·91	12·96
4	88·85	94·61

Comment on the outcome of this sum. (See Subsection

20. (i) Explain the process of estimating the duration of a portfolio of bonds.
(ii) A portfolio consists of two bonds with the following features :

Features	Bond-1	Bond-2
(a) Current market price :	₹ 115·75	₹ 138·25
(b) Duration (Years) :	3·75	4·55

Calculate the duration of the portfolio. (See Subsection 2.3.9)

21. (i) Explain the immunisation technique of portfolio management. (See Section 2.4)
(ii) State some of the problems associated with this immunisation technique. (See Subsection 2.4.1)

22. Explain three standard explanations (or theories) for the Term Structure of interest rates. [C.U., B.Sc. (H), Sem-V, 2020]
(See Sections 2.5 & 2.6, and Subsections 2.6.1-2.6.3)

23. (i) Define spot rate of interest.
(ii) Consider a zero-coupon bond with a face value of ₹ 100, current price of ₹ 35·80, with a maturity period of 10 years. Calculate the spot rate of this bond when frequency of compounding in a year(m) is $m = 1$ and $m = 2$. (See Subsection 2.5.1)

24. (i) Explain the method of 'bootstrapping' for the estimation of spot rates. (See Subsection 2.5.3)
(ii) Consider the following two coupon bearing bonds and estimate the 2-year spot rate (s_2) :

Bond	Type	Current bond price	YTM (%)
Bond-1	1-Year 10% bond (Face value : ₹ 1,000)	1025·50	8%
Bond-2	2-Year 12% bond (Face value : ₹ 1,000)	1068·20	—

(See Subsection 2.5.3)

25. (i) What is meant by forward interest rate ?
(ii) Consider the following spot rates and calculate the forward rate ($f_{1,2}$).
(a) 1-Year spot rate (s_1) = 8%
(b) 2-Year spot rate (s_2) = 9%. (See Subsection 2.5.4)

26. Explain the generalised formula for the estimation of forward interest rate between the time period i and j ($i < j$).
(a) with yearly compounding of interest rate, and
(b) with multi-period compounding within a year. (See Subsection 2.5.4)

27. Explain the expectation theory or the unbiased expectation theory related to the term structure. (See Subsection 2.6.1)

28. (i) Explain the liquidity premium theory of the term structure of interest rates. (See Subsection 2.6.2)
(ii) If 1-Year spot rate (s_1) = 8%, 2-Year spot rate (s_2) = 8·52% and the 1-Year forward rate ($f_{1,2}$) [based on s_1 and s_2] is 9·04%, then estimate the expected spot rate ($s^e_{1,2}$) if the liquidity premium is ($l_{1,2}$) is 0·44. (See Subsection 2.6.2)

29. Discuss the market segmentation theory and the preferred habitat theory of term structure of interest rates. (See Subsections 2.6.3-2.6.4)

3

Random Cash Flow and Portfolio Analysis

3.1. Introduction

When any investor makes an investment at the current period by purchasing a financial asset, he knows the current price of that asset but the future price of the financial asset (say, a stock) is not known. Hence, the future cash flow from this investment is random in nature. In this chapter we shall discuss **single period random flows**, i.e., money is invested at the initial period and the payoff is received at the end of the period. For instance, the investor who purchases a zero-coupon bond, can hold it upto maturity. However, in reality, cash flows from an investment are not restricted to only single period random cash flows since the stocks (say, equity shares of different companies) generate periodic dividend payments and such stocks can be traded and liquidated in the stock market at any time after their listing with any stock exchange.

In this chapter, we shall discuss the random character of the return on any financial asset, return on portfolio of assets, portfolio mean and variance, mean-variance portfolio analysis, Markowitz Model and the two-fund theorem, risk-free assets and one-fund theorem, Capital Asset Pricing Model (CAPM), the beta of any financial asset, capital market line and security market line, and use of CAPM as a pricing formula in investment analysis.

3.2. Return on Investment

When any investor buys any financial asset (e.g., a bond or a stock) then the gain or loss from that investment is normally considered as the return on investment.

Let us first start with a situation where an investor has purchased 100 shares of ITC currently selling at ₹ 205 per share, i.e., he makes an initial investment of ₹ 20,500. Let us assume that this investment is being made for a period of 1 year after which the investor will make an exit by selling these shares.

Now, we are to estimate the return from this investment. Let us suppose that over the year, the stock paid a dividend of ₹ 5 per share. So, by the end of the year, the investor would receive a **dividend income** of ₹ 5 × 100 = ₹ 500. Let us also assume that the market price of ITC stock has increased to ₹ 235 per share over a year. Hence, by selling 100 shares in the stock market the investor will gain ₹ 235 – ₹ 205 = ₹ 30 per share. This is called as **capital gain**.

∴ Total return on investment

= Dividend income + Capital gains

= ₹ 500 + (₹ 30 × 100)

= ₹ 3,500

Absolute return on investment would be

= Amount realised – Amount invested

= [(₹ 5 × 100) + (₹ 235 × 100)] – ₹ 20,500

= 24,000 – 20,500

= ₹ 3,500

However, if the market price of ITC stock drops down to, say, ₹ 180 per share over a year then there will be a capital loss for the investor to the extent of ₹ 25 per share, i.e., capital loss

= (₹ 180 – ₹ 205) × 100

= (–) ₹ 25 × 100

= (–) ₹ 2,500

In this case, return on investment would be (dividend income + capital loss)

= ₹ 500 – ₹ 2,500

= (–) ₹ 2,000

Thus, the absolute return on investment would be

= [(₹ 5 × 100) – (₹ 25 × 100)] – ₹ 20,500

= (–) ₹ 22,500

Thus, we have an overall negative return.

Sometimes, the total return on investment is estimated as follows :

$$R = \frac{x_1}{x_0} \quad \text{......... (3.1)}$$

Where R = Total return on investment,

x_1 = Amount realised

x_0 = Amount invested.

Our example shows that

if x_0 = ₹ 20,500

and x_1 = ₹ 24,000

then $R = \frac{x_1}{x_0} = \frac{24,000}{20,500} = 1{\cdot}17$ or 117%

However, the rate of return (r) or the percentage return can be estimated as follows :

$$\text{Rate of return } (r) = \frac{x_1 - x_0}{x_0} \quad \text{......... (3.2)}$$

$$\text{So, } r = \frac{24,000 - 20,500}{20,500}$$

$$= \frac{3,500}{20,500} = 0{\cdot}170 = 17{\cdot}0\%$$

In our example, this rate of return can be divided into two parts :

(i) dividend yield

(ii) capital gains yield.

$$r = \frac{D_{t+1} + (P_{t+1} - P_t)}{P_t} \quad \text{......... (3.3)}$$

Where D_{t+1} = Dividend paid on stock at the time period $t + 1$.

P_t = Stock Price at the time period t

P_{t+1} = Stock Price at the time period $t + 1$.

So, $r = \frac{D_{t+1}}{P_t} + \frac{(P_{t+1} - P_t)}{P_t}$

In our example P_t = ₹ 205

and if P_{t+1} = ₹ 235

and D_{t+1} = ₹ 5 (per stock).

Then $r = \frac{5+(235-205)}{205}$

$= \frac{5}{205} + \frac{30}{205}$

$= 0{\cdot}024 + 0{\cdot}146$

$= 0{\cdot}17$

$= 17\%$

Here, dividend yield $= \frac{D_{t+1}}{P_t} = \frac{5}{205} = 0{\cdot}024 = 2{\cdot}4\%$

and capital gains yield $= \frac{P_{t+1} - P_t}{P_t} = \frac{235-205}{205}$

$= 0{\cdot}146$

$= 14{\cdot}6\%$

- **Relation between 'R' and 'r' :**

From our previous discussion and on the basis of formula (3.1) and (3.2), we can show the following relationship between overall return (*R*) and the rate of return (*r*) on an investment :

$$1 + \frac{x_1 - x_0}{x_0} = \frac{x_0 + x_1 - x_0}{x_0} = \frac{x_1}{x_0}$$

or, $1 + r = R$ .... (3.4)

Therefore, we can state

$$\frac{x_1}{x_0} = 1 + r$$

or, $x_1 = x_0(1 + r)$ .... (3.5)

So, the rate of return on investment is like an interest rate.

Example 3.1

Let us assume that an investor had purchased 100 shares of Axis Bank Ltd. @ ₹ 670 per share in 2020 and after 1 year those stocks could be sold at a price of ₹ 675 per share. The investor has also received a dividend payment of ₹ 10 per share. Calculate

(i) *overall return on investment,*

(ii) *rate of return on investment,*

(iii) *show that the sum total of dividend yield and capital gains yield would be equal to the rate of return on investment.*

Solution :

(i) Here, the initial investment (x_0)

= ₹ 670 × 100

= ₹ 67,000

Dividend income = ₹ 10 × 100 = ₹ 1,000

Capital gain = (₹ 675 − 670) × 100

= ₹ 500

∴ Total return on investment

= ₹ 1,000 + 500

= ₹ 1,500

Alternatively, absolute overall return on investment = Amount realised − Amount invested

= [(₹ 10 × 100) + (₹ 675 × 100)] − (₹ 670 × 100)

= (₹ 1,000 + 67,500) − 67,000

= ₹ 68,500 − 67,000

= ₹ 1,500

∴ Total return $(R) = \frac{x_1}{x_0} = \frac{\text{Amt. realised}}{\text{Amt. invested}}$

$= \frac{₹\ 68,500}{₹\ 67,000}$

$= 1{\cdot}022$

$= 102{\cdot}2\%$

(ii) Rate of return $(r) = \frac{x_1 - x_0}{x_0}$

$= \frac{₹\ 68,500 - ₹\ 67,000}{₹\ 67,000}$

$= \frac{₹\ 1,500}{₹\ 67,000}$

$= 0{\cdot}0223$

$= 2{\cdot}23\%$

(iii) Here, dividend yield $= \frac{D_{t+1}}{P_t} = \frac{10}{670}$

$= 0{\cdot}0149$

$= 1{\cdot}49\%$

and capital gains yield $= \frac{P_{t+1} - P_t}{P_t}$

$= \frac{675 - 670}{670} = \frac{5}{670}$

$= 0{\cdot}00746$

$= 0{\cdot}746\%$

∴ rate of return (r)

$= 1·48\% + 0·75\%$

$= 2·23\%$

Thus, the sum total of dividend yield and capital gains yield becomes equal to the rate of return on investment.

3.2.1. Average return

In many instances, the historical data regarding the movement of stock price and the rate of return help us in estimating the average return on investment.

If r_t = Rate of return on investment at the period t ($t = 1, 2, \ldots, n$) then the average rate of return can be estimated by the simple arithmetic mean of those rates of return over 'n' number of periods

$$\bar{r} = \frac{1}{n}\sum_{t=1}^{n} r_t \quad \text{—— (3.6)}$$

where $\bar{r}$ = Average rate of return

Example 3.2

Period (t)	Stock price (P_t)	Dividend during the Year (D_t)	Dividend yield $\left(\frac{D_t}{P_{t-1}}\right)$	Capital gains $\frac{(P_t - P_{t-1})}{P_t}$	Rate of return (r_t)
1	100	10	—	—	—
2	110	10	10 ÷ 100	10 ÷ 100	20·00
3	125	10	10 ÷ 110	15 ÷ 110	22·72
4	100	12	12 ÷ 125	−25 ÷ 125	−10·40
5	95	12	12 ÷ 100	−5 ÷ 100	7·00
6	120	12	12 ÷ 95	25 ÷ 95	38·94

[r_t = Dividend yield + capital gains]

Here, the average rate of return can be estimated as follows :

$$\bar{r} = \frac{1}{6}(20·00 + 22·72 - 10·40 + 7·00 + 38·94)$$

$$= \frac{1}{6}(78·26) = 13·04\%$$

This simple arithmetic mean shows that the average rate of return on that stock over a period of 6 years has been 13·04%.

However, in many cases geometric mean is used for the estimation of average return. This can be analysed with the help of a simple example.

Let us assume that the investor has purchased a stock at a price of ₹ 100 in the year t, and in the year $t + 1$, the market price of that stock declines to ₹ 50 (i.e., a 50% fall), and in the year $t + 2$, the market price of that stock again increases to ₹ 100 (i.e., a 100% rise compared to the year $t + 1$). Hence, following the simple arithmetic mean, the average rate of return on investment would be :

$$\bar{r} = \frac{1}{2}(-50·0 + 100·0) = 25\%$$

However, in case of geometric mean we follow the formula :

$$r_G = \left[(1+r_1)(1+r_2)(1+r_3) \text{——} (1+r_n)\right]^{\frac{1}{n}} - 1 \quad \text{—— (3.7)}$$

In fact, the geometric mean reflects the average compound return per year over a particular time period. Thus, it shows

$$\left(1+r_G\right)^n = (1+r_1)(1+r_2) \text{——} (1+r_n)$$

$$\text{or, } \left(1+r_G\right) = \left[(1+r_1)(1+r_2) \text{——} (1+r_n)\right]^{\frac{1}{n}}$$

$$= \sqrt[n]{(1+r_1)(1+r_2) \text{——} (1+r_n)}$$

$$\text{or, } r_G = \sqrt[n]{(1+r_1)(1+r_2) \text{——} (1+r_n)} - 1 \quad \text{—— (3.8)}$$

Thus, following our example, the geometric mean of returns would be as follows :

At first, the percentage return for each period has to be expressed in decimal terms, i.e., −50% should be expressed (−) 0·50 and 100% should be expressed as 1·0.

$$\therefore r_G = \left[(1-0·5)(1+1)\right]^{\frac{1}{2}} - 1$$

$$= \sqrt{2 \times 0·5} - 1$$

$$= \sqrt{1} - 1 = 0$$

So, following the geometric mean, the average return is observed to be zero. Now, the question is : which one is correct ? In fact, both are correct. The geometric mean method of estimating the average return answers to the question — 'What is the average compound return on investment per year over a particular time period' ? The simple arithmetic mean method, on the other hand, answers the question : 'What is the return on investment in any average year over a particular time period' ?

Example 3.3

On the basis of our previous example we consider the following rates of return on a stock :

Period (between t and $t + 1$)	Rate return (r_t)
1 & 2	20%
2 & 3	22·72%
3 & 4	(−) 10·40%
4 & 5	7·0%
5 & 6	38·94%

We have already shown that the average rate of return following the simple arithmetic mean is $\bar{r}$ = 13·04%. Now, we are to estimate the average compound return on investment. In determining the average compound return on investment, we have to follow the steps as noted in the next page :

Rate of return (r_i)	Rate of return (in decimal form)	$(1 + r_i)$
20%	0·20	1·2000
22·72%	0·2272	1·2272
(–) 10·40%	(–) 0·1040	0·896
7·0%	0·0700	1·0700
38·94%	0·3894	1·3894

So, average compound return or the geometric mean of the returns would be :

$$r_G = [(1{\cdot}20)(1{\cdot}2272)(0{\cdot}896)(1{\cdot}07)(1{\cdot}3894)]^{\frac{1}{5}} - 1$$

$$\text{or, } r_G = (1{\cdot}9626)^{\frac{1}{5}} - 1$$

$$= 1{\cdot}1442 - 1$$

$$= 0{\cdot}1442$$

$$= 14{\cdot}42\%$$

Thus, average compound return gives us a figure different from what we had derived on the basis of simple arithmetic mean.

3.2.2. Portfolio return

A portfolio of assets consists of many financial assets. Let the portfolio consist of 'n' number of assets (say, shares of different companies, government bonds etc.) and let us assume that the investor has an amount of investible fund that can be apportioned among 'n' number of assets. Now, if x_{0i} = Amount invested in i-th asset,

$$\text{then } \sum_{i=1}^{n} x_{0i} = x_0 = \text{Total fund available for investment.}$$

If w_i = Fraction of i-th asset in the portfolio, and $\sum_{i=1}^{n} w_i = 1$

then $x_{0i} = w_i . x_0$

If R_i = total return from i-th asset at the end of a period

then $R_i . x_{0i} = R_i . w_i . x_0$

= Total amount of money received at the end of the investment period, from i-th asset.

So, total amount of money realised from all the assets of the portfolio will be $\sum_{i=1}^{n} R_i w_i . x_0$

So, total return from portfolio (R) will be :

$$R = \frac{\sum_{i=1}^{n} R_i . w_i . x_0}{x_0}$$

$$\text{or, } R = \frac{x_0 \sum_{i=1}^{n} R_i . w_i}{x_0} \quad [\text{Since } x_0 \text{ is given constant}]$$

$$= \sum_{i=1}^{n} R_i . w_i \text{ .... (3.9)}$$

The portfolio rate of return can be determined from (3.9).

Since $R = 1 + r$ (from 3.4)

So, $R_i = 1 + r_i$

Hence, (3.9) can be expressed as follows :

$$R = \sum R_i . w_i$$

$$\text{or, } 1 + r = \sum_{i=1}^{n} (1 + r_i) . w_i$$

$$= \sum_{i=1}^{n} w_i . r_i + \sum_{i=1}^{n} w_i$$

$$\text{or, } 1 + r = \sum_{i=1}^{n} w_i . r_i + 1 \qquad \left[\text{Since } \sum_{i=1}^{n} w_i = 1\right]$$

$$\text{or, } r = \sum_{i=1}^{n} w_i . r_i \text{ .... (3.10)}$$

Thus, (3.10) shows that the portfolio rate of return is the weighted average rate of return of the individual assets in the portfolio, the weight being the proportion of i-th asset in the portfolio.

Example 3.4

Consider the following portfolio of financial assets of an investor, and calculate the portfolio rate of return.

Asset	No. of stocks	Price of stock (₹) (per unit)	Rate of return (r_i)
A	200	125	15·5%
B	300	120	14·8%
C	500	110	13·5%

Solution :

The total value of the portfolio consisting of three stocks, viz., A, B and C, and their respective weights (i.e., their respective shares in the total value of the portfolio) will be as follows :

Stocks	No. of stocks	Price of stock per unit (₹)	Total cost (₹)	Weights of the stocks in portfolio (w_i)
(a)	(b)	(c)	(d) = (b) × (c)	(e) = (d) ÷ 1,16,000
A	200	125	25,000	(25,000 ÷ 1,16,000) = 0·2155
B	300	120	36,000	(36,000 ÷ 1,16,000) = 0·3103
C	500	110	55,000	(55,000 ÷ 1,16,000) = 0·4742
		Total value of the portfolio =	1,16,000	Σw_i = 1·000

Now, the portfolio return can be estimated as follows :

Stocks	w_i	r_i	$w_i \cdot r_i$
A	0.2155	15.5%	$0.2155 \times 0.155 = 0.0334$
B	0.3103	14.8%	$0.3103 \times 0.148 = 0.0459$
C	0.4742	13.5%	$0.4742 \times 0.135 = 0.0640$
		Portfolio rate of return	$\Sigma w_i \cdot r_i = 0.1433$

Therefore, the portfolio rate of return $(r) = \Sigma w_i \cdot r_i = 14.33\%$.

3.2.3. Return from short selling

If an investor does not own an asset and sells that asset by borrowing the same from any agent (say, from a stock broker), then this process is considered as short selling.

Thus, at the beginning, the investor borrows the financial asset from any of the market players (say, a stock broker) and sells that borrowed asset and receives an amount x_0.

At a later date, normally when the market price of that asset declines then the investor can purchase that asset, say, at a cost x_1 (where $x_1 < x_0$) and can refund the borrowed asset to the stock broker. Hence, for the investor the amount of profit is $(x_0 - x_1)$. However, there is no guarantee that market price of that asset would always fall at a later date. If the market price increases then $x_1 > x_0$ and the investor would suffer a loss. Thus, short selling is a risky venture.

Now, the return associated with such short selling can be estimated as follows :

Since the investor gets x_0 amount by selling the borrowed asset at the initial period, it implies that the value of the borrowed asset is x_0. Hence, the initial outlay = (–) x_0. Again, when he purchases the asset (say, at a lower price) from the market at a later date at a value x_1 to refund the asset to the stock broker then this value also implies a cash outflow, i.e., (–) x_1.

$$\therefore \quad \text{Total return } (R) = \frac{-x_1}{-x_0} = \frac{x_1}{x_0}$$

Since the minus signs in the denominator and the numerator cancel out, we get the same result (see equation 3.1) as before.

Hence, the total return (R) received from the purchase of a financial asset by the investor (as shown in equation 3.1) and that from short selling of an asset become same. Obviously, the rate of return from short selling would also give similar expression to that in case of purchase of an asset. Here, we get $-x_1 = -x_0 R = -x_0 (1 + r)$ $[\because R = 1 + r]$

or, $x_1 = x_0 (1 + r)$

However, in case of short selling, the investor has to keep a margin money with the stock broker while borrowing the stock (say, 60% of the price per stock). In this case, if the short seller borrows the stock and the stock price is, say, ₹ 100 per stock, then he will have to keep 60% of ₹ 100, i.e., ₹ 60 with the stock broker. Further, if the stock pays dividend during the period for which it has been borrowed, the short seller will have to pay that dividend to the stock broker.

Example 3.5

Let us give a simple example showing the estimation of rate of return in case of such short selling.

Stock borrowed and sold			
Stock price per unit (₹)	No. of stocks borrowed	Total borrowed value (₹)	Value of loan (₹)
100	100	10,000	(a) With margin : 60% of ₹ 10,000 = ₹ 6,000
100	100	10,000	(b) Without margin facility : 100% of ₹ 10,000 = ₹ 10,000
Buy back of stock to repay the loan			
Stock price (₹)	No. of stocks	Purchase value (₹)	Refund of dividend received @ ₹ 1 per stock (₹)
90	100	9,000	₹ 1 × 100 = ₹ 100

Here, the rate of return to the short seller can be estimated as follows :

(a) Rate of return (with margin facility)

$$= \frac{[(100 \times 100) - (90 \times 100) - (1 \times 100)]}{(0.6 \times ₹\ 10{,}000)}$$

$$= \frac{10{,}000 - 9{,}100}{6{,}000} = 0.15 = 15\%$$

(b) Rate of return (without margin facility)

$$= \frac{[(100 \times 100) - (90 \times 100) - (1 \times 100)]}{₹\ 10{,}000}$$

$$= \frac{10{,}000 - 9{,}100}{10{,}000} = 0.09 = 9\%$$

However, it is interesting to note that for the person who had purchased this stock on margin @ ₹ 100 and sold at a price of ₹ 90 per stock (and received a dividend of ₹ 1 per stock in that period), the rate of return would be (with margin facility) :

$$\frac{[(90 \times 100) + (1 \times 100) - (100 \times 100)]}{0.6 \times ₹\ 10{,}000} = -0.15 = (-)\ 15\%$$

and without any margin facility, this rate of return would be

$$\frac{[(90 \times 100) + (1 \times 100) - (100 \times 100)]}{₹\ 10{,}000} = -0.09 = (-)\ 9\%$$

For a short seller (without using margin facility), this rate of return can also be estimated in the following manner :

Here, total return $(R) = \frac{-x_1}{-x_0} = \frac{-9{,}100}{-10{,}000} = 0{\cdot}91$

We know that $R = 1 + r$

but here $r = \frac{x_1 - x_0}{x_0} = \frac{9{,}100 - 10{,}000}{10{,}000} = -0{\cdot}09$

$\therefore \quad 1 - r = R \quad$ [Since $r < 0$]

or, $-r = R - 1 = 0{\cdot}91 - 1 = -0{\cdot}09$

or, $r = 0{\cdot}09 = 9\%$

3.3. Random returns to an asset

The random nature in returns to an asset implies that such returns are not equally likely. Some values occur fewer times under some particular economic environment, while some other values occur with frequenty. A graphical representation of the values of these random values of return along the horizontal axis and the frequency of occurence on the vertical axis is considred as the *frequency distribution*.

Example 3.6

Let us consider one example. We can collect data regarding the rate of returns from an asset on the basis of, say, 150 observations, and the information can be arranged as shown below :

Possible values of returns (%)	Frequency of occurence (f_i)	Probability of occurence (p_i)	P_iX_i
10	26	0·173	1·73
15	34	0·227	3·40
20	50	0·333	6·67
25	18	0·120	3·00
30	12	0·080	2·40
35	10	0·067	2·34
	$N = \Sigma f_i = 150$	$\Sigma p_i = \cdot 990 \cong 1$	$\Sigma p_i x_i = 19{\cdot}54$

This table shows that the rate of return from the asset may vary between 10% to 35%, and a 10% return has occured for 26 times out of 150 observations. Thus, the probability of getting 10% return is $\frac{26}{150} = 0{\cdot}173$ i.e., 17·3%. Similarly, the probability of getting 15% return from the asset over a period of time is $\frac{34}{150} = 0{\cdot}227$ or 22·7%, and so on. The discrete probability distribution of these returns can be shown with the help of a diagram (Fig.-3.1).

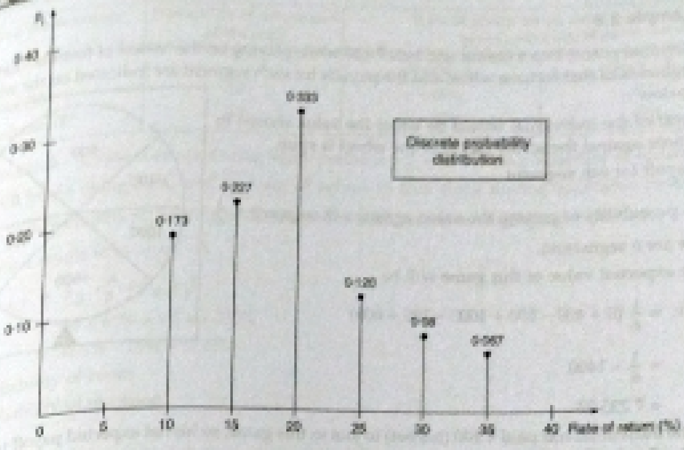


Fig.-3.1

The expected return from the asset is 19·54% and this expected value shows the sum of the product of probabilities of occurence of respective returns and the value of those returns, i.e., $\Sigma p_i x_i$.

Here, the rate of return from the asset is considered to be a random variable since the amount of money that the investor can obtain by selling that asset in the market is uncertain.

Thus, when any random variable x (say, the rate of return from any financial asset) can take any value $x_1, x_2, \ldots, x_n$ and if the probabilities of getting these values are $p_1, p_2, \ldots, p_n$ respectively, where $\Sigma p_i = 1$, then the expected value of x will be $E(x) = \sum_{i=1}^{n} x_i \cdot p_i$ .... (3.11)

This shows the mean value ($\bar{x}$) of the random variable.

Example 3.7

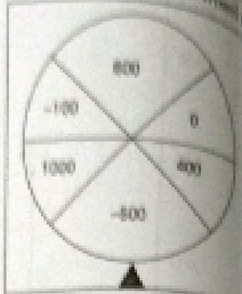
If a die is thrown, than the random outcomes would be 1, 2, 3, 4, 5, 6, with probability, $\frac{1}{6}$ for each of these outcomes. So, here, $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5$ and $x_6 = 6$, and $p_i = \frac{1}{6}$ where $\sum_{i=1}^{6} p_i = 1$.

Now, if the money value to be obtained by the gambler is equal to the number appeared on the die, then the expected pay off for this gambler is $\sum p_i x_i = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6)$

$$= \frac{1}{6} \times 21 = ₹\ 3{\cdot}50$$

Example 3.8

One individual enters into a casino and bets ₹ 100 while playing on the 'wheel of fortune'. Let there be six segments of that fortune wheel and the payoffs for each segment are indicated on the wheel as shown below :



The payoff of the individual would be equal the value shown in the segment against the arrow (▲) after the wheel is spun.

If x_i = payoff for i-th segment

and p_i = probability of getting the value against i-th segment = $\frac{1}{6}$

($\because$ there are 6 segments).

$\therefore$ The expected value of this game will be

$$\Sigma p_i x_i = \frac{1}{6}(0 + 400 - 500 + 1000 - 100 + 600)$$

$$= \frac{1}{6} \times 1400$$

$$= ₹\ 233.33$$

Since, the individual has paid ₹ 100 (his bet) to join in this game, so his net expected payoff would be ₹ 233.33 – ₹ 100 = ₹ 133.33.

3.5.1. Some important properties of expected value

Some of the important properties of the expected value of a random variable are as follows :

(a) The expected value of a constant 'c' is constant, i.e., $E(c) = c$.

(b) The expected value of the product of a constant 'c' and a random variable x would be equal to c times the expected value of the random variable, i.e., $E(c\cdot x) = c\cdot E(x)$.

(c) The expected value of a linear function of a random variable is same as the linear function of its expectation, i.e., $E(a + bx) = a + b\,E(x)$.

(d) The expected value of the product of two random variables is equal to the product of their individual expected values, i.e., $E(x\cdot y) = E(x)\cdot E(y)$.

(e) The expected value of the sum of two independent random variables is equal to the sum of their individual expected values, i.e., $E(x + y) = E(x) + E(y)$.

(f) If the value of the random variable is never less than zero then $E(x) \geq 0$.

From our previous discussion, it becomes clear that the returns from a financial asset are random in nature, and the probability weighted sum of these returns is regarded as the expected return from the financial asset. This expected value measures the central tendency of a probability distribution.

Example 3.9

Let us consider two financial assets, A and B, and the returns from these assets over a particular year have been found to be dependent upon the economic environment. Normally, during economic recession the rate of return (r) from a financial asset remains lower compared to that attained during economic boom period. Let us first assume that the probabilities of occurrence of recession and boom are 50 - 50 during a given period. Now, we are to calculate the expected rate of return from Stock - A and Stock - B respectively on the basis of the following information :

State of the economy	Probability of occurrence of the state	Rate of return on an asset under a given economic state	
		Stock - A	Stock - B
Boom (B)	0.5	+ 65%	+ 25%
Recession (R)	0.5	- 30%	+ 10%

Here, for Stock - A, rate of return during boom period is $r_B^A = 65\%$ with probability of occurrence of economic boom being 50%, and the rate of return of this stock during economic recession is $r_R^A = (-)30\%$ with 50% probability of occurrence of such recession during a period.

$\therefore$ Expected return from stock - A

$$E(r_A) = p_B \cdot r_B^A + p_R \cdot r_R^A$$

$$= 0.5 \times 65\% + 0.5 \times (-)30\%$$

$$= 32.5\% - 15\% = 17.5\%$$

p_B = probability of boom

p_R = probability of recession

For stock - B, We can estimate the expected return following similar procedure, i.e.,

$$E(r_B) = p_B \cdot r_B^B + p_R \cdot r_R^B$$

$$= 0.5 \times 25\% + 0.5 \times 10\%$$

$$= 12.5\% + 5\% = 17.5\%$$

$$\therefore\ E(r_A) = E(r_B)$$

Let us now change the probabilities of occurrence of economic boom and recession during a given period.

State of the economy	Probability of occurrence of the state	Rate of return on an asset under a given economic state	
		Stock - A	Stock - B
Boom (B)	0.3	+ 65%	+ 25%
Recession (R)	0.7	– 30%	+ 10%

With this changed probability of occurrence of a state of the economy, the expected return from a stock will also change.

$$\therefore\ E(r_A) = 0.3 \times 65\% + 0.7 \times (-)\,30\%$$

$$= (-)\,1.5\%$$

$$\therefore\ E(r_B) = 0.3 \times 25\% + 0.7 \times 10\%$$

$$= 14.5\%$$

Thus, in this case $E(r_B) > E(r_A)$

However, if we assume that the probability of occurrence of economic boom is 0.7, and that of economic recession is 0.3 then

$$E(r_A) = 0.7 \times 65\% + 0.3 \times (-) 30\%$$
$$= 45.5 - 9 = 36.5\%$$

and $$E(r_B) = 0.7 \times 25\% + 0.3 \times 10\%$$
$$= 17.5 + 3 = 20.5\%$$

Thus, here $E(r_B) < E(r_A)$.

3.3.2. Variance of a random Variable

So far we have discussed the expected return from a financial asset where such returns are random in nature. Now, if we want to measure the risk involved in the investment where the investor invests the fund in purchasing financial assets, we can use the concept of variance of a random variable. One way of measuring the risk involved in the purchase of financial assets is to estimate the variance of returns on these assets. The variance of a random variable estimates the dispersion or variability among possible values of a random variable from its expected value.

The variance of a random variable denoted by Var (x) is expressed as

$$\text{Var}(x) = E(x-\bar{x})^2$$
$$= E\left[x^2 - 2x\bar{x} + \bar{x}^2\right]$$
$$= E\left(x^2\right) - 2E(x).\bar{x} + \bar{x}^2$$
$$= E\left(x^2\right) - 2\bar{x}^2 + \bar{x}^2$$
$$\therefore \sigma_x^2 = E\left(x^2\right) - \bar{x}^2 \quad [\because E(x) = \bar{x}] \text{ .... } (3.12)$$

Further, the standard deviation of the random varible x would be

$$S.D_x = \sigma_x = \sqrt{\sigma_x^2} = \sqrt{E(x-\bar{x})^2} \text{ .... } (3.13)$$

This concept of variance can be used to measure the risk involved in an investment in financial assets. Thus, the variance of the rate of return (r) from a financial asset would indicate this risk component.

$$\therefore \text{Var}(r) = E(r-\bar{r})^2 \text{ .... } (3.14) \quad \text{where } \bar{r} = E(r)$$

and $$S.D. (r) = \sqrt{E(r-\bar{r})^2} \text{ .... } (3.15)$$

Example 3.10

Let us consider our previous example where we have estimated the expected rate of return on a stock under different possible states of an economy over a period of time. Now, we are to estimate the risk involved in the investment of each of these stocks through the estimation of variance in the rate of returns on these stocks.

Estimate the variance of the rate of returns on Stock - A and B respectively on the basis of the following information :

State of the economy	Probability of occurrence of the state	Rate of return on a financial asset under a given state of the economy	
		Stock - A	Stock - B
Boom (B)	0.5	+ 65%	+ 25%
Recession (R)	0.5	− 30%	+ 10%

Here, the expected return from Stock - A

$$\bar{r}_A = E\left(r_A\right) = p_B.r_B^A + p_R.r_R^A$$
$$= 0.5 \times 65\% + 0.5 \times (-) 30\%$$
$$= 32.5\% - 15\% = 17.5\%$$

Similarly, the expected return from Stock - B

$$\bar{r}_B = E\left(r_B\right) = p_B.r_B^B + p_R.r_R^B$$
$$= 0.5 \times 25\% + 0.5 \times 10\%$$
$$= 12.5\% + 5\% = 17.5\%$$

Now, Variance of the rate of return on Stock - A will be as follows :

State of the economy	State probability (p_i)	r_i^A	$\left(r_i^A - \bar{r}_A\right)$	$p_i\left(r_i^A - \bar{r}_A\right)^2$
Boom (B)	0.5	+ 65%	0.65 − 0.175 = 0.475	0.1128125
Recession (R)	0.5	(−) 30%	−0.3 − 0.175 = (−) 0.475	0.1128125

$$\therefore \text{Var}(r_A) = \sum_i p_i\left(r_i^A - \bar{r}_A\right)^2 = E\left(r_i^A - \bar{r}_A\right)^2$$
$$= 0.1128125 + 0.1128125$$
$$= 0.225625$$
$$= 22.56\%$$

$$\therefore SD(r_A) = \sqrt{Var\left(r_A\right)}$$
$$= \sqrt{0.225625}$$
$$= 0.475$$
$$= 47.5\%$$

Similarly, the variance in the rate of return on Stock - B can be estimated as follows :

State of the economy	State probability (p_i)	r_i^B	$\left(r_i^B - \bar{r}_B\right)$	$p_i\left(r_i^B - \bar{r}_B\right)^2$
Boom (B)	0.5	+ 25%	0.075	0.0028125
Recession (R)	0.5	+ 10%	(−) 0.075	0.0028125

$$\therefore \text{Var}(r_B) = \sum p_i\left(r_i^B - \bar{r}_B\right)^2 = E\left(r_i^B - \bar{r}_B\right)^2$$
$$= 0.0028125 + 0.0028125$$
$$= 0.005625$$

$= 5.62\%$

$\therefore SD(r_B) = \sqrt{Var(r_B)}$

$= \sqrt{0.005625}$

$= 0.075$

$= 7.5\%$

Since $Var(r_A) > Var(r_B)$

hence Stock - A is considered to be more risky than Stock - B. The standard deviations of the rate of returns for Stock-A and B also reflect the same result.

Here it is observed that though the expected rate of returns on Stock-A and B are same, i.e., $E(r_A) = E(r_B) = 17.5\%$, the variance in the rate of return becomes higher in case of stock-A than stock-B.

However, whether the investor would invest in Stock-A or Stock-B would depend upon his/her attitude towards financial risks. If the investor is 'risk-averter' then he/she would invest more in purchasing Stock-B. However, for any 'risk-lover' investor, investment in Stock-A would be preferred.

3.3.3. Mutual dependence of random variables : Covariance

The mutual dependence between two or more random variables can be expressed with the help of covariance. Let x_1 and x_2 be two random variables, say, rate of return on Stock - A and Stock - B respectively. In that case, the mutual dependence between x_1 and x_2 can be shown with the notion of covariance, often expressed as $Cov(x_1, x_2)$.

It is estimated as follows :

$$Cov(x_1, x_2) = E[(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)] \text{ ——— } (3.16)$$

where $E(x_1) = \bar{x}_1$, $E(x_2) = \bar{x}_2$

Now, (3.16) can be expressed as :

$$cov(x_1, x_2) = E[x_1 . x_2 - x_1\bar{x}_2 - \bar{x}_1 x_2 + \bar{x}_1\bar{x}_2]$$

$$\text{or, } \sigma_{12} = E(x_1 . x_2) - E(x_1)\bar{x}_2 - E(x_2)\bar{x}_1 + E(\bar{x}_1\bar{x}_2)$$

$$\text{or, } \sigma_{12} = E(x_1 . x_2) - \bar{x}_1 . \bar{x}_2 - \bar{x}_2 . \bar{x}_1 + \bar{x}_1 . \bar{x}_2$$

$$\text{or, } \sigma_{12} = E(x_1 . x_2) - \bar{x}_1 . \bar{x}_2 \text{ ——— } (3.17)$$

If x_1 and x_2 are uncorrelated then

$Cov(x_1, x_2) = \sigma_{12} = 0$

Similarly, if x_1 and x_2 are positively correlated then $\sigma_{12} > 0$, e.g., rate of return on Stock - 1 may rise with an increase in the rate of return on stock - 2.

Further, if x_1 and x_2 are negatively correlated then $\sigma_{12} < 0$

If $\sigma_{12} = \sigma_1 . \sigma_2$ Where σ_1 = Standard deviation of x_1, σ_2 = Standard deviation of x_2,

then x_1 and x_2 are said to be perfectly correlated.

This is because, the correlation coefficient between x_1 and x_2 is estimated as :

$$\text{Correlation coefficient } (\rho_{12}) = \frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{Cov(x_1, x_2)}{SD(x_1).SD(x_2)}$$

Therefore, if $\sigma_{12} = \sigma_1 . \sigma_2$

then $\rho_{12} = 1$

Similarly, if $\sigma_{12} = -\sigma_1 . \sigma_2$

then $\rho_{12} = -1$

It implies that x_1 and x_2 have a perfect negative correlation.

Again, the covariance of a random variable with itself (σ_{11} or σ_{22}) would be equal to the variance (σ_1^2 or σ_2^2) of that random variable.

$$\text{Here, } \rho_{11} = \frac{\sigma_{11}}{\sigma_1 . \sigma_1} = \frac{\sigma_{11}}{\sigma_1^2}$$

$$\text{Since } \rho_{11} = 1, \text{ so } \sigma_{11} = \sigma_1^2$$

$$\text{Similarly, } \rho_{22} = \frac{\sigma_{22}}{\sigma_2 . \sigma_2} = \frac{\sigma_{22}}{\sigma_2^2}$$

$$\text{or, } 1 = \frac{\sigma_{22}}{\sigma_2^2}$$

$$\text{or, } \sigma_{22} = \sigma_2^2$$

3.4. Portfolio of assets

Normally most of the investors hold different varieties of financial assets which are called as portfolio of assets. Thus, the investors tend to own more than just a single stock, bond or other asset. This portfolio consists of variety of bonds, stocks and other assets. Some of these bonds might be almost risk-free (say, the government bonds) while some other bonds and stocks might be more risky. The maturity periods, coupon rates, redemption value etc. might also be different for different bonds in the portfolio. Again, stocks or equity shares of some renowned companies with high credit rating along with the stocks of some not so renowned companies with low credit rating might be included in the portfolio. The main objective of holding a large variety of assets in a portfolio is to diversify the risk, i.e., probable loss in some cases might be compensated by the gain in other assets.

Thus, for any investor, expected return from a portfolio and the expected risks involved with a portfolio are relevant.

- **Portfolio weights :** There are several ways of describing a portfolio of financial assets. However, the most convenient way is to indicate the weightage of each asset in that po[illegible]tfolio of assets. The share of each financial asset in the total value of portfolio investment can be considered as portfolio weights.

For instance, if the investor invests ₹ 55,000 to hold Asset-1, ₹ 45,000 to purchase Asset -2 and ₹ 50,000 to hold Asset - 3, then his total portfolio is worth ₹ 1,50,000. Thus, the weightage of each asset in this portfolio would be as follows :

(i) Portfolio weight for Asset-1 $= \frac{55,000}{1,50,000}$

$= 0.37$

$= w_1$

(ii) Portfolio weight for Asset-2 $= \frac{45,000}{1,50,000}$

$= 0.30$

$= w_2$

(iii) Portfolio weight for Asset-3 $= \frac{50,000}{1,50,000}$

$= 0.33$

$= w_3$

$\therefore \sum_{i=1}^{3} w_i = 0.37 + 0.30 + 0.33 = 1$

3.4.1. Expected return from a portfolio or Portfolio mean

Let us suppose that an investor has 'n' number financial assets in his portfolio, and the random rates of return of these assets are $r_1, r_2, \ldots r_n$; the expected values of each of these rates of return would be $E(r_1) = \bar{r}_1, E(r_2) = \bar{r}_2, \ldots E(r_n) = \bar{r}_n$ respectively.

Now, if the weights of individual assets in this portfolio are denoted by $w_1, w_2, \ldots w_n$ then the portfolio rate of return would be the weighted sum of all individual rates of returns, viz.

$r = w_1 r_1 + w_2 r_2 + \ldots + w_n r_n$,

where $\sum_{i=1}^{n} w_i = 1$

By taking expected values on both sides of the above expression, we get

$$E(r) = w_1 E(r_1) + w_2 E(r_2) + \ldots + w_n E(r_n)$$

$$= \sum_{i=1}^{n} w_i E(r_i)$$

$$= \sum_{i=1}^{n} w_i p_i r_i \text{ ...... (3.18)} \quad [\because E(r_i) = p_i r_i]$$

It shows the expected rate of return of a portfolio or the portfolio mean. So, the portfolio mean signifies the weighted sum of expected rates of returns of individual assets in a portfolio.

Example 3.11

Let a portfolio consist of equity shares of 5 companies with a proportion of 1 : 2 : 3 : 4 : 5.

Now, the rates of return for each of these assets under three different economic situations, viz. good (with a probability of 25%), average (with a probability of 50%) and bad (with a probability of 25%) are shown below :

Share	Rate of return (%) under		
	Good economic environment (p_1 = 25%)	Average economic environment (p_2 = 50%)	Bad economic environment (p_3 = 25%)
Share - 1	20	18	15
Share - 2	30	24	20
Share - 3	25	12	−6
Share - 4	12	24	30
Share - 5	40	30	20

Here, we are to find out expected return of each share as well as the expected return of the portfolio.

Since 5 types of stocks in this portfolio are held in a proportion 1 : 2 : 3 : 4 : 5,

So weightage of share-1 in the portfolio would be $\frac{1}{1+2+3+4+5} = \frac{1}{15} = w_1$

Similarly, weightage of Share-2 $= \frac{2}{15} = w_2$

weightage of Share-3 $= \frac{3}{15} = w_3$

weightage of Share-4 $= \frac{4}{15} = w_4$, and

weightage of Share-5 $= \frac{5}{15} = w_5$.

Now, expected return from share-1, will be $E(r_1) = \sum_{j=1}^{3} p_j r_{1j}$ [Here, $p_1 = 25\%$, $p_2 = 50\%$, $p_3 = 25\%$]

$= (0.25 \times 20) + (0.5 \times 18) + (0.25 \times 15)$

$= 17.75\%$

Similarly,

$E(r_2) = (0.25 \times 30) + (0.5 \times 24) + (0.25 \times 20)$

$= 24.5\%$

$E(r_3) = (0.25 \times 25) + (0.5 \times 12) + (0.25 \times -6)$

$= 13.25 - 1.5 = 10.75\%$

$E(r_4) = (0.25 \times 12) + (0.5 \times 24) + (0.25 \times 30)$

$= 22.5\%$

$E(r_5) = (0.25 \times 40) + (0.5 \times 30) + (0.25 \times 20)$

$= 30\%$

So, the expected return from the portfolio would be $E(r) = \sum_{i=1}^{5} w_i \cdot E(r_i)$

$= \left(\frac{1}{15} \times 17.75\right) + \left(\frac{2}{15} \times 24.5\right) + \left(\frac{3}{15} \times 10.75\right) + \left(\frac{4}{15} \times 22.5\right) + \left(\frac{5}{15} \times 30\right)$

$= 22.6\%$

3.4.2. Portfolio risk or portfolio variance

The risk of a portfolio of assets is measured by the variance of its return. The variance of the portfolio return is the expected squared deviation from the expected return. It can be expressed as follows :

$$\sigma_p^2 = E(r-\bar{r})^2 = \sum_{i=1}^{n} p_i (r_i - \bar{r})^2 \text{ ------ } (3.19)$$

Where σ_p^2 = Variance of the portfolio rate of return

r = Actual rate of return from the portfolio

$\bar{r}$ = Expected rate of return from the portfolio.

i = Economic environment (say: good, average and worst)

p_i = Probability of occurence of an economic environment

Example 3.12

Let us consider the information given in Example - 3.11, i.e., the investor has a portfolio consisting of 5 stocks (say, of 5 companies) in the proportion of 1 : 2 : 3 : 4 : 5, and the rates of return of this portfolio are 26·87%, 23·20% and 17·13% under good, average and worst business environments respectively with probabilities of occurrence of such business environment being 25%, 50% and 25% respectively. Here, we are to calculate the portfolio risk.

Solution :

Here, portfolio risk can be calculated on the basis of portfolio variance. Let us arrange the information in the following way :

State of the business environment	Good	Average	Worst
(i) Probability of occurrence of the business environment (p_i) :	25%	50%	25%
(ii) Rate of return from portfolio (r_i) :	26·87%	23·2%	17·13%
(iii) Expected rate of return from portfolio ($\bar{r}$) :	(0·25×26·87) + (0·50×23·2) + (0·25×17·13) = 22·6		
(iv) Deviations from expected rate of return $(r_i - \bar{r})$:	4·27	0·60	−5·47
(v) Square of deviation $(r_i - \bar{r})^2$:	18·20	0·36	29·88
(vi) $p_i \cdot (r_i - \bar{r})^2$:	(0·25×18·2) = 4·55	(0·5×0·36) = 0·18	(0·25×29·88) = 7·47
(vii) Portfolio Variance $\sigma_p^2 = \sum p_i (r_i - \bar{r})^2 = 4{\cdot}55 + 0{\cdot}18 + 7{\cdot}47 = 12{\cdot}20$			
(viii) Portfolio standard deviation $\sigma_p = \sqrt{\sigma_p^2} = \sqrt{12{\cdot}2} = 3{\cdot}49$			

The estimation of portfolio as shown in the formula (3.19) can also be expressed as follows :

$$\sigma_p^2 = E\left[(r-\bar{r})^2\right]$$

$$= E\left[\left(\sum w_i r_i - \sum w_i \bar{r}_i\right)^2\right]$$

$$= E\left[\left(\sum w_i r_i - \sum w_i \bar{r}_i\right)\left(\sum w_j r_j - \sum w_j \bar{r}_j\right)\right]$$ [w_i = weight of i-th asset in the portfolio]

[w_j = weight of j-th asset in the portfolio]

$$= E\left[\left\{\sum w_i (r_i - \bar{r}_i)\right\}\left\{\sum w_j (r_j - \bar{r}_j)\right\}\right]$$

$$= E\left[\sum_{i,j=1}^{n} w_i . w_j (r_i - \bar{r}_i)(r_j - \bar{r}_j)\right]$$

$$= \sum_{i,j=1}^{n} w_i . w_j E\left[(r_i - \bar{r}_i)(r_j - \bar{r}_j)\right]$$

$$= \sum_{i,j=1}^{n} w_i . w_j Cov(r_i, r_j)$$

$$= \sum_{i,j=1}^{n} w_i . w_j . \sigma_{ij} \text{ .... } (3.20)$$

If a portfolio consists of only two assets 1 and 2 then ($i, j = 1, 2$)

$$\sigma_p^2 = \sum_{i,j=1}^{2} w_i . w_j . \sigma_{ij}$$

$$= w_1 . w_1 \sigma_{11} + w_1 w_2 \sigma_{12} + w_2 w_1 \sigma_{21} + w_2 w_2 \sigma_{22}$$

$$= w_1^2 . \sigma_1^2 + 2 w_1 w_2 \sigma_{12} + w_2^2 \sigma_2^2$$

Since $\sigma_{12} = \sigma_{21}$ and $\sigma_{ii} = \sigma_i^2$ [we have already noted that the correlation coefficient $\rho_{11} = \frac{\sigma_{11}}{\sigma_1 \sigma_1} = \frac{\sigma_{11}}{\sigma_1^2}$ and since $\rho_{11} = 1$, hence $\sigma_{11} = \sigma_1^2$; and in a similar way, $\sigma_{22} = \sigma_2^2$]

Example 3.13

Let us suppose that the investor has only two financial assets (say, stock-1 & stock - 2) in his portfolio. Now based on the following information, calculate the variance of the portfolio as well as the portfolio mean.

(i) Expected return from Stock-1 : $E(r_1) = \bar{r}_1 = 15\%$

(ii) Expected return from Stock-2 : $E(r_2) = \bar{r}_2 = 10\%$

(iii) Standard deviation of Stock-1 : $\sigma_1 = 22\%$

(iv) Standard deviation of Stock-2 : $\sigma_2 = 18\%$

(v) Weightage of Stock-1 (w_1) in the portfolio = 30%

(vi) Weightage of Stock-2 (w_2) in the portfolio = 70%

(vii) Covariance of Stock-1 and 2 : Cov (1, 2) = σ_{12} = 1%

Solution :

We know that portfolio Variance can be calculated on the basis of the following formula :

$$\sigma_p^2 = \sum_{i,j=1}^{2} w_i w_j \sigma_{ij}$$

$$= w_1^2\sigma_1^2 + 2\sigma_{12}w_1w_2 + w_2^2\sigma_2^2$$

$$= (0.3)^2(0.22)^2 + 2 \times (0.01) \times 0.30 \times 0.70 + (0.7)^2(0.18)^2$$

$$= (0.09 \times 0.0484) + 0.0042 + (0.49 \times 0.0324)$$

$$= 0.004356 + 0.0042 + 0.015876$$

$$= 0.024432$$

$$= 2.44\%$$

Therefore, portfolio S.D. $= \sigma_p = \sqrt{\sigma_p^2} = \sqrt{0.024432}$

$$= 0.15631$$

$$= 15.63\%$$

The portfolio mean ($\bar{r}$) can be estimated by the following formula :

$$\bar{r} = \sum_{i=1}^{2} w_i E(r_i) = w_1 E(r_1) + w_2 E(r_2)$$

$$= w_1\bar{r}_1 + w_2\bar{r}_2 \text{ .... (3.21)}$$

$$\therefore \bar{r} = (0.30 \times 0.15) + (0.7 \times 0.10)$$

$$= 0.045 + 0.07$$

$$= 0.115$$

$$= 11.5\%$$

3.4.3. Systematic & Unsystematic risks

Since the rate of return from a financial asset cannot often be predicted in an accurate way, there always remains an unanticipated part of the return from an asset. This part reflects the true risk associated with any investment. According to Stephen A. Ross, Radolph W. Westerfield and Bradford D. Jordan, "The risk of owning an asset comes from surprises – unanticipated events."

The risk which is exogenous to any particular firm or business enterprise and effects almost all financial assets in a similar way, can be termed as **'systematic risk'**. For instance, the impact of sudden changes in the fiscal or monetary policies of the government would affect almost all securities in the financial market in an equal manner. Further, an inflationary or deflationary conditions in an economy can also be considered as an example of systematic risk because such environments generate an impact upon all the stocks or financial assets in a similar fashion. Thus, such systematic risks have an impact upon the whole financial market. So, such risks are also called as **market risks**.

On the other hand, when the risks which are very much specific to any particular industry or any particular group of firms or the financial assets of a particular group, then such risks are considered as unsystematic risks or **asset-specific risks** or **unique risks** or **residual risks** or **idiosyncratic risks**. For example, inadequate adoption of modern technology in an industry will mainly affect that industry. Such unsystematic risks can be avoided or minimised by the investors through portfolio diversification but the systematic risks cannot be avoided or minimised through portfolio diversification. Hence, unsystematic risks are also called as **diversifiable risks**. However, the systematic risks would be considered as **undiversifiable risks**.

3.4.4. Diversification and portfolio risk

The process of spreading an investment across assets (and thereby forming a portfolio) is treated as diversification. It has been observed by several financial analysts that the riskiness associated with individual financial assets (say, the shares of Tata Steel, ONGC, NTPC etc.) can be reduced by forming a portfolio consisting of different types of financial assets. The principle of diversification suggests that an investor can eliminate some of the risks by investing in different types of securities.

For example, the market portfolio's standard deviation (an estimation of risk) for NIFTY (consisting of 50 stocks of National Stock Exchange of India) was estimated to be about 15.28% for the period January 2011 -December 2017. However, the standard deviations of some selected stocks included in the calculation of stock index NIFTY were found to be higher than the standard deviation (based on NIFTY) of the market portfolio as noted below :

Stock	Standard deviation	Stock	Standard deviation
Tata Steel	38.65%	Bharti	28.81%
ONGC	26.25%	Infosys	27.40%
Yes Bank	41.39%	Hindustan Unilever	20.62%
MARKET PORTFOLIO	15.28%		

So we face an important question : if market portfolio is made up of individual stocks then why does not the variability of market portfolio reflect the average variability of the individual stocks included in the portfolio? The only answer is that **diversification can reduce the variance or standard deviation of the rate of returns on a portfolio.**

Diversification becomes effective in reducing the variability of returns from a portfolio since prices of different stocks included in that portfolio do not move in similar fashion, i.e., changes in stock prices are not perfectly correlated. For example, it has been observed that over a period of 85 months (December, 2010 - December, 2017) the stand and deviation of monthly returns of both ONGC and Hero Motocorp stocks remained at about 26%. However, the market prices of these two stocks did not move in exact lockstep, i.e., in several occasions an increase in the market price of ONGC was offset by a decline in the market price of Hero Motocorp (or the vice versa). Hence, if an investor can invest 50% of his portfolio fund in ONGC stock and the remaining 50% in purchasing Hero Motocorp stock then the investor can obviously reduce the monthly fluctuation in the rate of return on portfolio (fig-3.1).

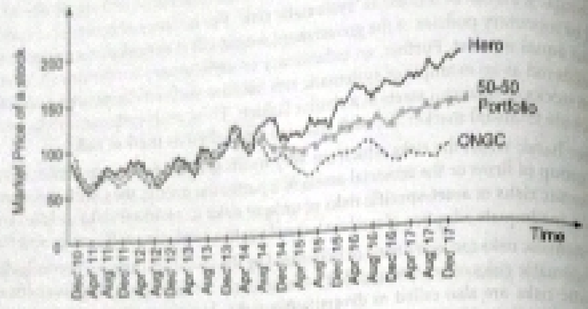


Fig.-3.1

Our previous discussion suggests that with an increase in the number of stocks in a portfolio, standard deviation of the rate of return on portfolio per time period (say, per month or per year) will fall. However, there remains a minimum level of risk which cannot be eliminated through diversification in the portfolio. This minimum level of risk is considered as 'undiversifiable risk' or 'market risk'. (Fig.-3.2)

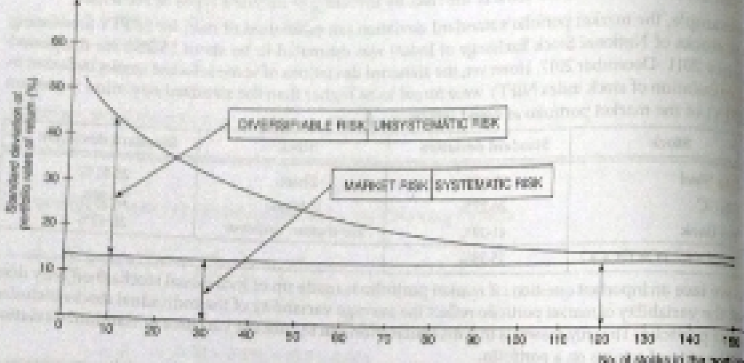


Fig.-3.2

Let us first suppose that there are 'n' number assets in the portfolio and they are mutually uncorrelated i.e., rate of return on any financial asset within the portfolio is not correlated to the rate of return of any other asset in the said portfolio. Further, let us suppose that the average rate of return on each of those assets be denoted by 'm', and the variance of the rate of return on each of those assets be denoted by σ^2.

We can think of a portfolio of assets where weightage of i-th aset may be denoted by $w_i = \frac{1}{n}$ where n = number of assets in the portfolio.

Thus, the average rate of return or the overall rate of return on the portfolio of assets can be expressed as

$$E(r) = \sum_{i=1}^{n} w_i E(r_i) \text{ where } E(r_i) = m,\ w_i = \frac{1}{n} \text{ (by assumption)}$$

$$\text{or, } \bar{r} = \frac{1}{n} \cdot (nm) = m$$

Thus, the mean value of the portfolio rate of return becomes independent of the number of assets (n) in the portfolio.

Similarly, the corresponding portfolio variance can be expressed as :

$$\text{Var}(r) = \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} \qquad \text{[See (3.20)]}$$

$$= \frac{1}{n} \cdot \frac{1}{n} \sum_{i,j=1}^{n} \sigma_{ij} \qquad \left[\because w_i = \frac{1}{n},\ w_j = \frac{1}{n}\right]$$

$$= \frac{1}{n^2}\left(n.\sigma^2\right) \qquad [\because \text{Var}\left(r_i\right) = \sigma^2 \text{by assumption and Cov } (i, j) = 0$$

as the assets are assumed to be uncorrelated]

$$\text{Var}(r) = \frac{\sigma^2}{n}$$

So, in this case, the portfolio risk (σ^2) will fall with an increase in the number of assets (n) in the portfolio.

However, if the return from different assets are correlated then the result will be a bit different.

If there are two stocks in a portfolio then the variance of the portfolio will be sum total of the four boxes as shown below (we have already explained it in formula 3.20) :

	Stock-1	Stock-2
Stock-1	$w_1^2\sigma_1^2$	$w_1 w_2 \sigma_{12} = w_1 w_2 \rho_{12} \sigma_1 \sigma_2$
Stock-2	$w_2 w_1 \sigma_{21} = w_2 w_1 \rho_{12} \sigma_1 \sigma_2$	$w_2^2\sigma_2^2$

[$\because$ correlation coefficient between Stock-1 and Stock-2 = $\rho_{12} = \frac{\text{Cov }(1, 2)}{\text{SD }(1)\text{SD }(2)} = \frac{\sigma_{12}}{\sigma_1 . \sigma_2}$]

Here, the top-left box shows the variance of Stock-1 weighted by the square of the proportion of the portfolio invested in Stock-1. Similarly, the bottom-right box shows the variance of Stock-2 weighted by the square of the proportion of portfolio spent on Stock-2. However, the entries in other two boxes depend on the co-variance of Stock-1 and Stock-2, and hence, upon the correlation coefficient (ρ_{12}) between Stock-1 and Stock-2.

If ρ_{12} is positive then σ_{12} is also positive. Again, if the prospects or the rate of returns from these two stocks move in opposite directions then ρ_{12} becomes negative and hence, σ_{12} also becomes negative.

However, if the rates of return from Stock-1 and Stock-2 are completely uncorrelated then $\rho_{12} = 0$, and hence, $\sigma_{12} = 0$.

Now, if we assume that $E(r_i) = m$, $w_i = \frac{1}{n}$ (just like our previous discussion), $Var(r_i) = \sigma^2$ and in addition we consider $Cov(i, j) = \sigma_{ij} > 0$ for $(i \neq j)$. Then the variance of the rate of return on the portfolio would be expressed as

$$Var(r) = E\left[\sum_{i,j} w_i (r_i - \bar{r})\right]^2$$

$$\sigma_p^2 = E\left[\sum_{i,j} \frac{1}{n}(r_i - \bar{r})\right]^2 \qquad \left[\because w_i = \frac{1}{n}\right]$$

$$= \frac{1}{n^2} E\left[\left\{\sum_{i=1}^{n}(r_i - \bar{r})\right\}\left\{\sum_{j=1}^{n} r_j - \bar{r}\right\}\right]$$

$$= \frac{1}{n^2}\sum_{i,j}\sigma_{ij}$$

$$= \frac{1}{n^2}\left\{\sum_{i=j}\sigma_{ij} + \sum_{i\neq j}\sigma_{ij}\right\}$$

$$= \frac{1}{n^2}\left\{n\sigma^2 + \sum_{i\neq j}\sigma_{ij}\right\} \qquad \left[\because \sigma_{ii} = \sigma^2\right]$$

$$\sigma_p^2 = \frac{1}{n}\left(\sigma^2\right) + \frac{1}{n^2}\left(n^2 - n\right)Cov(i, j)$$

$$\sigma_p^2 = \frac{1}{n}\sigma^2 + \left(1 - \frac{1}{n}\right)\bar{\sigma}_{ij} \quad \text{........ (3.22)}$$

Here σ^2 = average variance of each stock

$\bar{\sigma}_{ij}$ = average covariance between i-th and j-th stocks.

In this case, as the number of stocks (n) rises in the portfolio then in the limit as $n \to \infty$, $\frac{1}{n} \cong 0$, i.e. $\frac{\sigma^2}{n}$ term will vanish in (3.22) but the average covariance term $\bar{\sigma}_{ij}$ will remain. Hence, **portfolio risk cannot be brought down to zero through diversification if the rates of returns on stocks are correlated.**

[Note : Here, $\sum_{i\neq j}\sigma_{ij} = \left(n^2 - n\right)Cov(i, j)$ because products of 'n' terms taking 2 at a time (i and j) can be formed in ${}^nc_2$ ways. Out of ${}^nc_2$ product terms, each term can again be arranged in two ways (say, ab and ba, $a \neq b$). So ${}^nc_2$ number of product terms can be arranged in ${}^nc_2 \cdot 2!$ ways.

or, $\frac{n!}{2!(n-2)!} \cdot 2!$ ways

or, $\frac{n!}{(n-2)!}$ ways

or, $\frac{n(n-1)(n-2)!}{(n-2)!} = n(n-1)$

$= n^2 - n$ ways.]

3.4.5. Risk-return profile of a two asset portfolio

In case of a two asset portfolio, the portfolio rate of return is expressed as

$$r = \sum_{i=1}^{2} r_i w_i = w_1 r_1 + w_2 r_2 \text{ (See 3.10)}$$

and the expected return from this portfolio is expressed as

$$E(r) = \sum_{i=1}^{2} w_i E(r_i) = \sum_{i=1}^{2} w_i \cdot P_i r_i \text{ (See 3.18)}$$

Similarly, the portfolio risk of a two-asset portfolio can be estimated by the variance of the rate of return on this portfolio $\left(\sigma_p^2\right)$ where

$$\sigma_p^2 = \sum_{i,j} w_i w_j \sigma_{ij} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \sigma_{12}$$

$$= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \rho_{12}\sigma_1\sigma_2 \text{ (See 3.20)}$$

$$\left[\because \rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}\right]$$

Let us now consider different possible mix of two assets (say, Stock-1 and Stock-2) in the portfolio, and the corresponding expected rate of return and risk [Table - 3.1]

Table - 3.1

	Different proportions of two stocks						
Weightage	A	B	C	D	E	F	G
w_1	100	80	60	50	40	20	0
w_2	0	20	40	50	60	80	100
	Expected return and risk						
$E(r) = \bar{r}$	15	18	21	22.5	24	27	30
$SD(r) = \sigma_p$	10	12	14	15	16	18	20

If the investor invests only in Stock-1 then $w_1 = 100\%$, $w_2 = 0$; and the corresponding portfolio rate of return ($\bar{r}$) is 15%, and portfolio risk (σ_p) is 10% [portfolio risk has been estimated by the standard

deviation of the expected rate of return]. Here, $\bar{r}$ and σ_p actually indicate the expected return and associated with Stock-1 [shown by portfolio mix-A]. Similarly, if the investor invests only in Stock-2 (shown by the portfolio mix G). Then $w_1 = 0$, $w_2 = 100\%$, and the corresponding $\bar{r}$ and σ_p are 30% and 20% respectively. Here, we observe that as the risk (σ_p) is doubled (from 10% to 20%), the expected rate of return ($\bar{r}$) is also doubled (from 15% to 30%). It implies that there remains a perfect positive correlation between Stock-1 and Stock-2 (i.e., $\rho_{12} = 1$).

It is to be noted that here, we have computed the portfolio return ($\bar{r}$) and risk (σ_p) in the following manner :

$$\bar{r} = w_1 r_1 + w_2 r_2 \qquad [\text{Here } r_1 = 15\%, r_2 = 30\%]$$

$$\therefore \bar{r}(B) = 0{\cdot}8 \times 0{\cdot}15 + 0{\cdot}2 \times 0{\cdot}30 = 0{\cdot}12 + 0{\cdot}06 = 0{\cdot}18 = 18\%$$

$$\bar{r}(C) = 0{\cdot}6 \times 0{\cdot}15 + 0{\cdot}4 \times 0{\cdot}30 = 0{\cdot}09 + 0{\cdot}12 = 0{\cdot}21 = 21\%$$

$$\bar{r}(D) = 0{\cdot}5 \times 0{\cdot}15 + 0{\cdot}5 \times 0{\cdot}30 = 0{\cdot}075 + 0{\cdot}15 = 0{\cdot}225 = 22{\cdot}5\%$$

$$\bar{r}(E) = 0{\cdot}4 \times 0{\cdot}15 + 0{\cdot}6 \times 0{\cdot}30 = 0{\cdot}06 + 0{\cdot}18 = 0{\cdot}24 = 24\%$$

$$\bar{r}(F) = 0{\cdot}2 \times 0{\cdot}15 + 0{\cdot}8 \times 0{\cdot}30 = 0{\cdot}03 + 0{\cdot}24 = 0{\cdot}27 = 27\%$$

In this case, portfolio risk will be :

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2$$

Since we have assumed $\rho_{12} = 1$ in this case, so

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2$$

$$\sigma_p^2 = (w_1\sigma_1 + w_2\sigma_2)^2$$

$$\sqrt{\sigma_p^2} = \sigma_p = (w_1\sigma_1 + w_2\sigma_2) \text{ .... } (3.23)$$

So, in this case, the portfolio risk becomes the weighted average of risks of individual assets. So, for different possible portfolio mix, the σ_p has been computed in the following manner :

[Here $\sigma_1 = 10\%$, $\sigma_2 = 20\%$]

$$\sigma_p(B) = 0{\cdot}8 \times 0{\cdot}10 + 0{\cdot}2 \times 0{\cdot}20 = 0{\cdot}08 + 0{\cdot}04 = 0{\cdot}12 = 12\%$$

$$\sigma_p(C) = 0{\cdot}6 \times 0{\cdot}10 + 0{\cdot}4 \times 0{\cdot}20 = 0{\cdot}06 + 0{\cdot}08 = 0{\cdot}14 = 14\%$$

$$\sigma_p(D) = 0{\cdot}5 \times 0{\cdot}10 + 0{\cdot}5 \times 0{\cdot}20 = 0{\cdot}05 + 0{\cdot}10 = 0{\cdot}15 = 15\%$$

$$\sigma_p(E) = 0{\cdot}4 \times 0{\cdot}10 + 0{\cdot}6 \times 0{\cdot}20 = 0{\cdot}04 + 0{\cdot}12 = 0{\cdot}16 = 16\%$$

$$\sigma_p(F) = 0{\cdot}2 \times 0{\cdot}10 + 0{\cdot}8 \times 0{\cdot}20 = 0{\cdot}02 + 0{\cdot}16 = 0{\cdot}18 = 18\%$$

It is important to note in this connection that when prospects from two stocks move in exact lockstep, i.e., when $\rho_{12} = +1$ (perfectly correlated), there would be no possibility of gain from portfolio diversification.

Now, all those possible risks and returns for different mix of Stock-1 and Stock-2 in the portfolio can be plotted in a diagram (Fig.-3.3).

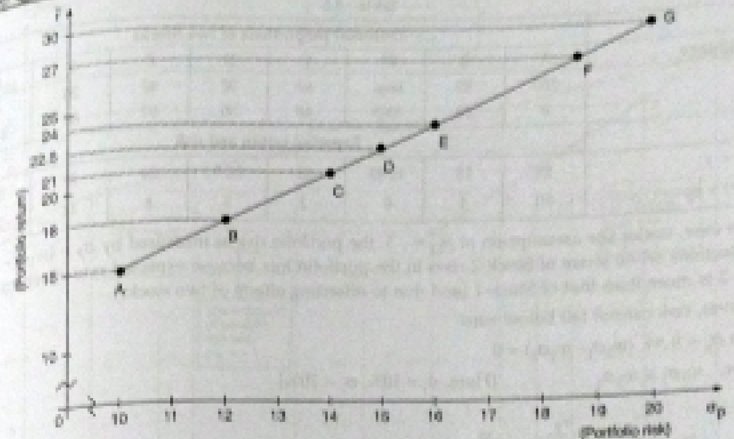


Fig.-3.3

In Fig.-3.3, we find that the upper bound of the portfolio 'risk and return' lies on a straight line showing a linear relation between portfolio risk and return (with the assumption that $\rho_{12} = 1$). Here, the expression for the upper bound, i.e., $(w_1\sigma_1 + w_2\sigma_2)$ is linear in 'w_i', just like the expression for mean. Here any portfolio consisting of these two stocks (Stock-1 and Stock-2) cannot have a standard deviation (σ_p) that plots to the right of the straight line connecting A and G. Instead the standard deviation must lie on this straight line (AG) or to the left of it. Here formula (3.21) and (3.23), i.e., linear expressions for the estimation of portfolio rate of return ($\bar{r}$) and portfolio risk (σ_p), indicate that portfolio mean and standard deviation move proportionately to w_i between their values $w_i = 0$ and $w_i = 100\%$ provided that $\rho_{12} = 1$. As w_i varies from 0 to 100%, the portfolio point traces out a straight line between points A and G as shown in Fig.-3.3.

- **Perfect negative correlation between Stock-1 and Stock-2 :**

Let us now assume that the prospects of those two stocks or the expected rate of return from Stock-1 and Stock-2 move in opposite directions, so that $\rho_{12} < 0$. We assume that there remains perfect negative correlation between the prospects of these two stocks so that $\rho_{12} = -1$. In that case, the variance of the portfolio rate of return would be

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 - 2w_1w_2\sigma_1\sigma_2 \quad [\because \rho_{12} = -1]$$

$$\sigma_p^2 = (w_1\sigma_1 - w_2\sigma_2)^2$$

$$\text{or,} \quad \sqrt{\sigma_p^2} = \sigma_p = (w_1\sigma_1 - w_2\sigma_2) \text{ .... } (3.24)$$

Here, the diversifiable risk or the unsystematic risk of the portfolio can be made zero. This can be shown with the help of the following table. [Table - 3.2]

Table - 3.2

Weightage	Different proportions of two Stocks							
	A	B	B*	C	D	E	F	G
w_1	100	80	66·6	60	50	40	20	0
w_2	0	20	33·3	40	50	60	80	100
	Expected return and risk							
$E(r) = \bar{r}$	15	18	19·98	21	22·5	24	27	30
$SD(r) = \sigma_p$	10	4	0	2	5	8	14	20

In this case, under the assumption of $\rho_{12} = -1$, the portfolio risk as measured by $\sigma_p = (w_1\sigma_1 - w_2\sigma_2)$ first declines when share of Stock-2 rises in the portfolio mix because expected rate of return from Stock-2 is more than that of Stock-1 (and due to offsetting effects of two stocks).

However, risk cannot fall below zero.

When $\sigma_p = 0 \Rightarrow (w_1\sigma_1 - w_2\sigma_2) = 0$

or, $w_1\sigma_1 = w_2\sigma_2$ [Here, $\sigma_1 = 10\%$, $\sigma_2 = 20\%$]

$$\text{or,}\quad \frac{w_1}{w_2} = \frac{\sigma_2}{\sigma_1} = \frac{\dfrac{\sigma_2}{\sigma_1+\sigma_2}}{\dfrac{\sigma_1}{\sigma_1+\sigma_2}} = \frac{\dfrac{20}{10+20}}{\dfrac{10}{10+20}}$$

$$\text{or,}\quad \frac{w_1}{w_2} = \frac{2/3}{1/3}$$

$\therefore$ if $w_1 = \frac{2}{3} = 66\cdot6\%$ and $w_2 = \frac{1}{3} = 33\cdot3\%$ then $\sigma_p = 0$

In that case, $E(r) = \bar{r} = 0\cdot666 \times 0\cdot15 + 0\cdot333 \times 0\cdot30$ Here, $r_1 = 15\%$

$= 0\cdot0999 + 0\cdot0999$ $r_2 = 30\%$

$= 0\cdot1998$ $w_1 = 66\cdot6\%$

$= 19\cdot98\%$ $w_2 = 33\cdot3\%$

It is to be noted that portfolio risk (viz. diversifiable risk) cannot fall below zero.

When the proportion of Stock-2 rises in the portfolio then there comes a point where (B) off-setting risk is complete. From there both risk and return start following the profile of Asset-2 (i.e., higher the risk, higher is the return), and in that case we calculate the modulus value of* $(w_1\sigma_1 - w_2\sigma_2)$, *i.e.,* $|w_1\sigma_1 - w_2\sigma_2|$. *This is shown below :*

$\sigma_p(B) = (0\cdot8 \times 0\cdot10) - (0\cdot2 \times 0\cdot2) = 0\cdot08 - 0\cdot04 = 0\cdot04 = 4\%$

$\sigma_p(B^*) = (0\cdot666 \times 0\cdot10) - (0\cdot333 \times 0\cdot2) = 0\cdot0666 - 0\cdot0666 = 0$

$\sigma_p(C) = |(0\cdot6 \times 0\cdot10) - (0\cdot4 \times 0\cdot2)| = |0\cdot06 - 0\cdot08| = 0\cdot02 = 2\%$

$\sigma_p(D) = |(0\cdot5 \times 0\cdot10) - (0\cdot5 \times 0\cdot2)| = |0\cdot05 - 0\cdot01| = 0\cdot05 = 5\%$

$\sigma_p(E) = |(0\cdot4 \times 0\cdot10) - (0\cdot6 \times 0\cdot2)| = |0\cdot04 - 0\cdot12| = 0\cdot08 = 8\%$

$\sigma_p(F) = |(0\cdot2 \times 0\cdot10) - (0\cdot8 \times 0\cdot2)| = |0\cdot02 - 0\cdot16| = 0\cdot14 = 14\%$

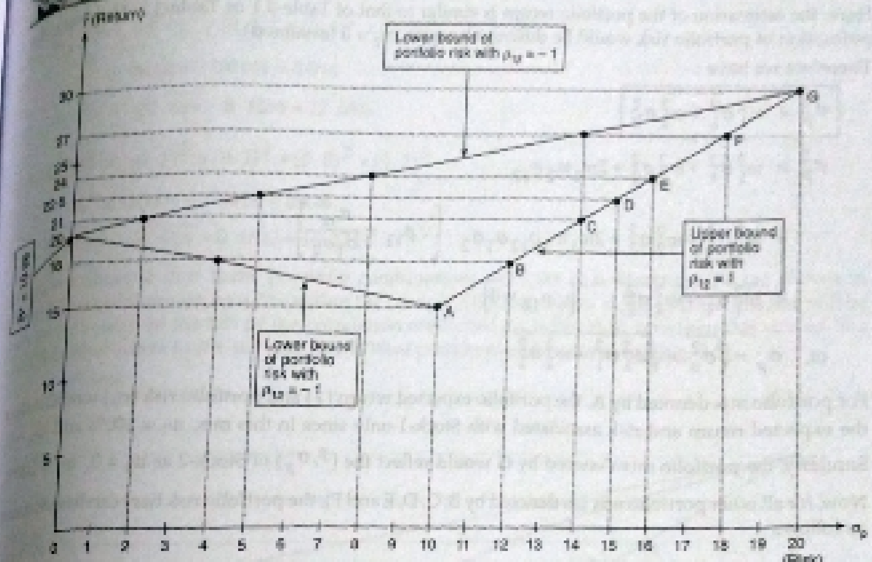


Fig.-3.4

It is interesting to note that when we plot the risk-return combinations for different portfolio mix with an assumption that there remains perfect negative correlation between the prospects of Stock-1 and 2 (i.e., $\rho_{12} = -1$) then the lower bounds of risk-return mix will lie on one of the two line segments i.e., either on the line segment AB* or on the segment B*G (Fig.-3.4).

Any portfolio consisting of Stock-1 and 2 cannot have standard deviation (σ_p) of the portfolio rate of return that plots to the left of either of these two line segments (AB* and B*G). For example, portfolio B must lie on the horizontal line going through the vertical axis at 18% ($\bar{r}$) but bounded between 4% (lower bound) and 12% (upper bound).

Thus, any portfolio consisting of securities (or stocks) denoted by point A and G will lie within or on the boundary of the triangle (AB*G) as shown in Fig.-3.4.

- **Prospects or the expected rate of returns from Stock-1 and Stock-2 are not correlated, i.e., $\rho_{12} = 0$**

Let us now assume that Stock-1 and Stock-2 within the portfolio are not correlated. In this case, the risk-return mix of the portfolio will be as follows (Table-3.3).

Table - 3.3

Weightage	Different proportions of two Stocks						
	A	B	C	D	E	F	G
w_1	100	80	60	50	40	20	0
w_2	0	20	40	50	60	80	100
	Expected return and risk						
$E(r) = \bar{r}$	15	18	21	22.5	24	27	30
$SD(r) = \sigma_p$	10	8·94	10	11·18	12·65	16·12	20

Here, the estimation of the portfolio return is similar to that of Table-3.1 or Table-3.2. However, estimation of portfolio risk would be different because $\rho_{12} = 0$ (assumed).

Therefore we have :

$$\boxed{\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2}$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$$

$$= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 \quad \left[\because \rho_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2}\right]$$

$$= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 \quad [\because \rho_{12} = 0]$$

$$\text{or, } \sigma_p = \sqrt{\sigma_p^2} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2}$$

For portfolio mix denoted by A, the portfolio expected return ($\bar{r}$) and portfolio risk (σ_p) would reflect the expected return and risk associated with Stock-1 only since in this mix, $w_1 = 100\%$ and $w_2 = 0$. Similarly, the portfolio mix denoted by G would reflect the ($\bar{r}, \sigma_p$) of Stock-2 as $w_1 = 0$, $w_2 = 100\%$.

Now, for all other portfolio mix (as denoted by B, C, D, E and F), the portfolio risk here can be estimated as follows :

(i) $\sigma_p^2(B) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2$ [Here, $w_1 = 80\%$, $w_2 = 20\%$, $\sigma_1 = 10\%$, $\sigma_2 = 20\%$]

$$= (0.8)^2 \times (0.1)^2 + (0.2)^2 \times (0.2)^2$$

$$= (0.64 \times 0.01) + (0.04 \times 0.04)$$

$$= 0.008$$

$$\sigma_p(B) = \sqrt{0.008} = 0.0894 = 8.94\%$$

(ii) $\sigma_p^2(C) = (0.6)^2 \times (0.1)^2 + (0.4)^2 \times (0.2)^2$ [Here $w_1 = 60\%$, $w_2 = 40\%$]

$$= 0.0036 + 0.0064$$

$$= 0.01$$

$$\sigma_p(C) = \sqrt{0.01} = 0.1 = 10\%$$

(iii) $\sigma_p^2(D) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2$

$$= (0.5)^2 \times (0.1)^2 + (0.5)^2 \times (0.2)^2 \quad [\text{Here } w_1 = w_2 = 50\%]$$

$$= 0.0025 + 0.01 = 0.0125$$

$$\sigma_p(D) = \sqrt{0.0125} = 0.1118 = 11.18\%$$

(iv) $\sigma_p^2(E) = (0.4)^2 \times (0.1)^2 + (0.6)^2 \times (0.2)^2$

$$= 0.0016 + 0.0144 = 0.016$$

$$\sigma_p(E) = \sqrt{0.016} = 0.1265 = 12.65\%$$

(v) $\sigma_p^2(F) = (0.2)^2 \times (0.1)^2 + (0.8)^2 \times (0.2)^2$

$$= 0.0004 + 0.0256 = 0.026$$

$$\sigma_p(F) = \sqrt{0.026} = 0.1612 = 16.12\%$$

Here, we observe that these portfolio combinations with the risk-return profile (as shown in Table-3.3) would remain on a line within the triangle AB*G (as shown in Fig.-3.4) but this line will be curved or bowed to the left (if the correlation coefficient ρ_{12} is less than zero then this curved line would further move to the left and North-West portion would be concave) [Fig.-3.5]

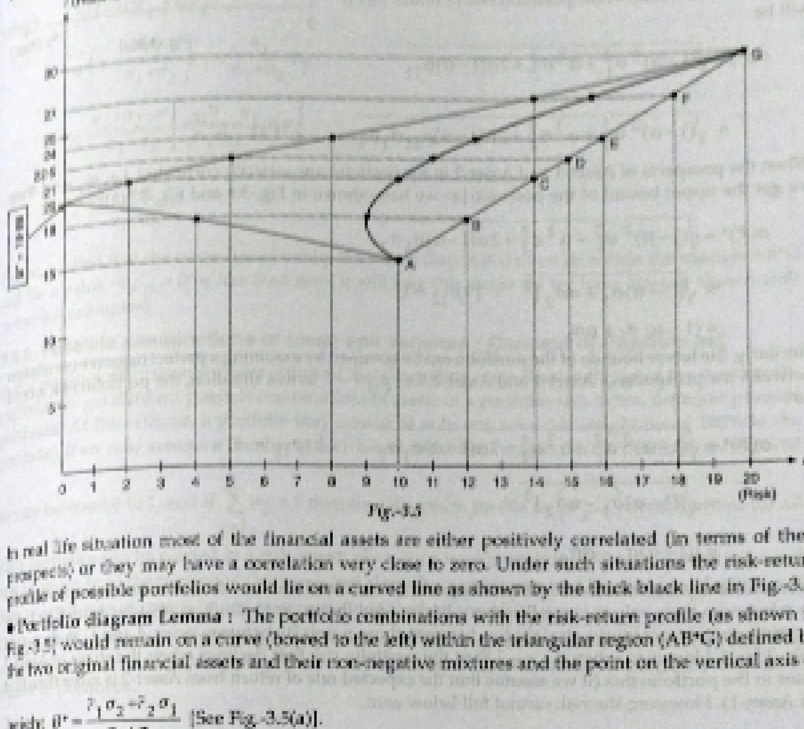


Fig.-3.5

In real life situation most of the financial assets are either positively correlated (in terms of their prospects) or they may have a correlation very close to zero. Under such situations the risk-return profile of possible portfolios would lie on a curved line as shown by the thick black line in Fig.-3.5.

• Portfolio diagram Lemma : The portfolio combinations with the risk-return profile (as shown in Fig.-3.5) would remain on a curve (bowed to the left) within the triangular region (AB*G) defined by the two original financial assets and their non-negative mixtures and the point on the vertical axis of height: $B^* = \dfrac{\bar{r}_1\sigma_2 + \bar{r}_2\sigma_1}{\sigma_1 + \sigma_2}$ [See Fig.-3.5(a)].

Proof : Let us consider a portfolio of two assets and let it be denoted by P.

The rate of return on this portfolio (consisting of asset-1 and asset-2) would be $r(P) = (1-\alpha)r_1 + \alpha r_2$, where α shows the weightage of i-th asset in the portfolio, and the mean value of this portfolio rate of return can be expressed as $\bar{r}(P) = (1-\alpha)\bar{r}_1 + \alpha\bar{r}_2$.

It implies that the portfolio mean lies in between the original means in proportion to the weightage of asset-1 and asset-2 in the portfolio. If asset-1 consists of 50% of the portfolio and the remaining 50% goes to asset-2 then the mean rate of return on portfolio will be on the midway between the original means.

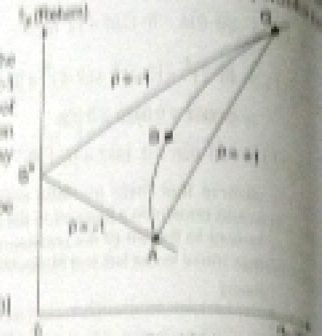


Fig.-3.5(a)

The risk involved in the portfolio rate of return can be measured by the variance of the portfolio return :

$$\sigma^2(P) = (1-\alpha)^2\sigma_1^2 + \alpha^2\sigma_2^2 + 2\alpha(1-\alpha)\sigma_{12}$$

∴ Standard deviation of the portfolio rate of return $[\sigma(P)]$ will be

$$\sigma(P) = \sqrt{(1-\alpha)^2\sigma_1^2 + \alpha^2\sigma_2^2 + 2\alpha(1-\alpha)\sigma_{12}}$$

$$= \sqrt{(1-\alpha)^2\sigma_1^2 + \alpha^2\sigma_2^2 + 2\alpha(1-\alpha)\rho_{12}\sigma_1\sigma_2} \qquad \left[\because \rho_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2}\right]$$

When the prospects of Asset-1 and Asset-2 in the portfolio are perfectly correlated, i.e. $\rho_{12} = +1$ then we get the upper bound of the portfolio [as we have shown in Fig.-3.4 and Fig.-3.5(a)] :

$$\sigma(P)^* = \sqrt{(1-\alpha)^2\sigma_1^2 + \alpha^2\sigma_2^2 + 2\alpha(1-\alpha)\sigma_1\sigma_2}$$

$$= \sqrt{[(1-\alpha)\sigma_1 + \alpha\sigma_2]^2} \qquad [\because \rho_{12} = 1]$$

$$= (1-\alpha)\sigma_1 + \alpha\sigma_2$$

Similarly, the lower bounds of the portfolio can be obtained by assuming a perfect negative correlation between the prospects of Asset-1 and Asset-2, i.e., $\rho_{12} = -1$. In this situation, the portfolio risk would be :

$$\sigma(P)^* = \sqrt{(1-\alpha)^2\sigma_1^2 + \alpha^2\sigma_2^2 - 2\alpha(1-\alpha)\sigma_1\sigma_2} \qquad [\because \rho_{12} = -1]$$

$$= \sqrt{[(1-\alpha)\sigma_1 - \alpha\sigma_2]^2}$$

$$= |(1-\alpha)\sigma_1 - \alpha\sigma_2|$$

From this analysis, it becomes clear that the portfolio mean and standard deviation move proportionally to α between their values at $\alpha = 0$ and $\alpha = 1$ (provided that $\rho_{12} = 1$). Thus, as α varies from 0 to 1, the portfolio point traces out a straight line between these points [in between point A and C as shown in Fig.-3.5(a)]. However, when $\rho_{12} = -1$ then the portfolio risk first declines when the share of Asset-2 rises in the portfolio mix (if we assume that the expected rate of return from Asset-2 is more than that of Asset-1). However, the risk cannot fall below zero.

when $\sigma(P) = 0$ then $(1-\alpha)\sigma_1 - \alpha\sigma_2 = 0$

or, $(1-\alpha)\sigma_1 = \alpha\sigma_2$

or, $$\frac{(1-\alpha)}{\alpha} = \frac{\sigma_2}{\sigma_1} = \frac{\frac{\sigma_2}{\sigma_1+\sigma_2}}{\frac{\sigma_1}{\sigma_1+\sigma_2}}$$

Thus until $\alpha = \dfrac{\sigma_1}{\sigma_1+\sigma_2}$, we get $[(1-\alpha)\sigma_1 - \alpha\sigma_2] > 0$ and after that the sign is reversed, and hence we take the modulus value $|(1-\alpha)\sigma_1 - \alpha\sigma_2|$.

This reversal occurs at point B' [Fig.-3.5(a)]

where $\bar{r} = (1-\alpha)\bar{r}_1 + \alpha\bar{r}_2$

$$= \left(1 - \frac{\sigma_1}{\sigma_1+\sigma_2}\right)\bar{r}_1 + \frac{\sigma_1}{\sigma_1+\sigma_2}\bar{r}_2$$

$$= \frac{\sigma_1+\sigma_2-\sigma_1}{\sigma_1+\sigma_2}\bar{r}_1 + \frac{\sigma_1}{\sigma_1+\sigma_2}\bar{r}_2$$

$$= \frac{\sigma_2\bar{r}_1 + \sigma_1\bar{r}_2}{\sigma_1+\sigma_2}$$

Hence, we find that the curve traced out by the points (Say, ABC) must be within the triangle $AB'C$ and for a value of $\rho_{12} = 0$ or less than zero, it will take this shape (as we have already shown with numerical examples).

3.4.6 Feasible combinations of mean and variance : Concept of Feasible set

In our previous discussion we have observed that depending upon the weightage of the assets in the portfolio we get different possible combinations of assets in a portfolio and hence, different possible portfolios. At the extreme, a portfolio may consist of only one asset (its weight being 100% in the portfolio). If we now assume n number of such assets and if their weights in the portfolio are allowed to vary between 0 to 1, and if $\sum_{i=1}^{n} w_i = 1$ then the risk-return profile or $(\sigma_p, \bar{r})$ combinations for all such portfolios can be plotted in a risk-return plane [in a two dimensional diagram where expected rate of return from the portfolio ($\bar{r}$) is measured along the vertical axis, and the standard deviation of the expected rate of return (σ_p) is measured along the horizontal axis].

The set of all such points representing risk-return profiles of different possible portfolios consisting of n number of assets in different proportions can be considered as possible set of portfolios or feasible region of portfolios.

3.4.7. Properties of a feasible set of portfolios

Some of the important properties of a feasible set of portfolios are as follows :

1. **The feasible set of portfolios will be a solid two-dimensional region.**

Let us assume that there are at least 3 assets in a portfolio which are not perfectly correlated and they have different expected rate of returns. Now, the feasible set of portfolios consisting of these three assets has been shown in Fig.-3.6.

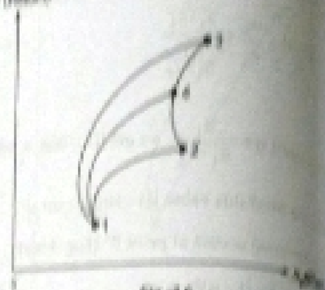


Fig.-3.6

Fig.-3.6 shows that the portfolios formed by asset-1 and 2 can be denoted by the curve 1-2; the portfolios formed by asset-1 and 3 can be denoted by the curve 1-3; and the curve 2-3 denotes the portfolio which can be formed by asset-2 and 3. Now, if the assets 2 and 3 can be combined at a particular proportion to form a new asset-4 then it can again be combined with asset-1 to trace out the possible portfolios denoted by the curve 1-4.

2. **The feasible set of portfolios or the feasible region becomes convex to the left (Fig.-3.7).**

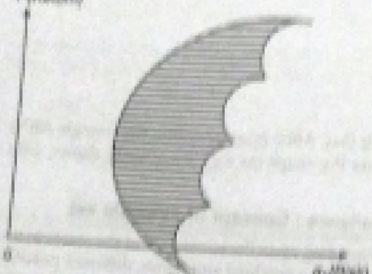


Fig.-3.7

Fig.-3.7 shows that the shaded region would imply the feasible set of portfolios. This feasible set is convex to the left, i.e., if any two points within the set is joined by a straight line then that line lies within the set and does not cross the left boundary. This happens because all possible portfolios formed by two assets (with positive weights) will lie on or to the left of the line connecting those two assets. It is to be kept in mind that while showing the feasible set in Fig.-3.7, we have assumed that there remains no short selling of assets. If there is a possibility of short selling then the investor would have some portfolios without possessing any particular asset at present. This possibility will expand the feasible set.

3.4.8. Minimum variance set and efficient frontier

The risk component associated with any portfolio of assets is measured by the variance of the rate of returns of the portfolio. So, any investor who is a risk-averter, wants to minimise the risk given a particular rate of return on portfolio. A portfolio's efficiency is also judged from this view point.

A portfolio is considered as an inefficient portfolio if :

(i) there exists another portfolio that generates more rate of return with similar risk, or

(ii) there exists another portfolio that generates same rate of return but involves lower risk.

Alternatively speaking, (i) an efficient portfolio can generate more return for a given risk or (ii) an efficient portfolio can reduce the risk for a given rate of return from the portfolio.

This can be shown with the help of a diagram (Fig.-3.8).

In Fig.-3.8, the curve 1-2 represents portfolios formed by different proportions of asset-1 and 2 (with given correlation coefficient). Here, A and D are two such portfolios. In a similar fashion, the curve 3-4 shows portfolios formed by different proportions of asset-3 and 4. In that case, portfolio B and C are two such portfolios. However, here we find that given the portfolio risk σ_P^2, portfolio return ($\bar{r}$) is higher for portfolio A ($\bar{r}_2 > \bar{r}_1$) compared to that of portfolio B. So, A will be considered as an efficient portfolio. Similarly, given the portfolio return $\bar{r}_0$, portfolio C signifies lower risk $\left(\sigma_P^0\right)$ compared to that of portfolio $D\left(\sigma_P^1\right)$. So, portfolio C would be considered as an efficient portfolio. Following this argument, we can show that all portfolios lying on the thick black line (Fig.-3.8) would imply efficient portfolios.

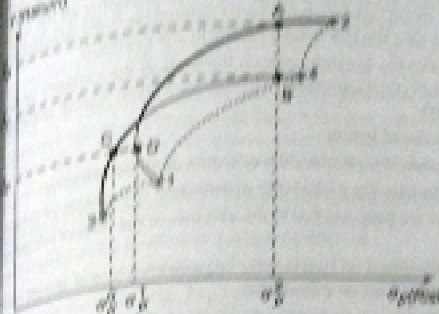


Fig.-3.8

We can show the **minimum variance portfolio set** with the help of a diagram (Fig.-3.9). We have already stated that most of the risk-averse investors prefer minimum variance in the rates of return on a portfolio or a minimum possible risk for a given rate of return.

Fig.-3.9 shows that the left boundary of a feasible set of portfolios can be regarded as the minimum variance set. This is because of the fact that given any portfolio rate of return ($\bar{r}$), the feasible portfolios with minimum variance or with minimum standard deviation (σ_P) would be the left boundary points.

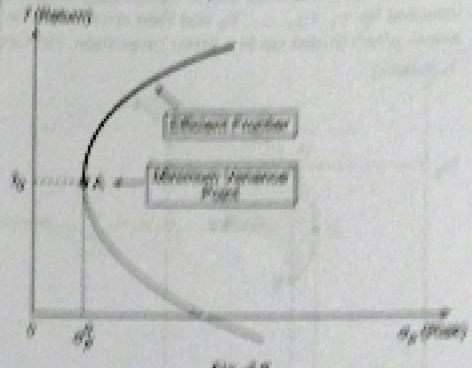


Fig.-3.9

Further, it can also be shown that the **upper portion of the minimum variance portfolio set** denoted by the thick black portion in Fig.-3.9 would be considered as the efficient frontier. This is because of the fact that investors also prefer higher expected rate of return from portfolio for a given risk. This attitude of the investor is termed as 'Nonsatiation' or 'More is preferred to less' (more rate of return is preferred).

3.5. Markowitz Model

In 1952, Harry Markowitz drew attention to the fact that any prudent investor would always try to reduce the risks involved in the rates of return from a portfolio of securities, i.e., minimise the standard

deviation of portfolio returns by choosing securities the prospects of which do not move together ("Portfolio Selection", Journal of Finance, March 1952).

Most of the investors agree that holding two stocks is less risky than holding just one stock. For instance, holding the stocks of textile, banking and IT companies is better than investing all the funds only in stocks of IT companies. So, the most difficult task is to build up an optimal portfolio. Markowitz provides an answer to this problem by analysing the risk-return profile of the portfolio.

This model assumes that the investor estimates the portfolio risk on the basis of the variance or standard deviation of the expected rates of return on the portfolio. Further, for a given level of portfolio return, an investor wants to minimise the risk.

Some of the **assumptions of this model** are stated below :

1. The rate of return on a portfolio adequately summarises the outcome of an investment.
2. The investors are prudent enough to estimate the probability distribution of the rates of return.
3. The risk estimates made by the investors are proportional to the variance of the rates of return on a portfolio.
4. Investment decisions of the investors are based on two criteria : (i) expected rate of return on a portfolio, and (ii) variance of the rates of return on a portfolio.
5. All the investors are risk-averters, i.e., for a given expected rate of return, they want to minimise the portfolio risk or alternatively speaking, given the portfolio risk, they want to maximise the portfolio return.
6. Investors' attitude follow the principle of 'non-satiation', i.e., given the level of risk, they always prefer higher expected rate of return on portfolio.

Let us assume that there are 'n' number of assets. Let the expected rates of return on these assets be denoted by $\bar{r}_1, \bar{r}_2, \text{------}, \bar{r}_n$ and their covariances be denoted by σ_{ij} for $i, j = 1, 2, 3, \ldots, n$. All these assets when mixed up at a given proportion, can form the portfolio of assets. So, a portfolio can be defined as a set of all such assets with n number of weights, i.e., $w_i (i = 1, 2, \ldots, n)$

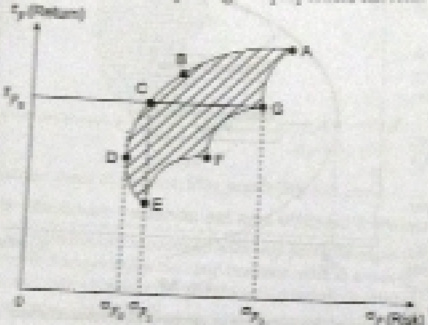


Fig.-3.10

and $\sum_{i=1}^{n} w_i = 1$, the weight of i-th asset (w_i) would be the value of this asset in the total portfolio value.

It is important to note in this connection that if short selling is allowed (where the investor can sell a stock without possessing that stock at present) then there may exist some negative weights.

Markowitz has formalised the risk-return relationship and developed the concept of efficient frontier. We have already discussed the concepts of feasible set of portfolios, minimum variance set and efficient frontier in Subsections 3.4.7 and 3.4.8.

Markowitz has defined the portfolio diversification as the process of combining assets whose prospects are not perfectly positively correlated. If all the possible portfolios of assets are plotted on the risk-return plane in accordance with the $\left(\bar{r}_p, \sigma_p\right)$ combinations of the respective portfolios then

we get portfolio points like A, B, C, D, E, F & G as shown in Fig.-3.10. Out of all these portfolio points the portfolio points A, B, C and D are lying on the efficient frontier.

In fact, an efficient portfolio has the highest return among all portfolios with identical risks; and this efficient portfolio is also characterised by lowest risk among all portfolios with identical expected rate of return. In Fig.-3.10, the shaded area represents all attainable or feasible portfolios, i.e., all possible combinations of risk and expected rate of return which may be achieved with the available securities. However, the efficient frontier contains all possible efficient portfolios and any point on the frontier dominates any other point situated either to the right of it or below it.

In Fig.-3.10, let us consider the portfolios represented by points C and G. Both C and G promise same expected rate of return on portfolio but the risk associated with portfolio G (σ_{p_G}) is higher than that with C (σ_{p_C}). Hence, a risk-averse investor would prefer C to G. Similarly, given the portfolio risk $\sigma_p = \sigma_{p_E}$, portfolio C would be preferred to portfolio E (Fig.-3.10) since C would result in higher expected rate of return $\left(\bar{r}_{p_0}\right)$ compared to that in case of E.

Finding out a minimum variance portfolio :

The Markowitz model suggests that the investor can fix an arbitrary value for the expected rate of return on portfolio (say, $\bar{r}_{p_0}$) and then try to find out minimum variance portfolio.

We know that the portfolio variance is expressed as

$$\sigma_p^2 = \text{Var}(r) = \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} \qquad \text{(See 3.20)}$$

and the expected rate of return on portfolio is expressed as

$$\bar{r}_p = E(r) = \sum_{i=1}^{n} w_i E(r_i) = \sum_{i=1}^{n} w_i \bar{r}_i \qquad \text{(See 3.18)}$$

$$\text{where } \sum_{i=1}^{n} w_i = 1$$

So, the problem before an investor is to minimise $\frac{1}{2} \sum_{i,j=1}^{n} w_i w_j \sigma_{ij}$ —— (i)

subject to $\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r}_p$ ——(ii)

and $\sum_{i=1}^{n} w_i = 1$ .... (iii)

Here, the factor of $\frac{1}{2}$ in front of the variance is put only for convenience, viz., to make the result more neat.

Now, this constrained optimization problem can be presented by the following Lagrange function:

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} - \lambda\left[\sum_{i=1}^{n} w_i \bar{r}_i - \bar{r}_p\right] - \mu\left[\sum_{i=1}^{n} w_i - 1\right] \text{ —— (iv)}$$

Now, for simplicity, we can present this problem for a 'two-asset' situation (i.e., i, j = 1, 2), and equation (iv) is expressed as:

$$L = \frac{1}{2}\left[w_1^2\sigma_1^2 + w_1 w_2 \sigma_{12} + w_2 w_1 \sigma_{21} + w_2^2\sigma_2^2\right] - \lambda\left[w_1\bar{r}_1 + w_2\bar{r}_2 - \bar{r}_p\right] - \mu\left[w_1 + w_2 - 1\right] \text{ —— (v)}$$

Now, differentiating equation (v) partially with respect to w_1, w_2, λ and μ, and setting them equal to zero, we get

$$\frac{\partial L}{\partial w_1} = \frac{1}{2}\left[2w_1\sigma_1^2 + \sigma_{12}w_2 + \sigma_{21}w_2\right] - \lambda\bar{r}_1 - \mu = 0 \text{ —— (vi)}$$

$$\frac{\partial L}{\partial w_2} = \frac{1}{2}\left[\sigma_{12}w_1 + \sigma_{21}w_1 + 2w_2\sigma_2^2\right] - \lambda\bar{r}_2 - \mu = 0 \text{ —— (vii)}$$

$$\frac{\partial L}{\partial \lambda} = w_1\bar{r}_1 + w_2\bar{r}_2 - \bar{r}_p = 0 \text{ —— (viii)}$$

$$\frac{\partial L}{\partial \mu} = w_1 + w_2 - 1 = 0 \text{ —— (ix)}$$

Now, from (vi) and (vii) [with $\sigma_{12} = \sigma_{21}$], we get

$$w_1\sigma_1^2 + \sigma_{12}w_2 - \lambda\bar{r}_1 - \mu = 0 \text{ —— (x)}$$

$$w_2\sigma_2^2 + \sigma_{12}w_1 - \lambda\bar{r}_2 - \mu = 0 \text{ —— (xi)}$$

So, we now have four equations, viz. (viii), (ix), (x) and (xi), and four unknowns, viz. w_1, w_2, λ and μ. Hence, we can solve for the values of these unknown variables.

Now, for n number of assets, the equations for the efficient set of portfolios would be as follows:

$$\sum_{j=1}^{n}\sigma_{ij}w_j - \lambda\bar{r}_i - \mu = 0 \text{ for } i = 1, 2, \ldots, n \text{ —— (xii)}$$

$$\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r}_p \text{ —— (xiii)}$$

$$\sum_{i=1}^{n} w_i = 1 \text{ —— (xiv)}$$

Here, we have '$n + 2$' equations with $n + 2$ unknowns [w_i ($i = 1, 2, \ldots, n$), λ and μ]. So, solution to this system will result the required weights for an efficient portfolio with $\bar{r}_p$.

Example 3.14

Let there be 3 uncorrelated assets with variance of the rates of return $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$. The expected rate of return for these three assets are $\bar{r}_1 = 1$, $\bar{r}_2 = 2$ and $\bar{r}_3 = 3$ respectively; and the covariances of the components of these uncorrelated assets are $\sigma_{12} = \sigma_{23} = \sigma_{13} = 0$. On the basis of these information, we have to determine the values of weights of these assets, viz. w_1, w_2 and w_3 for an efficient portfolio. Further, we can also determine the corresponding values of Lagrange multipliers and the minimum variance for a given value of portfolio return:

(a) $\bar{r}_p = 2$, and (b) $\bar{r}_p = 3$.

Solution:

We know that an efficient portfolio must satisfy the following three equations:

$$\sum_{j=1}^{3}\sigma_{ij}w_j - \lambda\bar{r}_i - \mu = 0 \text{ for } i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{3} w_i \bar{r}_i = \bar{r}_p$$

$$\sum_{i=1}^{3} w_i = 1$$

In our case, we have the following equations:

$$w_1\sigma_1^2 + w_1\sigma_{21} + w_1\sigma_{31} - \lambda\bar{r}_1 - \mu = 0 \text{ —— (1)} \quad [\because \sigma_{11} = \sigma_1^2]$$

$$w_2\sigma_2^2 + w_2\sigma_{12} + w_2\sigma_{32} - \lambda\bar{r}_2 - \mu = 0 \text{ —— (2)} \quad [\because \sigma_{22} = \sigma_2^2]$$

$$w_3\sigma_3^2 + w_3\sigma_{13} + w_3\sigma_{23} - \lambda\bar{r}_3 - \mu = 0 \text{ —— (3)} \quad [\because \sigma_{33} = \sigma_3^2]$$

$$w_1\bar{r}_1 + w_2\bar{r}_2 + w_3\bar{r}_3 = \bar{r}_p \text{ —— (4)}$$

$$w_1 + w_2 + w_3 = 1 \text{ —— (5)}$$

In this problem, it is assumed that

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1; \quad \sigma_{12} = \sigma_{23} = \sigma_{13} = 0;$$

$$\bar{r}_1 = 1, \bar{r}_2 = 2, \text{ and } \bar{r}_3 = 3$$

Therefore, from (1), (2), (3) and (4), we have:

$$w_1 - \lambda - \mu = 0 \text{ .... (6)}$$

$$w_2 - 2\lambda - \mu = 0 \text{ .... (7)}$$

$$w_3 - 3\lambda - \mu = 0 \text{ .... (8)}$$

$$w_1 + 2w_2 + 3w_3 = \bar{r}_p \text{ .... (9)}$$

Now, from (6), (7) and (8), we get :

$w_1 = \lambda + \mu$ ——— (6)′

$w_2 = 2\lambda + \mu$ ——— (7)′

$w_3 = 3\lambda + \mu$ ——— (8)′

Substituting these values of w_1, w_2 and w_3 in (9), we get :

$$(\lambda + \mu) + 2(2\lambda + \mu) + 3(3\lambda + \mu) = \bar{r}_p$$

or, $\lambda + \mu + 4\lambda + 2\mu + 9\lambda + 3\mu = \bar{r}_p$

or, $14\lambda + 6\mu = \bar{r}_p$ ——— (10)

Again, substituting those values of w_1, w_2 and w_3 [as shown in (6)′, (7)′ and (8)′ respectively] in equation (5), we get :

$$(\lambda + \mu) + (2\lambda + \mu) + (3\lambda + \mu) = 1$$

or, $6\lambda + 3\mu = 1$ ——— (11)

Now, multiplying both sides of equation (11) by 2 and then substracting it from equation (10), we get

$$\begin{array}{rcl} 14\lambda + 6\mu &=& \bar{r}_p \\ 12\lambda + 6\mu &=& 2 \\ - \quad\;\; - && - \\ \hline 2\lambda \qquad\;\; &=& \bar{r}_p - 2 \end{array}$$

or, $\lambda^* = \frac{\bar{r}_p}{2} - 1$ ——— (12)

Now, putting this value of λ in equation (11), we get :

$$6\left(\frac{\bar{r}_p}{2} - 1\right) + 3\mu = 1$$

or, $6\left(\frac{\bar{r}_p - 2}{2}\right) + 3\mu = 1$

or, $3\bar{r}_p - 6 + 3\mu = 1$

or, $\bar{r}_p - 2 + \mu = \frac{1}{3}$ [Dividing both sides by 3]

or, $\mu = \frac{1}{3} + 2 - \bar{r}_p$

or, $\mu^* = \frac{7}{3} - \bar{r}_p$ ——— (13)

Here, the portfolio variance would be as follows :

$$\sigma_p^2 = Var(r) = \sum_{i,j=1}^{n} w_i w_j \sigma_{ij}$$

$$= w_1^2\sigma_1^2 + w_1 w_2 \sigma_{12} + w_1 w_3 \sigma_{13} + w_2 w_1 \sigma_{21} + w_2^2 \sigma_2^2 + w_2 w_3 \sigma_{23}$$

$$+ w_3 w_1 \sigma_{31} + w_3 w_2 \sigma_{32} + w_3^2 \sigma_3^2$$

$\therefore \; \sigma_p^2 = w_1^2 + w_2^2 + w_3^2$ [$\because \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$; $\sigma_{12} = \sigma_{23} = \sigma_{13} = 0$; $\sigma_{12} = \sigma_{21}$; $\sigma_{13} = \sigma_{31}$; $\sigma_{23} = \sigma_{32}$]

or, $\sigma_p = \sqrt{\sigma_p^2} = \sqrt{w_1^2 + w_2^2 + w_3^2}$ ——— (14)

Now, substituting the values of λ^* and μ^* of (11) and (12) in equation (6)′, we get

$$w_1 = \lambda + \mu$$

$$= \left(\frac{\bar{r}_p}{2} - 1\right) + \left(\frac{7}{3} - \bar{r}_p\right)$$

$$= \frac{\bar{r}_p}{2} - \bar{r}_p + \frac{7}{3} - 1$$

$$= \frac{\bar{r}_p - 2\bar{r}_p}{2} + \frac{7-3}{3}$$

$$= \frac{-\bar{r}_p}{2} + \frac{4}{3}$$

or, $w_1^* = \frac{4}{3} - \frac{\bar{r}_p}{2}$ ——— (15)

Again, substituting the values of λ^* and μ^* in equation (7)′, we get :

$$w_2 = 2\lambda + \mu$$

$$= 2\left(\frac{\bar{r}_p}{2} - 1\right) + \left(\frac{7}{3} - \bar{r}_p\right)$$

$$= \bar{r}_p - 2 + \frac{7}{3} - \bar{r}_p$$

$\therefore \; w_2^* = \frac{7}{3} - 2 = \frac{1}{3}$ ——— (16)

Similarly, substituting the values of λ^* and μ^* in equation (8)′, we get

$$w_3 = 3\lambda + \mu$$

$$= 3\left(\frac{\bar{r}_p}{2} - 1\right) + \left(\frac{7}{3} - \bar{r}_p\right)$$

$$= \frac{3\bar{r}_p - 6}{2} + \frac{7 - 3\bar{r}_p}{3}$$

$$= \frac{9\bar{r}_p - 18 + 14 - 6\bar{r}_p}{6}$$

$\therefore \; w_3^* = \frac{3\bar{r}_p - 4}{6} = \frac{\bar{r}_p}{2} - \frac{2}{3}$ ——— (17)

Now, substituting the values of w_1^*, w_2^* and w_3^* [From equation (15), (16) and (17) respectively] in equation (14), we get :

$$\sigma_p = \sqrt{w_1^2 + w_2^2 + w_3^2}$$

$$= \sqrt{\left(\frac{4}{3} - \frac{\bar{r}_p}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{\bar{r}_p}{2} - \frac{2}{3}\right)^2}$$

$$= \sqrt{\left\{\left(\frac{4}{3}\right)^2 - 2\left(\frac{4}{3}\right)\left(\frac{\bar{r}_p}{2}\right) + \left(\frac{\bar{r}_p}{2}\right)^2\right\} + \frac{1}{9} + \left\{\left(\frac{\bar{r}_p}{2}\right)^2 - 2\left(\frac{\bar{r}_p}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2\right\}}$$

$$= \sqrt{\frac{16}{9} - \frac{8\bar{r}_p}{6} + \frac{\bar{r}_p^2}{4} + \frac{1}{9} + \frac{\bar{r}_p^2}{4} - \frac{2\bar{r}_p}{3} + \frac{4}{9}}$$

$$= \sqrt{\left(\frac{16}{9} + \frac{1}{9} + \frac{4}{9}\right) + \frac{2\bar{r}_p^2}{4} - \left(\frac{8\bar{r}_p}{6} + \frac{2\bar{r}_p}{3}\right)}$$

$$= \sqrt{\frac{21}{9} + \frac{2\bar{r}_p^2}{4} - \left(\frac{8\bar{r}_p + 4\bar{r}_p}{6}\right)}$$

$$\therefore \sigma_p = \sqrt{\frac{21}{9} + \frac{2\bar{r}_p^2}{4} - 2\bar{r}_p}$$

$$\therefore \sigma_p = \sqrt{\frac{7}{3} - 2\bar{r}_p + \frac{\bar{r}_p^2}{2}} \quad \text{.........} (18)$$

Now, (i) when $\bar{r}_p = 2$ then $\sigma_p = \sqrt{\frac{7}{3} - (2 \times 2) + \frac{4}{2}}$

$$= \sqrt{\frac{7}{3} - 4 + 2}$$

$$\sigma_p = \sqrt{\frac{7-12+6}{3}} = \sqrt{\frac{1}{3}} = 0.577$$

$$= 0.58$$

(ii) When $\bar{r}_p = 3$, then $\sigma_p = \sqrt{\frac{7}{3} - (2 \times 3) + \frac{9}{2}}$

$$= \sqrt{\frac{7}{3} - 6 + \frac{9}{2}}$$

$$= \sqrt{\frac{14-36+27}{6}}$$

$$= \sqrt{\frac{41-36}{6}}$$

$$= \sqrt{\frac{5}{6}} = \sqrt{0.833}$$

$$= 0.913$$

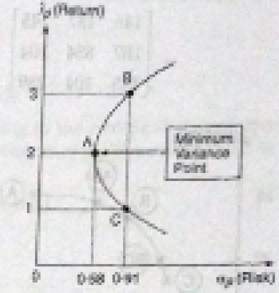


Fig.-3.11

[It is to be noted that in this case, if

$\bar{r}_p = 1$ then $\sigma_p = \sqrt{\frac{7}{3} - (2 \times 1) + \frac{1}{2}}$

$$= \sqrt{\frac{14-12+3}{6}} = \sqrt{\frac{5}{6}}$$

$$= 0.913.]$$

Thus, for $\bar{r}_p = 2$, we get the minimum variance (0.58) point (A).

3.5.1. Two-fund Theorem

We have already discussed that out of a feasible set of portfolios, the investor should be concerned only with the efficient set of portfolios. The efficient set of portfolios as shown by Markowitz includes an infinite number of funds or portfolios. The two fund theorem suggests that if two efficient portfolios or funds can be formed then several other efficient portfolios or funds can be formed in terms of portfolio mean and variance as a combination of those two efficient funds or portfolios.

The process of identifying any efficient portfolio (or fund) as a combination of two efficient portfolios, as suggested by Markowitz, is known as **critical-line method.** This method involves the use of a quadratic programming algorithm. However, this is beyond the scope of this text book. Hence, we can provide simple examples to explain this process.

Let there be three securities or stocks of different companies ; stock-1, stock-2 and stock-3, and the expected rate of return vector on these stocks can be expressed as :

$$E(\bar{r}) = \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_2 \end{bmatrix} = \begin{bmatrix} 16.2 \\ 24.6 \\ 22.8 \end{bmatrix}$$

On the other hand, the variance-covariance (VC) matrix of these stocks can be expressed as :

$$VC(r) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix} \quad \left[\because \sigma_{ii} = \sigma_i^2\right]$$

$$= \begin{bmatrix} 146 & 187 & 145 \\ 187 & 854 & 104 \\ 145 & 104 & 289 \end{bmatrix}$$

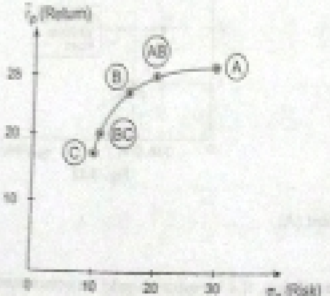


Fig.-3.12

Now, this alogorithm can be used to identify a number of corner portfolios which are associated with these stocks and therefore, can completely trace out the efficient set of portfolios.

A corner portfolio is considered to be an efficient portfolio and any combination of two adjacent corner portfolios will trace out a portfolio that lies on the efficient set of portfolios between the two corner portfolios. In our example, we find that Stock-2 gives highest expected rate of return, i.e., $\bar{r}_2 = 24{\cdot}6\%$.

Now, we can think of a corner portfolio, say, A (as shown in Fig.-3.12) that consists of only Stock-2. Therefore, the corresponding weight vector can be expressed as :

$$w(A) = \begin{bmatrix} 0{\cdot}00 \\ 1{\cdot}00 \\ 0{\cdot}00 \end{bmatrix}$$

In this case, the portfolio return and risk will correspond to the expected return and standard deviation of rates of return on Stock-2 itself, i.e., $\bar{r}_p = 24{\cdot}6\%$ and $\sigma_p = \sqrt{\sigma_2^2} = \sqrt{854} = 29{\cdot}22\%$.

Our algorithm then identifies the second corner portfolio with the following weight vector :

$$w(B) = \begin{bmatrix} 0{\cdot}00 \\ 0{\cdot}22 \\ 0{\cdot}78 \end{bmatrix}$$

It implies that the investor puts 22% of his fund in Stock-2 and the remaining 78% in Stock-3. So, the expected rate of return from this portfolio will be :

$$E(\bar{r}_p)_B = \bar{r}_2 w_2 + \bar{r}_3 w_3 = (0{\cdot}246 \times 0{\cdot}22) + (0{\cdot}228 \times 0{\cdot}78)$$
$$= 0{\cdot}05442 + 0{\cdot}17784$$
$$= 0{\cdot}23226$$
$$= 23{\cdot}23\%.$$

Similarly, the corresponding portfolio risk will be as follows :

$$\text{Var}(r)_B = \left(\sigma_p^2\right)_B = w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_2w_3\sigma_{23}$$
$$= (.22)^2 \times 854 + (.78)^2 \times 289 + (2 \times 0{\cdot}22 \times .78 \times 104)$$
$$= 252{\cdot}84$$

$$\therefore \left(\sigma_p\right)_B = \sqrt{\left(\sigma_p^2\right)_B} = \sqrt{252{\cdot}84} = 15{\cdot}9\%$$

Hence, this corner portfolio has been denoted by point B in Fig.-3.12.

Here, portfolios A and B are adjacent portfolios. Therefore, according to the two-fund theory, any other portfolio formed by the combinations of these two funds (or two corner portfolios) will also lie on the efficient set.

Let us form a weight vector by taking 50% from w(A) and 50% from w(B) :

$$0{\cdot}5 \times w(A) + 0{\cdot}5 \times w(B) = 0{\cdot}5\begin{bmatrix} 0{\cdot}00 \\ 1{\cdot}00 \\ 0{\cdot}00 \end{bmatrix} + 0{\cdot}5\begin{bmatrix} 0{\cdot}00 \\ 0{\cdot}22 \\ 0{\cdot}78 \end{bmatrix}$$

$$= \begin{bmatrix} 0{\cdot}00 \\ 0{\cdot}50 \\ 0{\cdot}00 \end{bmatrix} + \begin{bmatrix} 0{\cdot}00 \\ 0{\cdot}11 \\ 0{\cdot}39 \end{bmatrix}$$

$$w(AB) = \begin{bmatrix} 0{\cdot}00 \\ 0{\cdot}61 \\ 0{\cdot}39 \end{bmatrix}$$

The expected rate of return from this new portfolio will be

$$E(\bar{r})_{AB} = (0{\cdot}61 \times 0{\cdot}246) + (0{\cdot}39 \times 0{\cdot}228)$$
$$= 0{\cdot}2389 = 23{\cdot}9\%$$

and the portfolio risk will be :

$$\text{Var}(r)_{AB} = (\sigma_p^2)_{AB} = (0{\cdot}61)^2 \times 854 + (0{\cdot}39)^2 \times 289 + 2 \times 0{\cdot}61 \times 0{\cdot}39 \times 104$$
$$= 317{\cdot}77 + 43{\cdot}96 + 49{\cdot}48$$
$$= 411{\cdot}21$$

$$\left(\sigma_p\right)_{AB} = \sqrt{\left(\sigma_p^2\right)_{AB}} = \sqrt{411{\cdot}21} = 20{\cdot}28\%$$

This new portfolio AB also lies on the efficient set (as shown in Fig.-3.12).

Thus, the two-fund theorem clearly shows that a combination portfolio of two corner portfolios also represents a point on the minimum variance set.

Here, we can also identify another corner portfolio (denoted by point C in Fig.-3.12) with the following weight vector :

$$w(C) = \begin{bmatrix} 0{\cdot}99 \\ 0{\cdot}00 \\ 0{\cdot}01 \end{bmatrix}$$

So, $E(\bar{r}_p)_C = (0{\cdot}99 \times 0{\cdot}162) + (0{\cdot}01 \times 0{\cdot}226)$

$$= 0{\cdot}16266$$
$$= 16{\cdot}27\%$$

and $\left(\sigma_p^2\right)_C = (0.99)^2 \times 146 + (0.01)^2 \times 289 + 2 \times 0.99 \times 0.01 \times 145$

$$= 143.09 + 0.0289 + 2.871$$

$$= 145.98$$

$$\therefore (\sigma_p)_C = \sqrt{145.98} = 12.08\%$$

Now, from the corner portfolio B and C, we can form another efficient portfolio (denoted by BC in Fig.-3.12) with the following weight vector :

$$0.5 \times \begin{bmatrix} 0.00 \\ 0.22 \\ 0.78 \end{bmatrix} + 0.5 \begin{bmatrix} 0.99 \\ 0.00 \\ 0.01 \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.11 \\ 0.39 \end{bmatrix} + \begin{bmatrix} 0.495 \\ 0.00 \\ 0.005 \end{bmatrix} = \begin{bmatrix} 0.495 \\ 0.11 \\ 0.395 \end{bmatrix}$$

$$\therefore E(\bar{r}_p)_{BC} = w_1 \bar{r}_1 + w_2 \bar{r}_2 + w_3 \bar{r}_3$$

$$= (0.495 \times 0.162) + (0.11 \times 0.246) + (0.395 \times 0.228)$$

$$= 0.08 + 0.027 + 0.09 = 0.197 = 19.7\%$$

and $\left(\sigma_p^2\right)_{BC} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_{12} + 2w_1w_3\sigma_{13} + 2w_2w_3\sigma_{23}$

$$= [(0.495)^2 \times 146] + [(0.11)^2 \times 854] + [(0.395)^2 \times 289]$$
$$+ (2 \times 0.495 \times 0.11 \times 187) + (2 \times 0.495 \times 0.395 \times 145)$$
$$+ (2 \times 0.11 \times 0.395 \times 104)$$
$$= 35.77 + 10.33 + 45.09 + 20.36 + 56.70 + 9.04$$
$$= 177.29.$$

$$\therefore \left(\sigma_p\right)_{BC} = \sqrt{\left(\sigma_p^2\right)_{BC}} = \sqrt{177.29} = 13.31\%$$

This two-fund theory has a spectacular implication. In real life situation, any mutual fund (where the fund is invested in several stocks to diversity the risk) can be considered as a portfolio of assets for any investor. Now, two such mutual funds can be combined to form another efficient portfolio. However, this analysis is based on some particular assumptions such as

(i) every investor is concerned only with the portfolio mean and variance;

(ii) every investor has similar assessment or views regarding portfolio mean, variance and covariance.

(iii) all investments are made for a single period.

3.5.2. Inclusion of a risk-free asset in the portfolio

Our previous discussion suggests that all the assets in the portfolio of an investor constitute risky assets. However, a portfolio can also consist of risk-free assets. In case of risk-free asset, the rate of return is almost certain, e.g., the rate of return on a treasury bill or a dated government security.

We are particularly interested in knowing the implication of a risk-free asset from the view point of Markowitz model. This model assumes a single holding period. Hence, the investor who purchases a risk-free asset at the beginning of a holding period is supposed to know with certainty what the exact value of that asset will be at the end of the holding period.

As there remains no uncertainty regarding the terminal vlaue of the risk-free asset, so $E(r_f) = \bar{r}_f = r_f$ i.e., the expected rate of return on the risk-free asset will be the given rate of return (or assured rate of return) on this asset, i.e., r_f.

So $Var(r_f) = E(r_f - \bar{r}_f) = 0$. Since, $\bar{r}_f = r_f$ and therefore the standard deviation of the rate of return on risk-free asset (σ_f) will be also zero.

Further, the covariance of the prospects of a risk-free asset and a risky asset will be

$$Cov(r, r_f) = E\left[(r - \bar{r})\left(r_f - \bar{r}_f\right)\right]$$

$$= E\left[(r - \bar{r})\left(r_f - r_f\right)\right] = 0 \text{ ......(3.25)} \qquad \left[\because \bar{r}_f = r_f\right]$$

Alternatively,

$$Cov(r, r_f) = \sigma_{if} = \rho_{if}\,\sigma_i\,.\,\sigma_f = 0 \text{ .... (3.26)} \qquad \left[\because \sigma_f = 0\right]$$

Here, r_i = rate of return on i-th risky asset

r_f = rate of return on a risk-free asset.

- **Government bonds may not always be considered as risk-free :**

Although we believe that the Treasury Bill or a dated government bond is a risk-free asset but if the dated government bond has a maturity period greater than the holding period of the investor then this asset cannot be considered as a risk-free asset. For instance, if a dated government bond has a maturity period of 5 years, and the holding period of the investor is 1 year then that asset will be risky for the investor because the market price of that asset may change (because of the change in the market rate of interest) at the end of his holding period. It involves a price risk or an interest rate risk. Similarly, if the maturity period of the government bond is less than the holding period of the investor then also the investor faces the risk of reinvesting the proceeds from the maturity value of that bond as the interest rate that will prevail at the time of reinvestment cannot be estimated beforehand. Therefore, a **government bond can be considered as risk-free only when the maturity of that bond is exactly equal to the holding period of the investor.** When the investor purchases such risk-free asset it is considered as risk-free lending since the investor lends that amount to the government.

When such risk-free asset is added to the portfolio of the investor (consisting of risky assets), it would enlarge the feasible set of portfolios in Markowitz model. Further, the inclusion of such risk-free asset also implies the possibility of lending and borrowing money by the investor at the risk-free rate.

Let us first assume that there remains one risky asset and one risk-free asset in the portfolio of the investor. Therefore, the portfolio return can be expressed as :

$$E(r_p) = \alpha \bar{r}_i + (1 - \alpha)\,\bar{r}_f \text{ .... (3.27)}$$

Where α = proportion of risky asset (i-th asset) in the portfolio

$(1 - \alpha)$ = proportion of risk-free asset in the portfolio.

The variance of the rates of return on this portfolio will be :

$$Var(r_p) = \sigma_p^2 = \alpha^2\sigma_s^2 + (1-\alpha)^2\sigma_f^2 + 2\alpha(1-\alpha)\,\sigma_{sf}$$

$$= \alpha^2\sigma_s^2 \;[\because \sigma_f^2 = 0;\ \sigma_{sf} = 0]$$

or $\quad \sigma_p = \sqrt{\sigma_p^2} = \alpha\sigma_s$ .... (3.28)

From (3.28), we get $\alpha = \dfrac{\sigma_p}{\sigma_s}$

Again, from (3.27), we can write

$$r_p = \alpha r_s + r_f - \alpha r_f$$

$$= r_f + \alpha(r_s - r_f)$$

$$= r_f + \frac{\sigma_p}{\sigma_s}(r_s - r_f) \quad [\because \alpha = \frac{\sigma_p}{\sigma_s}]$$

$$\therefore \quad r_p = r_f + \frac{(r_s - r_f)}{\sigma_s}\,\sigma_p \text{ .... (3.29)}$$

Here, equations (3.27) and (3.28) signify that portfolio mean and standard deviation will vary linearly with α (asset weightage). Again, equation (3.29) shows that the risk-return combination of any portfolio will lie upon a straight line, and the slope of this straight line would suggest the amount of risk premium (when $r_s - r_f > 0$) per 1% risk (σ_s) taken by the investor.

The straight line as shown by equation (3.29) is the locus of all such portfolio mix of a risk-free asset and a risky asset with their risk-return profiles. Sometimes this straight line is also called as *Capital allocation line*.

Example 3.15

Let us consider that an investor prepares a portfolio with one risky asset and one risk-free asset. Let us assume that the given risk-free rate of return (r_f) = 4% and the expected rate of return on the risky asset $E(r_s)$ = 16·2%, and the variance of the rate of return on risky asset is $Var(r_s) = \sigma_s^2 = 146$. The weightages of risk-free asset and risky asset in the portfolio mix are as follows :

Portfolio :	A	B	C	D	E
(i) Risk-free asset :	1·00	0·75	0·50	0·25	0·00
(ii) Risky asset :	0·00	0·25	0·50	0·75	1·00

Estimate the corresponding risk-return profile of these portfolios.

Solution :

For the risk-free asset, the expected rate of return $E(r_f) = \bar{r}_f = r_f = 4\%$

and $Var(r_f) = E(r_f - \bar{r}_f) = 0 \quad [\because \bar{r}_f = r_f]$.

So, the standard deviation of the rate of return on risk-free asset (σ_f) would be zero. (i.e., $\sigma_f = 0$).

In case of risky asset,

$E(r_s)$ = 16·2% and $\sigma_s^2 = 146$; $\sqrt{\sigma_s^2} = \sigma_s = \sqrt{146}$

$= 12·08\%$

Here, the expected rate of return on portfolio will be

$E[r_p] = \alpha r_s + (1-\alpha) r_f$

When $\alpha = 0$ then for portfolio A, we get $E(r_p)_A$ = 4%

When $\alpha = 0·25$ then for portfolio B, $E(r_p)_B$ = (0·25 × 16·2) + (0·75 × 4) = 7·05%

When $\alpha = 0·50$ then for portfolio C, $E(r_p)_C$ = (0·50 × 16·2) + (0·50 × 4) = 10·10%

When $\alpha = 0·75$ then for portfolio D, $E(r_p)_D$ = (0·75 × 16·2) + (0·25 × 4) = 13·15%

When $\alpha = 1·00$ then for portfolio E, $E(r_p)_E$ = 16·2%

Here, the standard deviation of the rate of return on each portfolio is estimated as follows :

$$\sigma_p^2 = \alpha^2\sigma_s^2 + (1-\alpha)^2\sigma_f^2 + 2\alpha(1-\alpha)\sigma_{sf}$$

$$= \alpha^2\sigma_s^2 \;[\because \sigma_f^2 = 0;\ \sigma_{sf} = 0]$$

$$\therefore \sqrt{\sigma_p^2} = \sigma_p = \alpha\,\sigma_s$$

$\therefore$ For portfolio A, $\quad \sigma_p$ (A) = 0 [$\because \alpha = 0$]

For portfolio B, $\quad \sigma_p$ (B) = 0·25 × 12·08 = 3·02

For portfolio C, $\quad \sigma_p$ (C) = 0·50 × 12·08 = 6·04

For portfolio D, $\quad \sigma_p$ (D) = 0·75 × 12·08 = 9·06

For portfolio E, $\quad \sigma_p$ (E) = 1·00 × 12·08 = 12·08

If we plot (r_p, σ_p) combinations against each portfolio, we get the capital allocation line as shown in Fig. - 3.13.

In Fig.-3.13, the given slope of the capital allocation line is derived as follows :

Here,

$r_p = \alpha r_s + (1-\alpha) r_f$

or, $r_p = \alpha r_s + r_f - \alpha r_f$

or, $r_p = r_f + \alpha(r_s - r_f)$

or, $r_p = r_f + \dfrac{\sigma_p}{\sigma_s}(r_s - r_f) \quad [\because \alpha = \dfrac{\sigma_p}{\sigma_s}]$

or, $r_p = r_f + \dfrac{(r_s - r_f)}{\sigma_s}\cdot\sigma_p$.

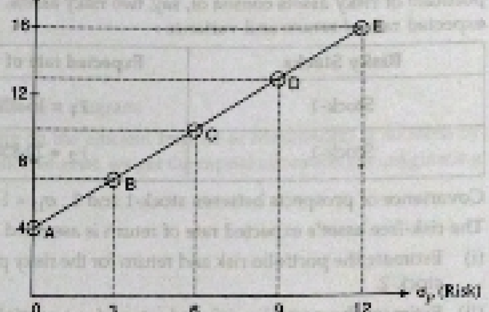


Fig. - 3.13

Hence, the slope of the capital allocation line is :

$$\frac{(\bar{r}_s - \bar{r}_f)}{\sigma_s} = \frac{16{\cdot}2 - 4{\cdot}0}{12{\cdot}08} = 1{\cdot}0$$

This slope implies that the investor receives a risk premium of 1% for taking an additional 1% risk.

3.5.3. Investing in both risk-free asset and a risky portfolio

In our previous discussion we have considered only one risky asset and a risk-free asset. Now, we can think of a situation where the investor combines a portfolio of some risky assets with the risk-free asset.

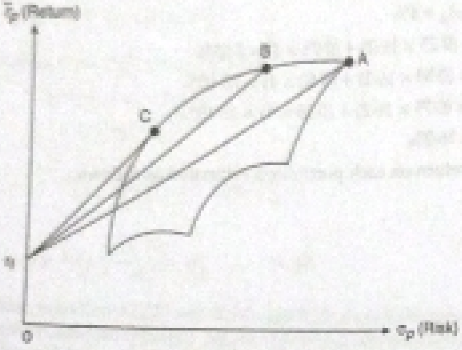


Fig. - 3.14

If we assume only risk-free lending (i.e. risk-free borrowing is not allowed or short selling of risk-free asset is not allowed) on the one hand and the Markowitz feasible set of portfolios on the other, then the posibilities of several combinations of risky portfolios with the risk-free asset can be shown, and depending upon the values of $\bar{r}_s$ and σ_s we get several capital allocation lines originating from the vertical intercept (viz. the r_f).

Fig.-3.14 shows that the efficient portfolios in the Markowitz feasible set can now be combined with the risk-free asset.

Now, any portfolio that consists of an investment in both a portfolio of risky assets (say, denoted by point A or B or C in Fig.-3.14) and the risk-free asset will have an expected return ($\bar{r}_p$) and standard deviation (σ_p) which can be estimated in a way similar to what we have shown in case of the combination of one risky asset and the risk-free asset. Let us consider a simple example.

Example 3.16

An investor wants to diversify his fund between a portfolio of risky assets and the risk-free asset. The portfolio of risky assets consist of, say, two risky assets, viz., asset-1 and asset-2 with the following expected rate of return and variance :

Risky Stocks	Expected rate of return	Variance of the rate of return
Stock-1	$\bar{r}_1 = 16{\cdot}2\%$	$\sigma_1^2 = 146$
Stock-2	$\bar{r}_2 = 22{\cdot}8\%$	$\sigma_2^2 = 289$

Covariance of prospects between stock-1 and 2 : $\sigma_{12} = 145$

The risk-free asset's expected rate of return is assumed to be 4%.

(i) Estimate the portfolio risk and return for the risky portfolio assuming 80% of stock-1 and 20% of stock-2.

(ii) Estimate the portfolio risk and return for a portfolio created out of a combination of the risky portfolio and the risk-free asset with the following weights :

Asset	Weightage	
risky portfolio :	0·25	0·50
risk-free asset :	0·75	0·50

Solution :

(i) The expected rate of return of the portfolio consisting of risky stocks (Stock-1 and 2) will be

$\bar{r}_s = (0{\cdot}80 \times 16{\cdot}2) + (0{\cdot}20 \times 22{\cdot}8) = 17{\cdot}52$ (Since $\bar{r} = \bar{r}_1 w_1 + \bar{r}_2 w_2$)

The variance of the rate of return on risky portfolio will be :

$$\sigma_s^2 = [(0{\cdot}8)^2 \times 146] + [(0{\cdot}2)^2 \times 289] + [2 \times 0{\cdot}8 \times 0{\cdot}2 \times 145]$$

(Since $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2 w_1 . w_2 \sigma_{12}$).

$$\therefore \sigma_s = \sqrt{\sigma_s^2} = \sqrt{93{\cdot}44 + 11{\cdot}56 + 46{\cdot}4} = \sqrt{151{\cdot}4}$$

$$= 12{\cdot}30\%$$

(ii) When risk-free asset is combined with the risky portfolio then the expected rate of return on the new portfolio will be :

$\bar{r}_p = (0{\cdot}25 \times 17{\cdot}52) + (0{\cdot}75 \times 4)$ [Considering first 25% weightage for risky portfolio and 75% weightage for risk-free asset].

$= 7{\cdot}38\%$

The standard deviation of the rate of return of this new portfolio (σ_p) will be :

$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{w_s^2 \sigma_s^2}$ Where σ_s^2 = Variance of rate of return of risky portfolio.

[Since $\sigma_f^2 = 0$ for risk-free asset, and $\sigma_{sf} = 0$]

$\therefore \sigma_p = w_s \sigma_s = 0{\cdot}25 \times 12{\cdot}30 = 3{\cdot}08\%$

Again, for $w_s = 0{\cdot}50$ and $w_f = 0{\cdot}50$,

$\bar{r}_p = (0{\cdot}50 \times 17{\cdot}52) + (0{\cdot}50 \times 4) = 10{\cdot}76$

$\sigma_p = 0{\cdot}50 \times 12{\cdot}30 = 6{\cdot}15\%$

We can now plot these risk-return profiles in a diagram.

In Fig.-3.15, the risky portfolio (say, lying on the efficient frontier of Markowitz) is denoted by point A. When it is combined with the risk-free asset, we get the capital allocation line originating from the risk-free rate of return (r_f).

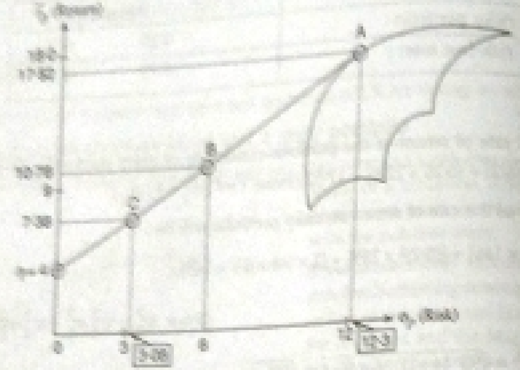


Fig. - 3.15

We have already explained that this capital allocation line is expressed as :

$$\bar{r}_p = r_f + \frac{(\bar{r}_s - r_f)}{\sigma_s}\sigma_p \text{ (See 3.29)}$$

In this example, the slope of the capital allocation line will be :

$$\frac{(\bar{r}_s - r_f)}{\bar{\sigma}_s} = \frac{17{\cdot}52 - 4{\cdot}0}{12{\cdot}3} = 1{\cdot}099 \approx 1{\cdot}1$$

3.5.4. Risk-free lending and the efficient portfolio

From our previous discussion it becomes clear that any risky portfolio can be combined with the risk-free asset to form a new portfolio. The inclusion of the risk-free asset (when we assume only risk-free lending) can have a significant impact upon the efficient portfolio in Markowitz model. In Fig.-3.16 we observe that within the feasible set of risky portfolios (as suggested by Markowitz), the efficient frontier includes the portfolios denoted by points A, B, C, D, E and F.

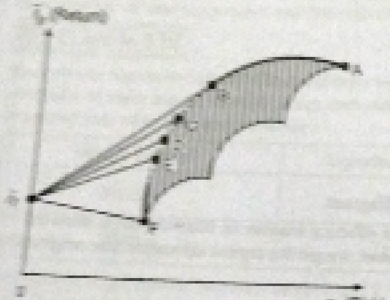


Fig. - 3.16

When a risk-free asset is combined with each of these efficient portfolios, we get several capital allocation lines originating from the risk-free rate of return (r_f), the equation of the capital allocation line being

$$\bar{r}_p = r_f + \frac{(\bar{r}_s - r_f)}{\sigma_s}\cdot\sigma_p \text{ (See 3.29)}$$

It is important to note that if expected rate of return on risky portfolio ($\bar{r}_s$) becomes higher than the risk-free return (r_f) then for the given values of portfolio risk (σ_p), $\bar{r}_s$ and r_f, the capital allocation line becomes an upward sloping straight line originating from the risk-free rate of return (r_f). However, if

for some reason or other $\bar{r}_s < r_f$ then the capital allocation line will be negatively sloped (as shown by $r_f F$ in Fig.-3.16).

In Markowitz model, the efficient frontier of the portfolio includes all the portfolios having risk-return profiles denoted by A, B, C, D, E and F in Fig.-3.16. But when one risk-free asset can be combined with each of these efficient portfolios, the efficiency locus or the efficient frontier will change.

The slope of each capital allocation line also implies the risk premium for the investor for undertaking additional risk.

In Fig.-3.16, among all the capital allocation lines, $r_f B$ has the highest slope, i.e., if a risky portfolio B can be combined with the risk-free asset it will generate highest possible risk premium for undertaking additional risk. Hence, in that case, given the feasible set of risky portfolios in Markowitz model, the efficient frontier will be $r_f BA$ where $r_f B$ portion will be a straight line and BA portion will be curved (i.e., the portion of the original Markowitz efficient frontier).

3.5.5. One-fund theorem

Our previous discussion clearly shows that any efficient portfolio in the feasible set (as shown by Markowitz model) can be combined with risk-free asset, and any such combination that lies on the capital allocation line (originating from the risk-free rate of return, r_f) can also be considered as an efficient portfolio.

Now we can think of a situation when the investor can borrow or lend at the risk-free rate. Thus, the capital allocation line can then extend beyond the feasible set of risky portfolios (i.e., the investor can borrow at the risk-free rate and can follow short selling).

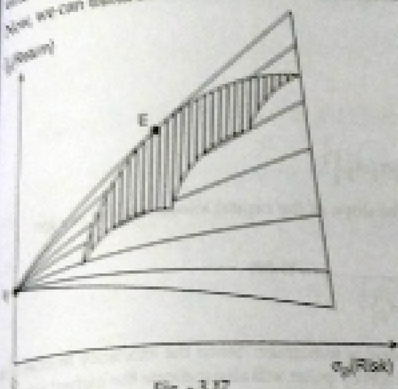


Fig. - 3.17

In that case, we can trace out the capital allocation line with highest possible slope (i.e., with highest possible risk premium for the investor for undertaking additional risk) and having a point of tangency with the efficient frontier of the risky portfolios.

Fig.-3.17 shows that the capital allocation line $r_f E$ has the highest slope among all possible capital allocation lines, and it has a point of tangency (at point E) with the efficient frontier of the risky portfolios. **One fund theorem suggests that** there remains a single efficient fund (or portfolio) of risky assets (here it is denoted by E) such that any efficient portfolio can be constructed as a combination of fund E and the risk-free asset.

- Optimal Portfolio :

In the one fund theorem, the objective of the investor is to find a portfolio that lies on the efficient frontier of risky assets and to combine it with a risk-free asset such that he gets highest possible excess rate of return (over the risk-free rate) for undertaking additional risk. Thus, the optimal portfolio will take place at the point of tangency between the capital allocation line and the efficient portfolio frontier (as shown by point E in Fig.-3.17).

We know that the equation of capital allocation line is :

$$r_p = r_f + \frac{(r_s - r_f)}{\sigma_s}\sigma_p \text{ (See 3.29)}$$

Thus, the slope of the capital allocation line is : $\theta = \frac{r_s - r_f}{\sigma_s}$

Where r_s = rate of return on risky asset

r_f = rate of return on risk-free asset.

σ_s = standard deviation of rate of return on risky asset.

Again, we know that expected return on portfolio is : $r_p = w_1\bar{r}_1 + (1-w_1)\bar{r}_2$ $[\because w_1 + w_2 = 1]$

Here, we assume that in the risky portfolio, there are two risky assets, Asset-1 and Asset-2.

The variance of the rate of return on portfolio is :

$$\sigma_p^2 = w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho_{12}\sigma_1\sigma_2 \quad [\because \rho_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2}]$$

Now, at the optimal point, the slope of the capital allocation line can be expressed as

$$\theta = \frac{r_E - r_f}{\sigma_E}$$

Where $r_E = w_1\bar{r}_1 + (1-w_1)\bar{r}_2$, and

$$\sigma_E = \sqrt{\sigma_E^2} = \{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2\}^{\frac{1}{2}}$$

Now substituting these values of r_E and σ_E in the slope of the capital allocation line, we get

$$\theta^1 = \frac{\{w_1\bar{r}_1 + (1-w_1)\bar{r}_2\} - r_f}{\{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho_{12}\sigma_1\sigma_2\}^{\frac{1}{2}}} \text{ ...... (3.30)}$$

The optimal portfolio is attained where this slope is maximum. Since the efficiency frontier of the risky portfolio is concave downward, so this tangency point will also indicate the highest achievable point on the frontier. Since $w_1 + w_2 = 1$, so the slope θ^1 can be expressed only in terms of w_1. Now, differentiating θ^1 with respect to w_1 and setting that equal to zero (for maximising θ^1), we can find out the optimal value of $w_1 = w_1^*$ (say). Hence, $w_2^* = 1 - w_1^*$.

Here, $$\frac{d\theta^1}{dw_1} = \frac{\{\}^{\frac{1}{2}}(\bar{r}_1 - \bar{r}_2) - \left[\{w_1\bar{r}_1 + (1-w_1)\bar{r}_2 - r_f\}\frac{1}{2}\{\}^{-\frac{1}{2}}\{2w_1\sigma_1^2 - 2(1-w_1)\sigma_2^2 + 2(1-2w_1)\rho_{12}\sigma_1\sigma_2\}\right]}{\left[\{\}^{\frac{1}{2}}\right]^2} = 0$$

or $$\{\}^{\frac{1}{2}}(\bar{r}_1 - \bar{r}_2) - \left[\{w_1\bar{r}_1 + (1-w_1)\bar{r}_2 - r_f\}\{\}^{-\frac{1}{2}}\{w_1\sigma_1^2 - (1-w_1)\sigma_2^2 + (1-2w_1)\rho_{12}\sigma_1\sigma_2\}\right] = 0$$

or $$\frac{\{\}^{\frac{1}{2}}\{\}^{\frac{1}{2}}(\bar{r}_1 - \bar{r}_2) - \left[\{w_1\bar{r}_1 + (1-w_1)\bar{r}_2 - r_f\}\{w_1\sigma_1^2 - (1-w_1)\sigma_2^2 + (1-2w_1)\rho_{12}\sigma_1\sigma_2\}\right]}{\{\}^{\frac{1}{2}}} = 0$$

or $$(\bar{r}_1 - \bar{r}_2)\{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho_{12}\sigma_1\sigma_2\} - \left[\{w_1\bar{r}_1 + (1-w_1)\bar{r}_2 - r_f\}\right.$$
$$\left.\{w_1\sigma_1^2 - (1-w_1)\sigma_2^2 + (1-2w_1)\rho_{12}\sigma_1\sigma_2\}\right] = 0$$

or $$\bar{r}_1 w_1^2\sigma_1^2 + \bar{r}_1(1-w_1)^2\sigma_2^2 + 2\bar{r}_1 w_1(1-w_1)\rho_{12}\sigma_1\sigma_2 - \bar{r}_2 w_1^2\sigma_1^2 - \bar{r}_2(1-w_1)^2\sigma_2^2$$
$$-2\bar{r}_2 w_1(1-w_1)\rho_{12}\sigma_1\sigma_2 - \bar{r}_1 w_1^2\sigma_1^2 + \bar{r}_1 w_1(1-w_1)\sigma_2^2 - \bar{r}_1 w_1(1-2w_1)\rho_{12}\sigma_1\sigma_2$$
$$-w_1(1-w_1)\bar{r}_2\sigma_1^2 + (1-w_1)^2\bar{r}_2\sigma_2^2 - (1-w_1)(1-2w_1)\bar{r}_2\rho_{12}\sigma_1\sigma_2$$
$$w_f(\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2)w_1 + r_f(\rho_{12}\sigma_1\sigma_2 - \sigma_2^2) = 0$$

or $$-\{\bar{r}_2 w_1^2 + w_1(1-w_1)\bar{r}_2\}\sigma_1^2 + \{(1-w_1)^2\bar{r}_1 + \bar{r}_1 w_1(1-w_1)\}\sigma_2^2$$
$$+\{2\bar{r}_1 w_1(1-w_1) - 2\bar{r}_2 w_1(1-w_1) - \bar{r}_1 w_1(1-2w_1)$$
$$-(1-w_1)(1-2w_1)\bar{r}_2\}\rho_{12}\sigma_1\sigma_2 + r_f(\cdot)w_1 + r_f(\cdot) = 0$$

or $$-[\bar{r}_2 w_1\{w_1 + (1-w_1)\}]\sigma_1^2 + [\bar{r}_1(1-w_1)\{(1-w_1) + w_1\}]\sigma_2^2$$
$$+\bar{r}_1 w_1\{2(1-w_1) - (1-2w_1)\} - \bar{r}_2(1-w_1)\{2w_1 + (1-2w_1)\}\rho_{12}\sigma_1\sigma_2 + r_f(\cdot)w_1 + r_f(\cdot) = 0$$

or $$-\bar{r}_2 w_1\sigma_1^2 + \bar{r}_1(1-w_1)\sigma_2^2 + [\bar{r}_1 w_1 - \bar{r}_2(1-w_1)]\rho_{12}\sigma_1\sigma_2 + r_f(\cdot)w_1 + r_f(\cdot) = 0$$

or $$-\bar{r}_2 w_1\sigma_1^2 - \bar{r}_1 w_1\sigma_2^2 + \bar{r}_1\sigma_2^2 + (\bar{r}_1 w_1 + \bar{r}_2 w_1)\rho_{12}\sigma_1\sigma_2 - \bar{r}_2\rho_{12}\sigma_1\sigma_2 + r_f(\cdot)w_1 + r_f(\cdot) = 0$$

or $$-\bar{r}_2 w_1\sigma_1^2 - \bar{r}_1 w_1\sigma_2^2 + \bar{r}_1\sigma_2^2 - \bar{r}_2\rho_{12}\sigma_1\sigma_2 + (\bar{r}_1 + \bar{r}_2)w_1\rho_{12}\sigma_1\sigma_2$$
$$+r_f(\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2)w_1 + r_f(\rho_{12}\sigma_1\sigma_2 - \sigma_2^2) = 0$$

or $$[\bar{r}_2\sigma_1^2 + \bar{r}_1\sigma_2^2 - (\bar{r}_1 + \bar{r}_2)\rho_{12}\sigma_1\sigma_2 - r_f(\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2)]w_1$$
$$= r_f(\rho_{12}\sigma_1\sigma_2 - \sigma_2^2) + \bar{r}_1\sigma_2^2 - \bar{r}_2\rho_{12}\sigma_1\sigma_2$$

or $\quad w_1^* = \dfrac{(\bar{r}_1 - r_f)\sigma_2^2 - (\bar{r}_2 - r_f)\rho_{12}\sigma_1\sigma_2}{(\bar{r}_1 - r_f)\sigma_2^2 + (\bar{r}_2 - r_f)\sigma_1^2 - (\bar{r}_1 + \bar{r}_2 - 2r_f)\rho_{12}\sigma_1\sigma_2}$ .... (3.31)

$\therefore \quad w_2^* = 1 - w_1^*$

Example 3.17

An investor's portfolio consists of two risky assets, say, stock-1 and stock-2. The expected rate of return on these two stocks are $\bar{r}_1 = 20\%$ and $\bar{r}_2 = 25\%$ respectively. The correlation coefficient of the two stocks is $\rho_{12} = 0{\cdot}40$. The standard deviation of the rate of return on these stocks are $\sigma_1 = 20\%$, $\sigma_2 = 30\%$ respectively. The rate of return on the risk-free asset is $r_f = 8\%$.

(i) In what proportion stock-1 and stock-2 should be combined to make it an optimal portfolio so that it can be efficiently combined with a risk-free asset to get highest possible risk premium while undertaking additional risk?

(ii) What would be the risk and expected rate of return on this portfolio?

(iii) If this optimal portfolio is combined with a risk-free asset then what would be the risk premium for undertaking additional risk?

(iv) If only stock-1 or stock-2 is combined with the risk-free asset then what would be its impact on the risk premium?

Solution :

(i) When the investor combines a risky fund (or the risky portfolio) with the risk-free asset then the expected rate of return on combined portfolio is expressed as :

$\bar{r}_p = wr_s + (1-w)r_f \qquad$ Where w = proportion of risky assets in the portfolio.

r_s = expected rate of return on risky assets

$(1-w)$ = proportion of risk-free asset in the portfolio.

r_f = risk-free rate of return.

The variance of the rate of return on this portfolio will be :

$$\sigma_p^2 = w^2\sigma_s^2 + (1-w)^2\sigma_f^2 + 2w(1-w)\sigma_{sf}$$

$$= w^2\sigma_s^2 + (1-w)^2\sigma_f^2 + 2w(1-w)\rho_{sf}\sigma_s\sigma_f \qquad \left[\because \rho_{sf} = \frac{\sigma_{sf}}{\sigma_s\sigma_f}\right]$$

Where σ_{sf} = Covariance of the prospects of risky assets and risk-free asset.

ρ_{sf} = Correlation coefficient of the prospects of risky assets and risk-free asset.

Since, $\sigma_f^2 = 0$ and $\rho_{sf} = 0$, so $\sigma_p^2 = w^2\sigma_s^2$

$\therefore \quad \sigma_p = w.\sigma_s$

or $\quad w = \dfrac{\sigma_p}{\sigma_s}$

Hence, the capital allocation line becomes

$r_p = r_f + w(r_s - r_f)$

$= r_f + \dfrac{(r_s - r_f)}{\sigma_s}\cdot\sigma_p$

However, in the risky portfolio, the expected rate of return (r_s) will be :

$r_s = w_1\bar{r}_1 + w_2\bar{r}_2$, where $w_1 + w_2 = 1$

$= w_1\bar{r}_1 + (1-w_1)\bar{r}_2$

and the corresponding variance $\left(\sigma_p^2\right)$ will be :

$$\sigma_s^2 = w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho_{12}\sigma_1\sigma_2$$

At the optimal level, the slope of the capital allocation line will be tangent to the efficient frontier of the risky portfolio, and let that slope be expressed as

$\theta = \dfrac{r_s - r_f}{\sigma_s}$

Now, substituting the values of r_s and σ_s for the risky portfolio in this slope, we get

$$\theta^1 = \frac{w_1\bar{r}_1 + (1-w_1)\bar{r}_2 - r_f}{\left\{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho_{12}\sigma_1\sigma_2\right\}^{\frac{1}{2}}}$$

Now differentiating θ^1 with respect to w_1 and setting that equal to zero, we can solve for the optimal value of w_1 as follows :

$$w_1^* = \frac{(\bar{r}_1 - r_f)\sigma_2^2 - (\bar{r}_2 - r_f)\rho_{12}\sigma_1\sigma_2}{(\bar{r}_1 - r_f)\sigma_2^2 + (\bar{r}_2 - r_f)\sigma_1^2 - (\bar{r}_1 + \bar{r}_2 - 2r_f)\rho_{12}\sigma_1\sigma_2} \quad \text{(See 3.31)}$$

In this example, $\bar{r}_1 = 20\%$, $\bar{r}_2 = 25\%$,

$\sigma_1 = 20\%$, $\sigma_2 = 30\%$, $\rho_{12} = 0{\cdot}40$ and $r_f = 8\%$

$$\therefore \quad w_1^* = \frac{(20-8)(30)^2 - (25-8)\times 0{\cdot}4\times 20\times 30}{(20-8)(30)^2 + (25-8)(20)^2 - (20+25-2\times 8)\times 0{\cdot}4\times 20\times 30}$$

$$= \frac{10{,}800 - 4{,}080}{10{,}800 + 6{,}800 - 6{,}960} = \frac{6{,}720}{10{,}640} = 0{\cdot}6316$$

$$= 63{\cdot}16\%$$

$\therefore \quad w_2^* = 1 - w_1^* = 1 - 0{\cdot}6316 = 0{\cdot}3684 = 36{\cdot}84\%$

So, the investor should invest 63·16% of his fund in Stock-1, and the remaining 36·84% fund in Stock-2.

(ii) The expected rate of return and risk of the optimal portfolio will be :

$$\bar{r}_S = w_1^* \bar{r}_1 + (1 - w_1^*)^2 \bar{r}_2$$
$$= 0{\cdot}6316 \times 20 + 0{\cdot}3684 \times 25$$
$$= 12{\cdot}63 + 9{\cdot}21 = 21{\cdot}84\%$$

$$\sigma_S^2 = w_1^{*2}\sigma_1^2 + w_2^{*2}\sigma_2^2 + 2w_1^* w_2^* \rho_{12}\sigma_1\sigma_2$$
$$= (0{\cdot}6316)^2 \cdot (20)^2 + (0{\cdot}3684)^2 \cdot (30)^2 + 2 \times (0{\cdot}6316) \times (0{\cdot}3684) \times 0{\cdot}4 \times 20 \times 30$$
$$= 159{\cdot}57 + 122{\cdot}15 + 111{\cdot}69$$
$$= 393{\cdot}41$$
$$\therefore \sigma_S = \sqrt{393{\cdot}41} = 19{\cdot}83\%$$

(iii) Here, $\theta = \dfrac{\bar{r}_S - r_f}{\sigma_S}$

$$= \frac{21{\cdot}84 - 8}{19{\cdot}83} = \frac{13{\cdot}84}{19{\cdot}83} = 0{\cdot}6979$$
$$\approx 0{\cdot}70$$

It implies that if risk rises by 1%, the investor's risk premium would be 0·70%.

(iv) If only Stock-1 is combined with the risk-free asset then

$$\theta_1 = \frac{\bar{r}_1 - r_f}{\sigma_1} = \frac{20-8}{20} = \frac{12}{20} = 0{\cdot}60$$
$$\therefore \theta_1 = 0{\cdot}60 < 0{\cdot}70$$

If only Stock-2 is combined with the risk-free asset then

$$\theta_2 = \frac{\bar{r}_2 - r_f}{\sigma_2} = \frac{25-8}{30} = \frac{17}{30} = 0{\cdot}57$$
$$\therefore \theta_2 = 0{\cdot}57 < 0{\cdot}70.$$

3.5.6. Utility function of the investor and optimal portfolio

Given the efficient portfolio frontier in Markowitz model, the problem of finding the optimal portfolio can also be viewed in terms of the maximisation of the utility of the investor.

The utility (U) of the investor is assumed to be a function of both risk (σ_p) and expected rate of return ($\bar{r}_p$) on his portfolio of risky assets. Hence, the utility function of the investor can be expressed as :

$$U = f(\bar{r}_p, \sigma_p)$$

Here, it is assumed that the investor is risk-averter, and the marginal utility of the expected rate of return on portfolio ($MU_{\bar{r}_p}$) is positive, while the marginal utility of risk (the stand deviation of rate of return) on portfolio (MU_{σ_p}) is negative. Thus, return on portfolio is assumed to be 'good' commodity while the risk on portfolio is assumed to be a 'bad' commodity.

Thus, higher return on portfolio raises the utility of the investor while higher risk on portfolio reduces the utility. Hence, if return on portfolio rises with higher risk then aggregate utility may remain constant. Different combinations of return and risk ($\bar{r}_p, \sigma_p$) which generate same level of utility for the investor, can be presented with the help of an iso-utility curve or indifference curve (IC). Along any such IC, the level of utility remains constant. Thus, from the total differentiation of the utility function, we get

$$dU = \frac{\partial U}{\partial \bar{r}_p} \cdot d\bar{r}_p + \frac{\partial U}{\partial \sigma_p} \cdot d\sigma_p = 0$$

$$\text{or, } \frac{d\bar{r}_p}{d\sigma_p} = (-)\frac{\frac{\partial U}{\partial \sigma_p}}{\frac{\partial U}{\partial \bar{r}_p}} = (-)\frac{MU_{\sigma_p}}{MU_{\bar{r}_p}} > 0 \qquad [\text{since } MU\sigma_p < 0,\ MU_{\bar{r}_p} > 0].$$

It shows the slope of the indifference curve. So, in this case, the indifference curve becomes positively sloped.

The absolute slope of the IC, i.e., $\left|\dfrac{d\bar{r}_p}{d\sigma_p}\right|$ would indicate the Marginal Rate of Substitution of risk for return on portfolio for the investor ($MRS_{\sigma_p, \bar{r}_p}$). An increase in this $MRS_{\sigma_p, \bar{r}_p}$ signifies that the investor expects higher return for an increment in risk by 1%. Thus, if an investor is highly risk-averter then his/her $MRS_{\sigma_p, \bar{r}_p}$ would be higher, and the indifference curve would be relatively steep. However, if the investor is less averse to risk-taking then $MRS_{\sigma_p, \bar{r}_p}$ would be less, and the indifference curve would be flatter.

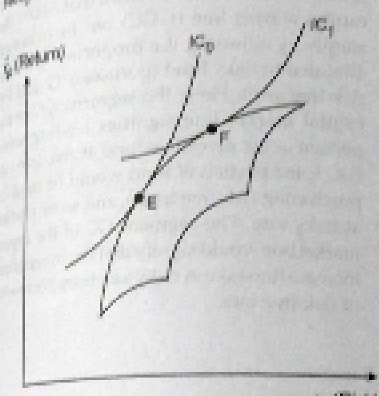


Fig. - 3.18

In Fig.-3.18, the indifference curve IC_0 shows higher $MRS_{\sigma_p, \bar{r}_p}$ and therefore, it is relatively steep. However, the indifference curve IC_1 shows lower $MRS_{\sigma_p, \bar{r}_p}$, and so this indifference curve is relatively flat. Here, IC_0 represents relatively risk-averse investor while IC_1 shows that the investor is less averse to risk-taking.

In this case, given the efficient portfolio frontier, the point of tangency between the indifference curve (IC) and the efficient portfolio frontier would determine the optimal portfolio.

In Fig.-3.18, the optimal portfolio for any investor who is more averse to risk can be denoted by E (with steeper IC_0), and the optimal portfolio for any investor who is less averse to risk can be denoted by F (with flatter IC_1).

3.5.7. Borrowing and lending at risk-free rate and optimal portfolio

While discussing one-fund theorem, we have already shown that an investor reaches at an optimal portfolio when the capital allocation line, with its highest possible slope, becomes tangent to the efficient frontier of the portfolio [See Subsection 3.5.5. & Fig.-3.17].

Now, we can incorporate the utility function or the indifference curve of the investor in that framework and find out the optimal portfolio.

In Fig.-3.19 the efficient frontier of the risky portfolio is denoted by MN. The capital allocation line, often called as the *Capital Market Line*, is denoted by the line $r_f C$. The capital market line (CML) is expressed by the equation :

$$\bar{r}_P = r_f + \frac{(\bar{r}_s - r_f)}{\sigma_s} \cdot \sigma_P \text{ [See 3.29]}$$

Where r_f = risk-free rate of return on a risk-free asset

$\bar{r}_s$ = expected rate of return on risky asset

σ_s = standard deviation of the rate of return on risky asset.

σ_p = standard deviation of the rate of return on portfolio.

$\bar{r}_P$ = expected rate of return on portfolio.

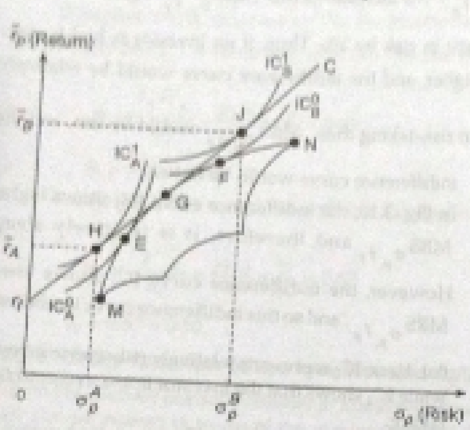


Fig.-3.19

We know that the slope of this capital market line $\left[\frac{(\bar{r}_s - r_f)}{\sigma_s}\right]$ implies the risk premium to the investor for taking additional risk. Any combination of risk and return that lies on the capital market line (r_fGC) can be obtained simply by adjusting the proportion of funds allocated to risky fund (portfolio) 'G' and the risk-free asset. Here, the segment r_fG of the capital market line signifies lending some portion of the investible fund at risk-free rate (i.e., some portion of fund would be used for purchasing risk-free asset), and some portion at risky rate. The segment GC of the capital market line would signify that the investor can increase the stake in risky assets by borrowing at risk-free rate.

The utility function of investor-A is :

$U_A = U_A(\bar{r}_P, \sigma_p)$ and the utility function of investor-B is :

$U_B = U_B(\bar{r}_P, \sigma_p)$.

It is assumed that investor-A is more averse to risks compared to investor-B, and therefore, $MRS^A_{\sigma_P, \bar{r}_P} > MRS^B_{\sigma_P, \bar{r}_P}$. So, the indifference curves of investor-A (denoted by IC_A^0 and IC_A^1) are steeper compared to the indifference curves of investor-B (denoted by IC_B^0 and IC_B^1).

It is important to note in this connection that when the investor can combine the risky portfolio (denoted by point G on the efficient portfolio frontier) with the risk-free asset then any portfolio having a risk-return profile denoted by any point on the capital market line (CML) would be treated as an efficient portfolio. Now, for investor-A who is more averse to risks, the optimal portfolio would be determined by the point of tangency (H) between the indifference curve (IC_A^1) and the CML. The risk-return combination for investor-A at the optimal portfolio would be (σ_A, $\bar{r}_A$) as shown in Fig.-3.19.

On the other hand, for investor-B who is less averse to risks, the optimal portfolio would be determined by the point of tangency (J) between his indifference curve (IC_B^1) and the CML. The risk-return combination for investor-B at the optimal portfolio would be (σ_B, $\bar{r}_B$). (See Fig.-3.19).

Example 3.18

Consider an investor who invests both in risky assets and risk-free asset. His utility function is given as $U = \bar{r}_P - 0{\cdot}005A\sigma_P^2$, where $\bar{r}_P$ = expected rate of return on risky portfolio, σ_P^2 = variance of the rates of return on risky portfolio and A = Coefficient of risk aversion. Let the risk-free rate of return is given as r_f = 8%. The optimal portfolio as determined by the point of tangency between the capital market line and the efficient frontier of risky portfolio results in $\bar{r}_P$ = 21·84% and σ_P = 19·83%.

Determine the optimal portfolio of the investor with its risk-return profile when (i) the investor is highly averse to risk with A = 5, and (ii) the investor has a comparatively low aversion to risk with A = 3.

Solution :

We know that the expected rate of return on a portfolio consisting of both risky and risk-free assets will be : $\bar{r}_P = r_f + w(\bar{r}_s - r_f)$ [See (3.29)]

where r_f = Risk-free rate = 8%

$\bar{r}_s$ = Expected rate of return on risky assets = 21·84%

w = Proportion of fund invested in risky assets.

Further, we know that the standard deviation (σ) of the expected rate of return on a portfolio consisting of both risky and risk-free assets will be :

$\sigma_P = w\,\sigma_S$ [See (3.28)]

where σ_P = Standard deviation of the rates of return on portfolio.

σ_S = Standard deviation of the rates of return on risky assets.

Now, Substituting these values of $\bar{r}_P$ and σ_P in the utility function of the investor, we get :

$$U = \bar{r}_P - 0{\cdot}005\,A\,\sigma_P^2$$

$$= r_f + w\,(\bar{r}_s - r_f) - 0{\cdot}005\,A\,(w{\cdot}\sigma_s)^2$$

$$= r_f + w\,(\bar{r}_s - r_f) - 0{\cdot}005\,A\,w^2 \cdot \sigma_S^2$$

Now, the problem is to choose that value of w which would maximise utility. Now, to maximise utility we differentiate the utility function partially with respect to w and setting that equal to zero, we get

$$\frac{\partial U}{\partial w} = (r_s - r_f) - 0.01\, Aw\sigma_s^2 = 0$$

$$\text{or, } w^* = \frac{r_s - r_f}{0.01 A\sigma_s^2}$$

(i) Here, according to this example,

$r_s = 21.84\%$, $r_f = 8\%$, $A = 5$, and $\sigma_s = 19.83\%$

$$\therefore w^* = \frac{21.84 - 8}{0.01 \times 5 \times (19.83)^2} = \frac{13.84}{19.66} = 0.70$$

It implies that the investor would allocate 70% of his fund in risky assets and the remaining 30% in risk-free asset.

In this case, $\bar{r}_p = r_f + w^*(r_s - r_f)$

$= 8 + 0.70\,(21.84 - 8)$

$= 17.69\%$

and $\sigma_p = w^*\sigma_s = 0.70 \times 19.83 = 13.88\%$

$$\therefore U = \bar{r}_p - 0.005\, A\sigma_p^2 = 17.69 - (0.005 \times 5 \times 13.88^2) = 12.87$$

(ii) When $r_s = 21.84\%$, $r_f = 8\%$, $A = 3$ and $\sigma_s = 19.83\%$ then $w^* = \dfrac{21.84 - 8}{0.01 \times 3 \times (19.83)^2}$

$$= \frac{13.84}{11.80}$$

$$= 1.17$$

Here, the investor would borrow funds equal to 17% of his own fund and then invest the entire amount in risky assets.

In this case, $\bar{r}_p = r_f + w^*(r_s - r_f)$

$= 8 + 1.17\,(21.84 - 8)$

$= 24.19\%$

and $\sigma_p = w^*\sigma_s = 1.17 \times 19.83 = 23.20\%$

$$\therefore U = \bar{r}_p - 0.005\, A\sigma_p^2 = 24.19 - (0.005 \times 3 \times 23.2^2) = 16.11$$

So, these results clearly show that an investor who is less averse to risk-taking, can take more risks and hence, can earn more return along with higher level of utility.

3.6. Capital Asset Pricing Model (CAPM)

This model, developed by William F. Sharpe and John Linter during 1960s, wants to show a relationship between the unavoidable risk and expected rate of return from a security. This model takes into account not only the risk differential between common stocks and risk-free assets, but also the risk differential between the common stock of a firm and the broad-based market portfolio. This model

logically follows from the mean-variance portfolio theory developed by Markowitz (which has already been discussed in earlier sections of this chapter).

- **Market equilibrium and market portfolio :**

Before entering into the CAPM, we shall first discuss the concepts of **market equilibrium and market portfolio.** Let us assume that every investor assigns same expected rate of return on risky portfolio, and same variance in the rates of return on risky portfolio. We can also assume that there remains a scope of combining the risky portfolio with risk-free asset and risk-free rate of return is given in the market. The borrowing and lending at risk-free rate are allowed without any transaction cost. The one-fund theorem that we have discussed in earlier section shows that an investor can invest in a single fund (portfolio) of risky assets and an efficient portfolio can be created by combining that single risky fund with risk-free asset. We have also shown that the proportion of risky assets and risk-free asset in the portfolio would depend upon the attitude of an investor towards risk-taking. Though we assume that an investor is a risk-averter, but the degree of risk-aversion may vary across individual investors.

Now, if every investor purchases the same fund of risky assets then that fund or portfolio would also reveal the **market portfolio.** This market portfolio is the summation of all risky stocks transacted in the stock market. Hence, in the aggregative sense, the purchase of risky stocks by all investors would signify the market portfolio. The weightage of each asset in the market portfolio is measured by the proportion of the asset's value in the total value of all assets in the stock market (often called as market capitalisation). The following example can help us in understanding the weightage of a risky asset in market portfolio :

Stocks	No. of stocks	Relative share in the market	Price per stock (₹)	Capitalisation (₹)	Weight of a stock
1. ITC	15,000	0·15	120	18,00,000	0·098
2. HUL	10,000	0·10	150	15,00,000	0·082
3. RIL	20,000	0·20	200	40,00,000	0·218
4. TCS	15,000	0·15	180	27,00,000	0·147
5. ONGC	25,000	0·25	250	62,50,000	0·341
6. INFOSYS	15,000	0·15	140	21,00,000	0·114
TOTAL :	1,00,000	1·00		1,83,50,000	1·000

[Imaginary figures have been used]

In a competitive market environment if all the investors behave in a similar fashion with regard to the selection of optimal portfolio then through their buying and selling activities, equilibrium will be attained in the stock market. In that situation, the stock prices will vary to drive the market towards an efficient one.

Some of the financial economists are also of the opinion that the stocks which are frequently traded in the stock market should be considered in analysing the market equilibrium.

Hence, when many prudent investors make adjustments in their expected rates of return with changes in stock prices, and select the optimum portfolio, then selection of such portfolio in the mean-variance analytical frame would also represent the market portfolio.

3.6.1. Basic assumptions of CAPM

The Capital Asset Pricing Model is based on some basic assumptions. These are as follows :

1. The portfolio of assets is evaluated by any investor on the basis of expected rate of return on portfolio and the standard deviation of the rates of return on portfolio over a single period time horizon.
2. The investors always prefer more return on portfolio to less of it, i.e., they are never satiated with the current rate of return on portfolio. Hence, if two portfolios possess same risk, the investors will choose that portfolio which generates higher expected rate of return.
3. The investors are risk averters. Hence, if two portfolios have identical expected rate of return, the investor will choose that portfolio which shows lesser standard deviation of the rates of return.
4. The individual assets can be divided into smaller units so that the investors can purchase a small fraction of a financial asset if she/he so desires.
5. Unrestricted borrowing and lending can take place at a risk-free rate.
6. There remain no taxes on the rate of return on portfolio, and transaction costs of buying and selling bonds also remain absent.
7. Capital markets are highly efficient where investors are well-informed, and the information is freely and instantly available to all investors.
8. Investors have homogeneous expectations regarding the rate of return on portfolio, variance of the rates of return and their covariance.
9. Any single investor cannot influence the market price of a stock out of his/her individual actions.
10. The risk-free rate of return remains same for all investors.

3.6.2. The Capital Market Line

The Capital Market Line (CML) indicates the relationship between the expected rate of return on portfolio and the risks involved (as measured by the standard deviation of the rates of return on portfolio) in possessing that portfolio of risky assets.

Our previous discussion shows that when any investor chooses a single efficient fund (or portfolio) of risky assets and combines it with risk-free asset with borrowing and lending possibilities at risk-free rate then in the aggregative sense, considering similar activities of all investors, that single fund would denote the market portfolio. If that single fund of risky assets on the efficient frontier is denoted by a risk-return combination point (e.g., point E in Fig-3.17) then any point lying on the linear combinations of risk-free asset (with risk-free return r_f) and that single fund E would be treated as efficient. This linear combinations of risky portfolio and the risk-free asset is represent by a straight line originating from r_f (see Fig.-3.17), and becomes tangent to the efficient frontier at point E. If all the investors want to purchase the securities included in E and ignore the other securities then the buying pressure in the capital market would cause a revision in the market prices of securities. In this situation, the prices of securities included in E would rise and therefore, expected rate of return on those securities will fall. However, the securities which were not included in E would experience a price-fall and hence, an increase in expected rate of return on those securities. This process continues until the aggregate demand for securities is just matched by the aggregate supply of securities. Some financial economists are of the opinion that this process of price adjustment will flatten the efficient frontier of risky portfolios (see Fig.-3.20).

Fig.-3.20 suggests that the buying pressure upon securities included in 'E' would lead to a fall in the expected rate of return on those securities so that the risk-return combination point will shift downward

from E to H. Ultimately, the set of security prices at equilibrium would be such that every security in the market (which are frequently traded) will enter into at least one portfolio on the linear segment THL. Obviously, the market portfolio would itself be a point on that linear segment (Fig.-3.20). For efficient portfolios (which also includes the market portfolio), the relation between the risk and return is expressed by the upward sloping straight line r_fHC. The equation of this straight line, called as **Capital Market Line** (CML) is given as

$$E(r_j) = \bar{r}_j = r_f + \lambda\sigma_j \text{ .... (3.32)}$$

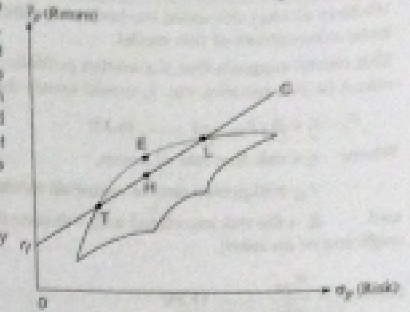


Fig. - 3.20

Where $\bar{r}_j$ = Expected rate of return on any arbitrary portfolio j.

r_f = Risk-free rate of return

σ_j = Standard deviation of the rates of return on j-th portfolio.

λ = Slope of the capital market line.

If the market portfolio has an expected rate of return $\bar{r}_M$ and a standard deviation σ_M then the slope of the CML will be :

$$\lambda = \frac{\bar{r}_M - r_f}{\sigma_M}$$

[Note : We know that

$$E(r_p) = \bar{r}_p = \alpha\bar{r}_S + (1-\alpha)r_f$$

and $$\text{Var}(r_p) = \sigma_p^2 = \alpha^2\sigma_S^2 + (1-\alpha)^2\sigma_f^2 + 2\alpha(1-\alpha)\sigma_{sf}$$

$$= \alpha^2\sigma_S^2 \quad [\because \sigma_f^2 = 0, \sigma_{sf} = 0]$$

$$\therefore \sigma_p = \alpha\sigma_s$$

So $$\bar{r}_p = r_f + \alpha(\bar{r}_S - r_f)$$

$$= r_f + \frac{(\bar{r}_S - r_f)}{\sigma_S}\sigma_p$$

If $\bar{r}_S = \bar{r}_M$, $\sigma_s = \sigma_M$, $\bar{r}_p = \bar{r}_j$ and $\sigma_p = \sigma_j$

then $$\bar{r}_j = r_f + \frac{(\bar{r}_M - r_f)}{\sigma_M}\sigma_j \text{]}$$

The slope of the capital market line is considered as the **'price of risk'** (or the risk premium for undertaking additional risk).

3.6.3. The pricing model : CAPM

We have already discussed the basic proposition of the Capital Asset Pricing Model (CAPM) and basic assumptions of this model.

This model suggests that if a market portfolio is considered to be efficient then the expected rate of return on j-th security, viz. $\bar{r}_j$ would satisfy the following relation :

$$\bar{r}_j - r_f = \beta_j(\bar{r}_M - r_f) \text{ .... } (3.33)$$

Where r_f = risk-free rate of return,

$\bar{r}_M$ = expected rate of return on market portfolio,

and β_j = the risk associated with j-th security relative to the stock market as whole. (called as beta coefficient of an asset).

$$\text{Here, } \beta_j = \frac{\sigma_{jM}}{\sigma_M^2} \text{ .... } (3.34)$$

It is a standardised measure of systematic risk of an asset.

Let us assume that α proportion of the investible fund has been invested for the purchase of asset, and $(1-\alpha)$ proportion has been invested in market portfolio M. (Here, $\alpha < 0$ if we allow riskless borrowing).

The expected rate of return on this portfolio can be expressed as follows :

$$\bar{r}_\alpha = \alpha\,\bar{r}_j + (1-\alpha)\,\bar{r}_M \text{ .... } (3.35)$$

Further, the standard deviation of the rates of return would be :

$$\sigma_\alpha = \sqrt{\alpha^2\sigma_j^2 + 2\alpha(1-\alpha)\sigma_{jM} + (1-\alpha)^2\sigma_M^2} \text{ .... } (3.36)$$

From (3.35), we get $\frac{d\bar{r}_\alpha}{d\alpha} = \bar{r}_j - \bar{r}_M$

Again, from (3.36), we get

$$\frac{d\sigma_\alpha}{d\alpha} = \frac{1}{2\sqrt{\cdot}}\left(2\alpha\sigma_j^2 + 2\sigma_{jM} - 4\alpha\sigma_{jM} - \sigma_M^2 + 2\alpha\sigma_M^2\right)$$

$$= \frac{1}{2\sqrt{\cdot}}\left[2\{\alpha\sigma_j^2 + (1-2\alpha)\,\sigma_{jM} + (\alpha-1)\,\sigma_M^2\}\right]$$

$$= \frac{\alpha\sigma_j^2 + (1-2\alpha)\sigma_{jM} + (\alpha-1)\sigma_M^2}{\sigma_\alpha} \quad [\because \sigma_\alpha = \sqrt{\cdot}\,]$$

Therefore, $\left.\frac{d\sigma_\alpha}{d\alpha}\right|_{\alpha=0} = \frac{\sigma_{jM} - \sigma_M^2}{\sigma_M}$ [Since all the investment is made on market portfolio so, $\sigma_\alpha = \sigma_M$]

So, we get $\frac{d\bar{r}_\alpha/d\alpha}{d\sigma_\alpha/d\alpha} = \frac{d\bar{r}_\alpha}{d\sigma_\alpha} = \frac{\bar{r}_j - \bar{r}_M}{\frac{\sigma_{jM} - \sigma_M^2}{\sigma_M}}$

$$= \frac{(\bar{r}_j - \bar{r}_M)}{\sigma_{jM} - \sigma_M^2}\cdot\sigma_M$$

This slope must be equal to the slope of the capital market line, i.e., $\frac{\bar{r}_M - r_f}{\sigma_M}$ (see 3.32).

$$\therefore \frac{(\bar{r}_j - \bar{r}_M)}{\sigma_{jM} - \sigma_M^2}\cdot\sigma_M = \frac{\bar{r}_M - r_f}{\sigma_M}$$

$$\text{or, } (\bar{r}_j - \bar{r}_M)\,\sigma_M^2 = (\bar{r}_M - r_f)(\sigma_{jM} - \sigma_M^2)$$

$$\text{or, } (\bar{r}_j - \bar{r}_M) = \frac{(\bar{r}_M - r_f)\sigma_{jM}}{\sigma_M^2} - \frac{(\bar{r}_M - r_f)\sigma_M^2}{\sigma_M^2}$$

$$\text{or, } \bar{r}_j - \bar{r}_M = \left(\frac{\bar{r}_M - r_f}{\sigma_M^2}\right)\sigma_{jM} - \bar{r}_M + r_f$$

$$\text{or, } \bar{r}_j = r_f + \left(\frac{\bar{r}_M - r_f}{\sigma_M^2}\right)\sigma_{jM}$$

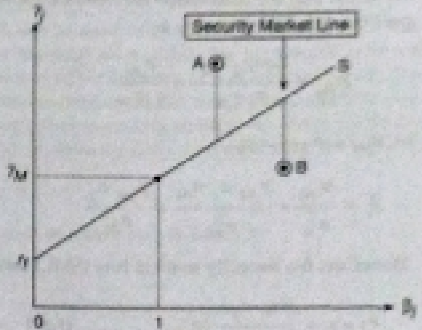


Fig. - 3.21

$$\text{or, } \bar{r}_j = r_f + \beta_j(\bar{r}_M - r_f) \text{ where } \beta_j = \frac{\sigma_{jM}}{\sigma_M^2}$$

So, we get the **security market line** (as shown in equation 3.31).

This security market line as suggested by CAPM shows :

Expected return on security j = Risk-free return + Market risk premium × Beta of security j

Fig-3.21 shows the security market line (r_f S). The slope of the security market line (SML), i.e., $\frac{d\bar{r}_j}{d\beta_j} = (\bar{r}_M - r_f)$ shows the market risk premium.

If $\beta_j = 1$, it signifies that the expected rate of return from j-th security exactly reflects the expected rate of return from the market portfolio as a whole. It also implies that the excess return on a stock $(\bar{r}_j - r_f)$ varies proportionately with the excess return on the market portfolio $(\bar{r}_M - r_f)$. Since $(\bar{r}_j - r_f) = \beta_j(\bar{r}_M - r_f)$, so $\beta_j = 1$ means $\frac{(\bar{r}_j - r_f)}{(\bar{r}_M - r_f)} = 1$.

On the other hand, if $\beta_j > 1$, it signifies that the expected rate of return on j-th security varies more than proportionately with that of the market portfolio as a whole.

Alternatively speaking, if $\beta_j > 1$ then it signifies that j-th security has more systematic risk than market portfolio as a whole. So, investment in this type of security would be considered as an aggressive investment.

Similarly, if $\beta_j < 1$ then it would imply that the expected rate of return on j-th security varies less than proportionately with that of the market portfolio, i.e., the systematic risk of j-th security is less than that of the market portfolio as a whole. Investment in this type of security is called as *defensive investment*.

The *beta coefficient* (β_j) can also be expressed as follows :

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2} = \frac{\text{Cov}(\bar{r}_j, \bar{r}_M)}{\text{Var}(\bar{r}_M)}$$

We know that the correlation between the expected rate of return on j-th security and that on the market portfolio can be expressed as :

$$\rho_{jM} = \frac{\text{Cov}(r_j, r_M)}{\sigma_j \sigma_M} = \frac{\sigma_{jM}}{\sigma_j \sigma_M}$$

or, $\sigma_{jM} = \rho_{jM} \cdot \sigma_j \sigma_M$

$$\therefore \beta_j = \frac{\sigma_{jM}}{\sigma_M^2} = \frac{\rho_{jM} \sigma_j \sigma_M}{\sigma_M^2} = \frac{\rho_{jM} \sigma_j}{\sigma_M}$$

Therefore, the security market line (SML) can be expressed as follows :

$$\bar{r}_j = r_f + \frac{(\bar{r}_M - r_f)}{\sigma_M} (\rho_{jM} \sigma_j) \text{ .... (3.37)}$$

In equation (3.37), ρ_{jM} measures the systematic risk (which arises from some macroeconomic variables such as inflation, recession, fiscal policy of the government etc.) which cannot be avoided by any investor. The systematic risk of j-th security will be $(\rho_{jM} \cdot \sigma_j)$. If the assets are fairly priced then they will be exactly on the SML. However, if an asset is underpriced (and hence, with higher expected rate of return for a given β_j value) then such assets are plotted above the SML (e.g., denoted by point A in Fig.-3.21). Similarly, when any asset is overpriced (and hence, with lower expected rate of return for a given β_j value) then such assets are plotted below the SML (e.g., denoted by point B in Fig.-3.21). The difference between the actual expected rate of return on a security and its fair return based on the SML is called as the *α (alpha) of the security*. For underpriced securities, $\alpha > 0$ and for overpriced securities, $\alpha < 0$. So, when $\alpha > 0$, the investor should purchase that security, and when $\alpha < 0$, the investor should sell that security.

3.6.4. Relationship between SML and CML

We have already shown the equation representing the capital market line (CML). This is shown below :

$$\bar{r}_j = r_f + \frac{(\bar{r}_M - r_f)}{\sigma_M} \cdot \sigma_j \text{ (see 3.32)}$$

On the other hand, the security market line (SML) as suggested by the CAPM is as follows :

$$\bar{r}_j = r_f + \frac{(\bar{r}_M - r_f)}{\sigma_M^2} \cdot \sigma_{jM} \text{ (see 3.33)}$$

or, $$\bar{r}_j = r_f + \frac{(\bar{r}_M - r_f)}{\sigma_M} (\rho_{jM} \cdot \sigma_j) \text{ (see 3.37)} \qquad \left[\because \rho_{jM} = \frac{\sigma_{jM}}{\sigma_j \sigma_M}\right]$$

If the expected rates of return on j-th security and those on the market portfolio (M) are perfectly correlated (which is true for efficient portfolios) then $\rho_{jM} = 1$. In that case, the SML and CML become similar.

Thus, CML can be treated as a special case of SML.

3.6.5. Systematic risks, Non-systematic risks and security market line

The security market line (SML), as shown in CAPM, can be used to show the systematic risks and non-systematic risks associated with any security or financial asset. While the systematic risks are associated with the market as a whole (say, arising out of an inflationary or recessionary trend), the non-systematic risks or security-specific risks are related to the factors affecting a particular firm (say, rising debt-equity ratio, adoption of improper technology, poor marketing strategy etc. of any particular firm). To show the influence of such systematic and non-systematic risks on any security, the security market line can be expressed as follows :

$$r_j = r_f + \beta_j (r_M - r_f) + u_j \text{ .... (i)}$$

Here, u_j = random factors which affect the expected rate of return on j-th security.

Now, averaging out the security market line, we get

$$E(r_j) = r_f + \beta_j E(r_M - r_f) + E(u_j)$$
$$= r_f + \beta_j E(r_M) - \beta_j r_f \text{ .... (ii)}$$

Since, it is assumed that $E(u_j) = 0$

Now, subtracting (ii) from (i), we get

$$r_j - E(r_j) = \beta_j [r_M - E(r_M)] + u_j$$

Here, $\text{Var}(r_j) = \sigma_j^2 = E[r_j - E(r_j)]^2$

$$= E[\beta_j \{r_M - E(r_M)\} + u_j]^2$$
$$= \beta_j^2 E\{r_M - E(r_M)\}^2 + E(u_j^2)$$
$$+ 2\beta_j E[\{r_M - E(r_M)\} u_j] \text{ .... (iii)}$$

Now, $\sigma_M^2 = E\{r_M - E(r_M)\}^2$

$$\sigma_u^2 = E(u_j - E(u_j))^2 = E(u_j^2)$$

$$\sigma_{uM} = \text{Cov}(u_j, r_M) = E[\{r_M - E(r_M)\}u_j]$$

Here, we assume that $\sigma_{uM} = 0$

$\therefore$ Equation (iii) can be stated as :

$$\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma_u^2 \text{ .... (iv)}$$

Here, $\sigma_j^2 = \text{Var}(r_j)$ = Total risk involved in j-th security.

This total risk has two parts : The first part, viz., $\beta_i^2\sigma_M^2$ indicates the systematic risks involved in holding i-th security, and these risks cannot be avoided through diversification of portfolio. The second part of the risk, viz, $\sigma_{u_i}^2$ = Var (u_i) indicates the non-systematic or security-specific risk. This type of risk can be avoided or minimised through diversification of the portfolio.

3.6.6. Beta (β) of a portfolio

The security market line shows the beta (β) of an individual security. A portfolio of assets consists of many securities. The portfolio beta can be estimated as the weighted average of the betas of individual assets of that portfolio. Let us assume that a portfolio consists of n number of assets, and the weights of those securities in the portfolio be denoted by $w_1, w_2, ..., w_n$ (here the weights denote the proportion of total fund invested in respective securities). The beta of a portfolio can then be calculated as

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i$$

where β_i denotes the beta of i-th security.

Example 3.19

The market portfolio consists of three securities, say, stock-A, stock-B and stock-C. The weightage of these stocks in the portfolio be 15%, 20% and 65% respectively. The market portfolio has an expected rate of return ($\bar{r}_M$) of 22·4% and a standard deviation (σ_M) of the rates of return of 15·2%. The risk-free rate of return (r_f) is given as 4%.

(i) Estimate the security market line.

(ii) Estimate the beta (β) of each stock in the portfolio on the basis of the following expected return vector, and variance-covariance (VC) matrix :

$$\bar{r} = \begin{bmatrix} 16\cdot2 \\ 24\cdot6 \\ 22\cdot8 \end{bmatrix} \qquad VC = \begin{bmatrix} 146 & 187 & 145 \\ 187 & 854 & 104 \\ 145 & 104 & 289 \end{bmatrix}$$

Solution :

(i) The security market line (SML) as per the CAPM is :

$$\bar{r}_i = r_f + \left(\frac{\bar{r}_M - r_f}{\sigma_M^2}\right)\sigma_{iM}$$

$$= 4 + \left[\frac{22\cdot4 - 4}{(15\cdot2)^2}\right]\sigma_{iM}$$

$$= 4 + \frac{18\cdot4}{231\cdot04}\sigma_{iM}$$

$$= 4 + 0\cdot08\ \sigma_{iM}.$$

(ii) In the security market line $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)}$

Again, the covariance of i-th security with market portfolio (σ_{iM}) can be expressed as the weighted average of the covariance of each security with security-i ;

$$\sigma_{iM} = \sum_{j=1}^{n} w_{jM}\sigma_{ij}$$

So, in our case,

$$\sigma_{1M} = \sum_{j=1}^{3} w_{jM}\sigma_{1j} = w_{1M}\sigma_{11} + w_{2M}\sigma_{12} + w_{3M}\sigma_{13}$$

$$= (0\cdot12 \times 146) + (0\cdot19 \times 187) + (0\cdot69 \times 145)$$

$$= 153$$

Similarly, $\sigma_{2M} = \sum_{j=1}^{3} w_{jM}\sigma_{2j} = w_{1M}\sigma_{21} + w_{2M}\sigma_{22} + w_{3M}\sigma_{23}$

$$= (0\cdot12 \times 187) + (0\cdot19 \times 854) + (0\cdot69 \times 104)$$

$$= 257$$

and $\sigma_{3M} = \sum_{j=1}^{3} w_{jM}\sigma_{3j} = w_{1M}\sigma_{31} + w_{2M}\sigma_{32} + w_{3M}\sigma_{33}$

$$= (0\cdot12 \times 145) + (0\cdot19 \times 104) + (0\cdot69 \times 289)$$

$$= 236$$

Now, the beta value of each security can be calculated as follows :

$$\beta_{1M} = \frac{\sigma_{1M}}{\sigma_M^2} = \frac{153}{(15\cdot2)^2} = 0\cdot66$$

$$\beta_{2M} = \frac{\sigma_{2M}}{\sigma_M^2} = \frac{257}{(15\cdot2)^2} = 1\cdot11$$

$$\beta_{3M} = \frac{\sigma_{3M}}{\sigma_M^2} = \frac{236}{(15\cdot2)^2} = 1\cdot02$$

[*Note* : The expected rate of return on Stock-1, 2 and 3 based on SML will be :

$\bar{r}_1 = 4 + 0\cdot08\ \sigma_{1M} = 4 + (0\cdot08 \times 153) = 16\cdot2$

$\bar{r}_2 = 4 + 0\cdot08\ \sigma_{2M} = 4 + (0\cdot08 \times 257) = 24\cdot6$

$\bar{r}_3 = 4 + 0\cdot08\ \sigma_{3M} = 4 + (0\cdot08 \times 236) = 22\cdot8$

Example 3.20

Let the risk-free rate of return be [illegible] [illegible] is 20%.

Solution :

According to CAPM,

$$\bar{r}_p = r_f + \left(\frac{\bar{r}_M - r_f}{\sigma_M}\right)\sigma_i$$

$$= [illegible]$$

$$\therefore \bar{r}_p = 12.8\%$$

Example 3.21

An investor manages a portfolio consisting of the following 4 stocks with their respective market values and beta (β) :

Stock	Market Value (₹)	β (beta)
A	2,00,000	1.16
B	1,00,000	1.20
C	1,50,000	0.80
D	50,000	0.50

If the risk-free return is 9% and the market rate of return is 15% then calculate the expected rate of return on this portfolio based on CAPM.

Solution :

Here, total investment made by the investor is ₹ 5,00,000 (= 2,00,000 + 1,00,000 + 1,50,000 + 50,000). Therefore, the weightage of each stock in the portfolio will be as follows :

$$w_A = \frac{2,00,000}{5,00,000} = 0.4$$

$$w_B = \frac{1,00,000}{5,00,000} = 0.2$$

$$w_C = \frac{1,50,000}{5,00,000} = 0.3$$

$$w_D = \frac{50,000}{5,00,000} = 0.1$$

As per CAPM, the expected rate of return on portfolio ($\bar{r}_p$) would be

$$\bar{r}_p = r_f + \beta_p(\bar{r}_M - r_f)$$

Here $r_f = 9\%$

Now we are to calculate the portfolio beta (β_p), where

$$\beta_p = \sum_{i=1}^{4} w_i\beta_i$$

$$= (0.4 \times 1.16) + (0.2 \times 1.2) + (0.3 \times 0.8) + (0.1 \times 0.5)$$

$$= 0.464 + 0.24 + 0.24 + 0.05$$

$$= 0.994$$

$$\therefore \bar{r}_p = 9 + 0.994\,(15 - 9)$$

$$= 9 + 5.964$$

$$= 14.964$$

Example 3.22

If the following assets are correctly priced on the security market line (SML) then (i) estimate the expected rate of return on market portfolio, and (ii) the risk-free rate of return.

The expected rate of returns and beta values for Stock-1 and 2 are given as :

$\bar{r}_1 = 9.40\%$, $\bar{r}_2 = 13.4\%$, $\beta_1 = 0.8$, $\beta_2 = 1.30$.

Solution :

The security market line (SML) is expressed as :

$$\bar{r}_i = r_f + \beta_i(\bar{r}_M - r_f)$$

According to this problem,

expected rate of return on stock -1 is

$$\bar{r}_1 = r_f + \beta_1(\bar{r}_M - r_f)$$

$$9.4 = r_f + 0.8(\bar{r}_M - r_f) \text{ .... (i)}$$

and $\bar{r}_2 = r_f + \beta_2(\bar{r}_M - r_f)$

$$13.4 = r_f + 1.3(\bar{r}_M - r_f) \text{ .... (ii)}$$

From (i), we get

$$9.4 = r_f - 0.8r_f + 0.8\,\bar{r}_M$$

$$9.4 = 0.2r_f + 0.8\,\bar{r}_M \text{ .... (iii)}$$

From (ii), we get

$$13.4 = r_f - 1.3r_f + 1.3\,\bar{r}_M$$

$$13.4 = -0.3r_f + 1.3\,\bar{r}_M \text{ .... (iv)}$$

$$\text{(iii)} \times 1.5 \Rightarrow 14.1 = 0.3r_f + 1.2\,\bar{r}_M \text{ .... (v)}$$

From (iv) + (v), we get

$27·5 = 2·5\bar{r}_M$

or, $\bar{r}_M = 11$

Now, substituting this value of $\bar{r}_M$ in (iii) we get

$9·4 = -0·2r_f + 0·8 \times 11$

or, $r_f = \frac{9·4-8·8}{0·2} = \frac{0·6}{0·2} = 3$

Example 3.23

Mr. Sen is planning to purchase the share of X-company Ltd. He expects that the share of X-company Ltd. would earn a return of 17% in the next year.

If the risk-free rate of return (r_f) is 7%, beta of that stock (β) is 1·3 and the expected rate of return on market portfolio ($\bar{r}_M$) is 15% then should Mr. Sen invest in this stock of X-company Ltd.?

Solution :

According to the CAPM,

$\bar{r}_i = r_f + \beta_i(\bar{r}_M - r_f)$

$= 7 + 1·3\,(15-7)$

$= 7 + 10·4$

$= 17·4\%$

Since the actual rate of return expected by Mr. Sen is 17% and it is less than the ideal rate of return, hence Mr. Sen should not invest in this stock.

Example 3.24

The beta of Stock-A is 1·4 ; the expected rate of return on market portfolio is 14% and the risk-free rate is 10%. (i) Estimate the expected rate of return on stock-A based on CAPM ; (ii) If the risk premium on market portfolio goes up by 2·5% what would be the revised expected rate of return on Stock-A?

Solution :

(i) The expected rate of return on stock -A based on CAPM is :

$\bar{r}_i = r_f + \beta_i(\bar{r}_M - r_f)$

$= 10 + 1·4\,(14 - 10)$

$= 10 + 5·6$

$= 15·6\%$

(ii) If the risk premium on market portfolio goes up by 2·5% then the risk premium would be $(14 - 10) + 2·5 = 6·5\%$

∴ The revised expected rate of return on Stock-A will be :

$\bar{r}_i = 10 + (1·4 \times 6·5)$

$= 10 + 9·1$

$= 19·1\%$

Example 3.25

The Treasury Bills give a rate of return of 5% and the expected rate of return on market portfolio is 13%.

(i) Estimate the risk premium over market rate of return.

(ii) Also calculate the β value and required rate of returns for the following combinations of investments :

Asset	Weightage				
Treasury bills	100	80	70	30	0
Risky assets	0	20	30	70	100

Solution :

(i) Here, risk premium $= \bar{r}_M - r_f = 13 - 5 = 8\%$.

(ii) The beta value of risk-free Treasury Bills is $\beta = 0$, and the beta value of risky market portfolio is $\beta = 1$. Now, the portfolio beta (β_p) [the weighted average of individual beta values] and the portfolio rate of return ($\bar{r}_p$) would be as follows : $\beta_p = \sum_{i=1}^{n} w_i\beta_i$

and $\bar{r}_p = r_f + \beta_i(\bar{r}_M - r_f)$

Portfolio	TB : Risky assets	$\beta_p = \sum w_i\beta_i$	$\bar{r}_p = r_f + \beta_i(\bar{r}_M - r_f)$
1	100 : 0	0	$5 + 0(13 - 5) = 5\%$
2	80 : 20	$0·8 \times 0 + 0·2 \times 1 = 0·2$	$5 + 0·2(13 - 5) = 6·6\%$
3	70 : 30	$0·7 \times 0 + 0·3 \times 1 = 0·3$	$5 + 0·3(13 - 5) = 7·4\%$
4	30 : 70	$0·3 \times 0 + 0·7 \times 1 = 0·7$	$5 + 0·7(13 - 5) = 10·6\%$
5	0 : 100	$1 \times 1 = 1$	$5 + 1·0(13 - 5) = 13\%$

Example 3.26

The Heritage Pvt. Ltd. expects that the equity share holders of this enterprise should get a return of at least 15·5% (based on current market prices of the stock of this enterprise). However, the present expected rate of return on market portfolio is 12%. The risk-free rate of return (on government bonds) is 4·5%. Estimate the beta (β) of the stock of the Heritage Pvt. Ltd.

Solution :

On the basis of CAPM, the expected rate of return on i-th asset would be

$\bar{r}_i = r_f + \beta_i(\bar{r}_M - r_f)$

Here, $\bar{r}_i = 15·5\%$, $r_f = 4·5\%$ and $\bar{r}_M = 12\%$

$\therefore \beta_i = \frac{\bar{r}_i - r_f}{\bar{r}_M - r_f} = \frac{15·5-4·5}{12-4·5} = \frac{11}{7·5} = 1·47$

3.6.7. Use of CAPM in investment analysis

The Capital Asset Pricing Model (CAPM) can be used by an investor for taking appropriate investment decision in the financial market. Our previous discussion indicates that a prudent investor should invest in purchasing financial assets in such a way that these assets would represent a maket portfolio. Even a small investor can have such market portfolio through investment in mutual funds which represent a diversified portfolio of the fund manager.

However, the CAPM can guide an investor in holding, buying and selling of stocks. This model provides the required rate of return on a security after considering the risk involved in an investment. The investors, on the basis of the current market price of the assets and some other judgemental factors (say, the stock market indices), can estimate the expected rate of return on any stock over a time period. The CAPM helps the investor to estimate the required rate of return on any asset. The investor becomes able to compare between the required rate of return and the expected rate of return for taking any investment decision. The possible decisions are noted below :

(i) If the CAMP-based required rate of return on an asset is less than the expected rate of return then it implies that the security is undervalued or underpriced, i.e., the stock gives more return than what it should give. In that situation, the investor should purchase that stock.

(ii) If the CAPM-based required rate of return on an asset becomes more than the expected rate of return on that asset then it signifies that the asset is overvalued or overpriced, i.e., the stock gives less return than what it should give. In that case, the investor should sell that stock.

(iii) If the CAPM-based *required* rate of return is just equal to the expected rate of return on an asset then it suggests that the stock gives same return as what it should give. In that case, the investor should hold that stock.

- ***CAPM and evaluation of the performance of a portfolio :***

The CAPM is often used to evaluate the performance of a portfolio, say, the performance of X-mutual fund.

*The performance evaluation is made in terms of the mean-variance portfolio theory and the CAPM. This evaluation helps the investor to decide whether X-fund can be regarded as a good fund and whether it can be regarded as the efficient market portfolio (or the **One fund** in Markowitz sense).*

To evaluate the performance of a mutual fund, the investor first collects the rate of return on X-mutual fund for a particular period, say, for the last 10 years. On the basis of this statistical information, the average rate of return ($\bar{r}_X$) on that fund can be calculated where $\bar{r}_X = \frac{1}{n}\sum_{i=1}^{n} r_i$

Then the investor can calculate the deviations of the actual rate of return from the average value.

i.e., $\sum_{i=1}^{n}(r_i - \bar{r}_X)$.

The investor then can collect the statistical information regarding the market rate of return based on, say, BSE SENSEX or NIFTY or BSE-500 etc. for the same period (i.e., for the last 10 years).

On the basis of this information, the investor can estimate average rate of return on market portfolio ($\bar{r}_M$) and the deviations of those returns from its average value during that period, i.e., $\sum_{i=1}^{n}(r_{Mi} - \bar{r}_M)$.

Then the investor can calculate the covariance between the rate of return on X-mutual fund and the market portfolio as follows :

$$Cov(r_X, r_M) = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r}_X)(r_{Mi} - \bar{r}_M)$$

The investor can also calculate the standard deviation of the market rate of return using the formula :

$$\sigma_M^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_{Mi} - \bar{r}_M)^2$$

Then, the investor can calculate the beta(β) of the X-fund as follows :

$$\beta_X = \frac{Cov(r_X, r_M)}{\sigma_M^2}$$

The average rate of return on 1-year Government Treasury Bill for that period can be regarded as the risk-free rate (r_f).

Then the investor can use '**Jensen Index**' (J) to evaluate the performance of X-fund. This estimation process (using J-index) is similar to that of CAPM formula as shown below :

$$\bar{r}_X - r_f = J + \beta_X(\bar{r}_M - r_f) \quad \text{......} \quad (3.38)$$

As opposed to CAPM pricing formula, the expected rates of return have been replaced by the measured average rates of return for X-fund and market portfolio based on available statistical information.

The CAPM measure shows that the J-index would be zero when true expected rates of returns are used. If J > 0 then it would imply that the fund (viz., X-fund) performs better than the CAPM prediction. So, this J-index shows how far the performance of X-fund has deviated from the theoretical value of zero.

We can use an example to explain the process of calculating J-index.

Example 3.27

Let us consider the following rates of returns on X-fund, BSE-500 (market portfolio) and 364-days Treasury Bills for a period of 10 years :

Year	Rate of return (%)		
	X-fund	BSE-500	T-Bill
1	14	12	7.0
2	10	7	7.5
3	19	20	7.7
4	– 8	– 2	7.5
5	23	12	8.5
6	28	23	8.0
7	20	17	7.3
8	14	20	7.0
9	– 9	– 5	7.5
10	19	16	8.0
Total	130	120	76

We have to calculate $\bar{r}_X$, $\bar{r}_M$, σ_M^2 and Cov (r_X, r_M)

Here, $\bar{r}_X = \frac{1}{n}\sum_{i=1}^{n} r_{iX} = \frac{130}{10} = 13$

$$\bar{r}_M = \frac{1}{n}\sum_{i=1}^{n} r_{iM} = \frac{120}{10} = 12$$

$$r_f = \frac{1}{n}\sum_{i=1}^{n} r_f = \frac{76}{10} = 7.6$$

Now, σ_M^2 and Cov (r_X, r_M) can be calculated as follows :

Year	$r_{iM} - \bar{r}_M$	$(r_{iM} - \bar{r}_M)^2$	$(r_{iX} - \bar{r}_X)$	$(r_{iX} - \bar{r}_X)(r_{iM} - \bar{r}_M)$
1	0	0	1	0
2	−5	25	−3	15
3	8	64	6	48
4	−14	196	−21	294
5	0	0	10	0
6	11	121	15	165
7	5	25	7	35
8	8	64	1	8
9	−17	289	−22	374
10	4	16	6	24
		$\sum(r_{iM} - \bar{r}_M)^2 = 800$		$\sum(r_{iX} - \bar{r}_X)(r_{iM} - \bar{r}_M) = 963$

$$\therefore \sigma_M^2 = \frac{1}{n-1}\sum(r_{iM} - \bar{r}_M)^2 = \frac{800}{9} = 88.89$$

$$\therefore \sigma_M = \sqrt{\sigma_M^2} = \sqrt{88.88} = 9.428$$

$$\text{Cov}(r_X, r_M) = \frac{1}{n-1}\sum(r_{iX} - \bar{r}_X)(r_{iM} - \bar{r}_M) = \frac{963}{9} = 107$$

$$\therefore \beta_X = \frac{\text{Cov}(r_X, r_M)}{\sigma_M^2} = \frac{107}{88.89} = 1.204$$

Hence, the J-index will be as follows :

$$\bar{r}_X - r_f = J + \beta_X(\bar{r}_M - r_f)$$

or, $13 - 7.6 = J + 1.204\,(12 - 7.6)$

or, $5.4 = J + 5.297$

or, $J = 5.4 - 5.297 = 0.103$

Here, the positive value of J-index implies that the X-fund has performed better than the value as predicted by the CAPM.

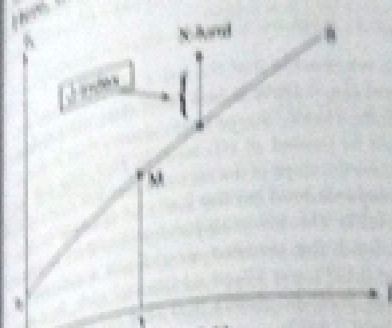


Fig - 3.22

According to CAPM, the ideal value of J should be zero. Thus, the J-index which has been derived on the basis of the observed values of the rates of return, shows that how far the performance of X-fund would deviate from the theoretical value (i.e. zero). Fig - 3.22 clearly shows that J-index for X-fund lies above the security market line (with a positive value). Hence, based on the J-index, it seems that X-fund is an excellent fund. But financial analysts are of the opinion that the quality of a fund should not only be judged by J-index. Its efficiency should also be tested by another index, viz. the Sharpe index (S).

The Sharpe index (S) is measured as follows : $\frac{\bar{r}_X - r_f}{\sigma_X} = S$

Where σ_X = standard deviation of the rates of return on X-fund (portfolio).

Based on our example, we can calculate the σ_X as follows :

Year	$(r_{iX} - \bar{r}_X)$	$(r_{iX} - \bar{r}_X)^2$
1	1	1
2	−3	9
3	6	36
4	−21	441
5	10	100
6	15	225
7	7	49
8	1	1
9	−22	484
10	6	36
		$\sum(r_{iX} - \bar{r}_X)^2 = 1382$

$$\therefore \sigma_X^2 = \frac{1}{n-1}\sum(r_{iX} - \bar{r}_X)^2 = \frac{1382}{9} = 153.55$$

$$\therefore \sigma_X = \sqrt{\sigma_X^2} = \sqrt{153.55} = 12.39$$

So, the Sharpe index (S) $= \frac{13 - 7.6}{12.38}$

$= 0.4369$

The Sharpe index measures the excess return earned on X-fund per unit of total risk (as measured by the σ_X).

This should be compared with the excess return per unit of risk taken for market portfolio (M), i.e.

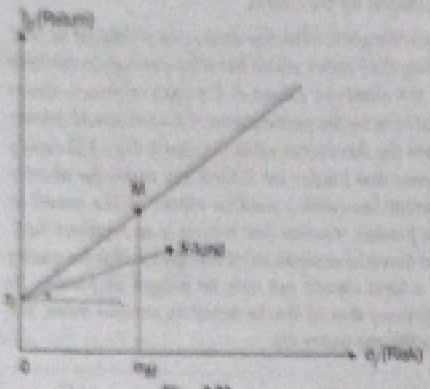


Fig - 3.23

$$\frac{\bar{r}_M - r_f}{\sigma_M} = \frac{12-7.6}{9.428} = 0.4666$$

Thus, we observe that the price of risk taken on X-fund (i.e. 0.4369) is less than that on market portfolio. Hence, Sharpe index shows that X-fund cannot be treated as efficient. In fact, we are to compare the slope of the security market line with the slope derived on the basis of Sharpe index (Fig -3.23). This lower slope signifies that in case of X-fund, the investor would get a return of about 0.437% per 1% of additional risk, but the market portfolio shows that this return should be about 0.467%.

3.6.8. Use of CAPM as pricing formula

The CAPM does not explicitly show the price of a security. However, we can use CAPM formula to estimate the price of a security. We know that the expected rate of return on an asset as per CAPM is expressed as :

$$\bar{r}_i = r_f + \beta_i(\bar{r}_M - r_f)$$

Now, this expected rate of return on *i-th* asset ($\bar{r}_i$) can be expressed as follows :

$$\bar{r}_i = \frac{\bar{P}_1 - P_0}{P_0}$$

where P_0 = Price at which an asset was purchased.

$\bar{P}_1$ = Expected price at which the asset has been sold.

Hence, the CAPM formula can then be expressed as :

$$\frac{\bar{P}_1 - P_0}{P_0} = r_f + \beta_i(\bar{r}_M - r_f)$$

or, $\bar{P}_1 - P_0 = P_0[r_f + \beta_i(\bar{r}_M - r_f)]$

or, $\bar{P}_1 = P_0 + P_0[r_f + \beta_i(\bar{r}_M - r_f)]$

or, $\bar{P}_1 = P_0[1 + \{r_f + \beta_i(\bar{r}_M - r_f)\}]$

or, $P_0 = \dfrac{\bar{P}_1}{[1 + r_f + \beta_i(\bar{r}_M - r_f)]}$ .... (3.39)

Here, $\bar{P}_1$ is a random value (since, it can take any value) and P_0 is a known value (since the investor knows the price at which the asset was purchased by him). However, this pricing formula resembles the discounting formula for a deterministic situation that we have already discussed in our earlier chapter.

...ever, in this case, we are trying to find out the present value of a non-deterministic or random ... flow. In case of a deterministic cash flow, the present value discount factor would have been $\frac{1}{1+r_f}$ (where r_f = risk-free rate of return). However, in case of this random cash flow, the risk-adjusted present value discount factor is $\dfrac{1}{1 + r_f + \beta_i(\bar{r}_M - r_f)}$.

Example 3.28

An investor invests in a mutual fund that has allocated 20% of the fund in risk-free asset and 80% of the fund in risky market portfolio. The risk-free rate of return is 5% (i.e, r_f = 0.05) and the expected rate of return on market portfolio is 12% (i.e., $\bar{r}_M$ = 0.12). Estimate the expected value of the unit of that mutual fund after one year. If the present price of 1 unit of the mutual fund is ₹ 100 then does it represent the present value of the future expected price of the unit? Estimate with the help of CAPM pricing formula.

Solution :

Here, the investor wants to invest in purchasing the units of a mutual fund which has allocated 20% of the fund in risk-free asset where risk-free rate of return (r_f) is 5%, and 80% of the fund in risky market portfolio with an expected rate of return ($\bar{r}_M$) of 12%.

Therefore, after 1 year, the expected value of one unit of that mutual fud ($\bar{P}_1$) will be :

$\bar{P}_1$ = 20 × (1 + 0.05) + 80 × (1 + 0.12)

or, $\bar{P}_1$ = 21 + 89.60 = ₹ 110.60

Thus, the present value of this expected future price of one unit of that mutual fund must be ₹ 100. This can be verified with the help of CAPM pricing formula as shown below :

$$P_0 = \frac{\bar{P}_1}{[1 + r_f + \beta_i(\bar{r}_M - r_f)]}$$

Since the investor knows P_0 = ₹ 100 and has calculated $\bar{P}_1$ = ₹ 110.60 so, he can calculate the *beta* (β_i) of the *i-th* security as follows :

$$\frac{110.60}{1 + 0.05 + \beta_i(0.12 - 0.05)} = 100$$

or, $\dfrac{110.60}{1.05 + 0.07\beta_i} = 100$

or, $\dfrac{110.60}{100} - 1.05 = 0.07\,\beta_i$

or, $\dfrac{0.056}{0.07} = \beta_i$

or, β_i = 0.8

Hence, considering $\dfrac{1}{1 + r_f + \beta_i(\bar{r}_M - r_f)}$ as the risk-adjusted present value discount factor, the present value of the expected cash flow of ₹ 110.60 would be :

$$\frac{110.60}{1+0.05+0.8(0.12-0.05)} = \frac{110.60}{1.106} = ₹\,100$$

Thus, the present price of the unit of that mutual fund correctly reflects the present value of expected future price of that unit.

Example 3.29

An investor has the following information regarding the expected value of a security after 1 year ($\bar{P}_1$), the risk-free rate of return (r_f), the expected rate of return on market portfolio ($\bar{r}_M$) and the beta (β) of the security :

$\bar{P}_1$	r_f	$\bar{r}_M$	β
₹ 800	6%	11%	0.55

Estimate the present price (P_0) of that security based on CAPM pricing formula. If, other things remaining same, β-value rises to 0.65 then what would be its impact on the present price of the security ?

Solution

Here, we can use the CAPM pricing formula, as shown below, to estimate the present price (P_0) of the security :

$$P_0 = \frac{\bar{P}_1}{1+r_f+\beta(\bar{r}_M-r_f)}$$

or, $$P_0 = \frac{800}{1+0.06+0.55(0.11-0.06)}$$

$$= \frac{800}{1.06+0.0275} = \frac{800}{1.0875} = ₹\,735.63$$

Now, if the beta (β) value, i.e., the relative riskiness of that security in relation to the market portfolio rises to 0.65

then, $$P_0 = \frac{800}{1+0.06+0.65(0.11-0.06)}$$

$$= \frac{800}{1.0925} = ₹\,732.26$$

Thus, with an increase in β value, the present value of the future expected pay off of that security will fall.

Example 3.30

Assume that the expected rate of return on the market portfolio is 23% and the risk-free return is 7%. The standard deviation of the market portfolio is 32%. Assuming that the market portfolio is efficient,

(a) Derive the equation of the capital market line. Interpret the slope of the line.

(b) What will be the standard deviation of this position if an expected return of 39% is desired ?

(c) If you invest ₹ 600 in the risk-free asset and ₹ 1,400 in the market portfolio, how much money should you expect to have at the end of the year ?

(d) Consider an asset with expected pay-off ₹ 1,000 and covariance of 0.154 with the market. Determine the current value of the asset based on CAPM pricing. [C.U. B.SC(H), Sem-V, 2012]

Solution :

(a) The equation of capital market line is expressed as :

$$E(r_j) = r_f + \frac{(\bar{r}_M - r_f)}{\sigma_M}\cdot\sigma_j$$

Here, $\bar{r}_M = 23\%$, $r_f = 7\%$, and $\sigma_M = 32\%$

$$\therefore E(r_j) = 7\% + \frac{23\%-7\%}{32\%}\cdot\sigma_j$$

$$= 0.07 + \frac{0.23-0.07}{0.32}\sigma_j$$

$$= 0.07 + \frac{0.16}{0.32}\sigma_j$$

$$= 0.07 + 0.5\,\sigma_j$$

The slope of this line (0.5) suggests that the investor would get a return of 0.5% for undertaking an additional risk of 1%.

(b) If $E(r_j) = 39\% = 0.39$ then from the capital market line, we get

$$0.39 = 0.07 + 0.5\,\sigma_j$$

$$\text{or, } \sigma_j = \frac{0.39-0.07}{0.5} = 0.64 \text{ or } 64\%$$

(c) If ₹ 600 is invested in risk-free asset then at the end of the year the return would be 7% of ₹ 600 = ₹ 42.

On the other hand, if ₹ 1,400 is invested in risky market portfolio then at the end of the year the return would be 23% of ₹ 1,400 = ₹ 322.

At the end of the year, the investor would get ₹ 42 + ₹ 322 = ₹ 364.

(d) According to CAPM pricing formula

$$P_0 = \frac{\bar{P}_1}{1+r_f+\beta_i(\bar{r}_M-r_f)}$$

$$\text{Where } \beta_i = \frac{\mathrm{Cov}(r_i, \bar{r}_M)}{\mathrm{Var}(\bar{r}_M)} = \frac{\mathrm{Cov}(r_i, \bar{r}_M)}{\sigma_M^2}$$

Here, $\bar{P}_1 = ₹\,1{,}000$, $r_f = 7\%$, $\bar{r}_M = 23\%$ and $\mathrm{Cov}(r_i, \bar{r}_M) = 0.154$, $\sigma_M = 32\%$

$$\therefore \beta_i = \frac{0.154}{(0.32)^2} = \frac{0.154}{0.1024} = 1.504$$

$$\therefore P_0 = \frac{1{,}000}{1+0.07+1.504(0.23-0.07)}$$

$$= \frac{1000}{1.31} = ₹\,763.36$$

Summary

When any investor buys any financial asset then total return on investment will be dividend income plus capital gains. If x_1 = Amount realised and x_0 = Amount invested then total return $(R) = \frac{x_1}{x_0}$, and the rate of return $(r) = \frac{x_1 - x_0}{x_0}$.

If P_t = Stock price at the time period t

P_{t+1} = Stock price at the time period $t+1$

D_{t+1} = Dividend paid at the time period $t+1$

then $r = \frac{D_{t+1} + (P_{t+1} - P_t)}{P_t}$

However,

$$1 + \frac{x_1 - x_0}{x_0} = \frac{x_0 + x_1 - x_0}{x_0} = \frac{x_1}{x_0}$$

So, $1 + r = R$

Further, $1 + r = \frac{x_1}{x_0}$

or, $x_1 = x_0(1 + r)$

Hence, the rate of return is just like an interest rate.

The average rate of return will be :

$$\bar{r} = \frac{1}{n}\sum_{t=1}^{n} r_t \quad \text{Where } r_t = \text{Rate of return on investment at period } t \text{ where } t = 1, 2, 3, \ldots n.$$

The average compound rate of return during any period is estimated as

$$r_C = \left[(1+r_1)(1+r_2)\cdots(1+r_n)\right]^{\frac{1}{n}} - 1$$

$$= \sqrt[n]{(1+r_1)(1+r_2)\cdots(1+r_n)} - 1$$

If x_{0i} = Amount invested in i-th asset of a portfolio

then $\sum_{i=1}^{n} x_{0i} = x_0$ = Total fund available for investment

If w_i = Fraction of i-th asset in the portfolio and $\sum_{i=1}^{n} w_i = 1$

then $x_{0i} = w_i \cdot x_0$

If R_i = Total return from i-th asset at the end of a period

Then total return from a portfolio will be $R = \frac{\sum_{i=1}^{n} R_i \cdot w_i \cdot x_0}{x_0} = \frac{x_0 \sum_{i=1}^{n} R_i \cdot w_i}{x_0}$ [Since x_0 is given constant]

$$= \sum_{i=1}^{n} R_i \cdot w_i$$

$$\text{or, } 1 + r = \sum_{i=1}^{n} (1 + r_i) \cdot w_i \qquad [\because R = 1 + r]$$

$$= \sum w_i r_i + \sum w_i = \sum w_i \cdot r_i + 1 \qquad [\because \sum w_i = 1]$$

$$\text{or, } r = \sum w_i \cdot r_i$$

In case of short selling the initial outlay will be (–) x_0 and the cash outflow while refunding the value will be (–) x_1 and therefore,

$$R = \frac{-x_1}{-x_0} = \frac{x_1}{x_0}$$

So, in this case also, we get $-x_1 = -x_0 R = -x_0(1 + r)$

or, $x_1 = x_0(1 + r)$.

If x_i is a random variable then the expected return will be $E(x) = \sum_{i=1}^{n} x_i \cdot p_i$

where p_i = Probability of getting x_i and $\sum_{i=1}^{n} p_i = 1$

The variance of a random variable will be $Var(x) = E(x - \bar{x})^2 = E(x^2) - \bar{x}^2$

The mutual dependence of random variables is measured by the covariance :

$$Cov(x_1, x_2) = E[(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)]$$

$$= E(x_1 \cdot x_2) - \bar{x}_1 \cdot \bar{x}_2$$

If x_1 and x_2 are uncorrelated then $Cov(x_1, x_2) = 0$

The expected return from a portfolio of assets will be :

$$E(r) = \sum_{i=1}^{n} w_i E(r_i) = \sum_{i=1}^{n} w_i \cdot p_i \cdot r_i \quad \text{where } \sum_{i=1}^{n} w_i = 1$$

The portfolio risk is measured as follows

$$Var(r_p) = E(r - \bar{r})^2 = \sum_{i=1}^{n} p_i (r_i - \bar{r})^2$$

This can also be measured as follows :

$$Var(r_p) = E[(r - \bar{r})^2]$$

$$= E\left[\left(\sum w_i r_i - \sum w_i \bar{r}_i\right)^2\right]$$

$$= \sum_{i,j=1}^{n} w_i \cdot w_j E[(r_i - \bar{r}_i)(r_j - \bar{r}_j)]$$

$$= \sum_{i,j=1}^{n} w_i \cdot w_j \, Cov(r_i, r_j)$$

$$= \sum_{i,j=1}^{n} w_i \cdot w_j \cdot \sigma_{ij}$$

The risk which cannot be avoided through portfolio diversification is called as **systematic risk** (like economic recession or boom which affect all the firms almost equally). However, the risk which is specific for any particular firm can be treated as **unsystematic risk** and this risk can be avoided through portfolio diversification. However, portfolio risk cannot be brought down to zero through diversification if the rates of returns on assets in the portfolio are correlated.

Again, if in a two-asset portfolio the two assets are perfectly correlated there would be no gain from portfolio diversification. In this case, the upper bound of the portfolio 'risk and return' would lie on a straight line showing a positive linear relationship between portfolio risk and return. However, if two assets in a portfolio have perfect negative correlation in terms of their risk and return then the lower bounds of risk-return mix will lie either on a negatively sloped line segment or on a positively sloped line segment. Further, if the prospects of two assets in a portfolio have zero correlation then we get a curved line (bowed to the left) showing the risk-return mix.

A set of all possible points representing the risk-return profiles of different possible portfolios consisting of n number of assets in different proportions can be considered as the *feasible set of portfolios*. A portfolio is considered to be an *efficient portfolio* if (i) there exists another portfolio that generates more rate of return with similar risk, or (ii) there exists another portfolio that generates same rate of return but involves lower risk.

In this way, we can trace out the efficient frontier of a portfolio in the risk-return plane, and in that frontier we can locate the **minimum variance point**.

Harry Markowitz has formalised the risk-return relationship on the basis of some assumptions and developed the notions of feasible set of portfolios, minimum variance set and the efficient portfolio frontier. He has defined portfolio diversification as a process of combining assets whose prospects are not perfectly positively correlated. Markowitz model suggests that the investor can fix an arbitrary value for the expected rate of return on portfolio and then try to find out minimum variance portfolio.

Two fund theorem suggests that if two efficient portfolios or funds can be formed then several other efficient portfolios or funds can be formed in terms of portfolio mean and variance as a combination of those two efficient funds.

When any risk-free asset is added to the portfolio of the risky assets, it would enlarge the feasible set of portfolios in Markowitz model. It also implies the possibility of lending and borrowing at risk-free rate. In this case, the **portfolio return** would be :

$$E(r_p) = \alpha r_i + (1 - \alpha) r_f$$

Where α = Proportion of risky assets in the portfolio

r_f = Risk-free rate of return.

The variance of the rates of return on this portfolio will be :

$$Var(r_p) = \sigma_p^2 = \alpha^2 \cdot \sigma_i^2 \quad [\because \sigma_f^2 = 0, \sigma_{if} = 0]$$

$$\therefore \sqrt{\sigma_p^2} = \alpha \cdot \sigma_i \quad or, \quad \alpha = \frac{\sigma_p}{\sigma_i}$$

Therefore, we get

$$\bar{r}_p = r_f + \frac{(\bar{r}_i - r_f)}{\sigma_i} \cdot \sigma_p$$

This equation is known as **Capital allocation line**.

Now given the efficient frontier of a portfolio, the highest possible slope of the capital allocation line (with a point of tangency with the efficient frontier) will show highest possible return over the risk-free rate for undertaking additional risk.

One Fund Theorem signifies that there remains a single efficient fund or portfolio of risky assets such that any efficient portfolio can be constructed as a combination of that 'one fund' and the risk-free asset. Now given the efficient portfolio in Markowitz model, the problem of funding an optimal portfolio can also be viewed in terms of the maximisation of utility of the investor. The utility function of the investor may be expressed as $U = f(\bar{r}_p, \sigma_p)$. The point of tangency between the indifference curve and the capital allocation line, viz. $\bar{r}_p = r_f + \frac{(\bar{r}_i - r_f)}{\sigma_i} \cdot \sigma_p$ can locate the equilibrium position of the investor.

The **Capital Asset Pricing Model (CAPM)** logically follows from the Markowitz model. This model, based on certain asumptions, suggests that if a market portfolio is considered to be efficient then the expected rate of return on j-the security or asset will be :

$$\bar{r}_j = r_f + \beta_j(\bar{r}_M - r_f)$$

Where β_j = Risk associated with j-th security relative to the stock market as a whole = $\frac{\sigma_{jM}}{\sigma_M^2}$

$$or, \quad \bar{r}_j = r_f + \left(\frac{\bar{r}_M - r_f}{\sigma_M^2}\right)\sigma_{jM}$$

This is called as **Security Market Line**.

On the other hand, the **Capital Market Line** is expressed as :

$$\bar{r}_j = r_f + \frac{(\bar{r}_M - r_f)}{\sigma_M} \cdot \sigma_j$$

The **portfolio beta** (β) will be :

$$\beta_p = \sum_{i=1}^{n} w_i \cdot \beta_i \quad \text{where } \beta_i = \text{Beta of } i\text{-th security.}$$

The CAPM is often used for evaluating an investment project.

Assignment

Short-answer type questions

1. What is meant by return on investment ? (See Section 3.2)
2. What are the constituents of total return on investment ? (See Section 3.2)
3. How can you estimate total return on investment ? (See Section 3.2)
4. How the rate of return on investment in financial assets can be estimated ? (See Section 3.2)

5. If a stock is purchased and sold at a price of ₹ 200 and ₹ 300 respectively and if the dividend income that accrues at the end of the year is ₹ 10 per stock then estimate the rate of return on investment.
(See Section …)

6. Show the relationship between overall return and the rate of return on investment in financial assets.
(See Section …)

7. Define average rate of return on investment. (See Subsection …)
8. How can an investor estimate the total return realised from a portfolio of assets ? (See Subsection …)
9. What is meant by short selling of a financial asset ? (See Subsection …)
10. If the money value to be obtained from a gamble is equal to the number appeared on the die then estimate the expected pay-off for the gamble. (See Section …)
11. How can you estimate the variance of a random variable ? (See Subsection …)
12. How do you estimate the mutual dependence between two or more random variables ?
(See Subsection …)

13. What is meant by portfolio of assets ? (See Section …)
14. What do you mean by portfolio weights ? (See Section …)
15. Define portfolio mean. (See Subsection …)
16. How can you estimate portfolio risk ? (See Subsection …)
17. What is meant by systematic risk ? (See Subsection …)
18. What are unsystematic risks ? (See Subsection …)
19. What is meant by a feasible set of portfolios ? (See Subsection …)
20. How can you define an efficient portfolio of financial assets ? (See Subsection …)
21. Show a minimum variance portfolio set using a diagram. (See Subsection …)
22. What is meant by 'nonsatiation' of an investor ? (See Subsection …)
23. State any two assumptions of Markowitz model. (See Section …)
24. What is the basic proposition of two-fund theorem ? (See Subsection …)
25. What is critical line method ? (See Subsection …)
26. What is meant by a corner portfolio ? (See Subsection …)
27. State the basic assumptions of two-fund theorem. (See Subsection …)
28. What would be the expected rate of return and the variance of the rate of return on a risk-free asset ?
(See Subsection …)
29. What would be the covariance of the prospects of a risk-free asset and a risky asset ?
(See Subsection 3.5.2)
30. What is a risk-free asset ? Give an example. (See Subsection 3.5.2)
31. State the expected rate of return on a portfolio that consists of both risky assets and a risk-free asset.
(See Subsection 3.5.2)
32. What would be the variance of the rate of return on a portfolio that consists of both risky assets and a risk-free asset ?
(See Subsection 3.5.2)
33. What is the main proposition of one-fund theorem ? (See Subsection 3.5.3)
or, State the one-fund theorem.
[C.U. B.Sc. (H), Sem-V, 2020]
34. In the risk-return plane, what would be the shape of the indifference curve for an investor (i) who is relatively more risk-averse, and (ii) who is relatively less averse to risk-taking.
35. State any one assumption of CAPM. (See Subsection 3.5.6)
36. State the security market line as shown in the CAPM. (See Subsection 3.6.1)
37. What is the implication of 'β' (beta) in the security market line as shown in CAPM ? (See Subsection 3.6.3)

38. How would a risk-averse individual minimise risk of portfolio return when there are n mutual funds that are (i) uncorrelated, (ii) positively correlated ?
[C.U. B.Sc. (H), Sem-V, 2020]
(See Subsection 3.4.3.3)

[Hint : When n mutual funds are uncorrelated (ρ_{ij} = 0) then the portfolio risk would be $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2$

or, $\sigma_p = \sqrt{\sigma_p^2} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2}$. In this case, the possible portfolios would lie on a curved line in the risk-return plane. So, given the portfolio rate of return, the investor can minimise portfolio risk by minimising the variance (σ_p^2) of the rate of return.

However, if the prospects of the n mutual funds are positively correlated (ρ_{ij} = + 1) then portfolio risk cannot be minimised through portfolio diversification.]

Long-answer type questions

1. (a) Explain the concepts of overall return on investment and rate of return on investment.
(b) An investor has purchased 100 shares of ICICI Bank @ ₹ 850 per share in 2000 and after 1 year he has sold those shares at a price of ₹ 920 per share. He has also received a dividend of ₹ 18 per share. Estimate (i) the overall return on investment, (ii) rate of return on investment, (iii) the dividend yield and capital gains yield, and show that the sum total of these yields would be equal to the rate of return on investment.
(See Section 3.2)

2. Explain the process of estimating
(a) the average rate of return on investment. (See Subsection 3.2.1)
(b) Consider the following rates of return on a stock and estimate the average compound return on investment :

Period (between t and t + 1)	1 & 2	2 & 3	3 & 4	4 & 5	5 & 6
Rate of return (r_t)	20%	33.73%	− 10.40%	7.0%	38.94%

(See Subsection 3.2.1)

3. Explain the process of estimating
(a) Return on a portfolio of assets. (See Subsection 3.2.2)
(b) Consider the following portfolio of assets and estimate the portfolio rate of return :

Asset	No. of stocks	Price of Stock (₹) (per unit)	Rate of return (%)
X	150	120	12.5%
Y	250	130	13.0%
Z	320	114	14.2%

(See Subsection 3.2.2)

4. Explain the process of estimating the return from short selling. (See Subsection 3.2.3)
5. Make a discrete probability distribution of the rate of returns on an asset on the basis of following information :

Possible values of returns (%)	10	15	20	25	30	35
Frequency of occurrence (f_i)	26	34	50	18	12	10
Probability of occurrence (p_i)	0.173	0.227	0.333	0.120	0.080	0.067

(See Section 3.3)

6. Explain the important properties of the expected value of a random variable. (See Subsection 3.3.1)

7. Estimate the expected rate of return on Stock-A and Stock-B on the basis of the following information :

State of the economy	Probability of occurrence of the state of economy	Rate of return on an asset under a given economic state	
		Stock-A	Stock-B
Boom	0·4	+ 45%	+ 20%
Recession	0·6	– 35%	+ 15%

(See Subsection 3.3.1)

8. Based on the information given in Q. No. 7, estimate the risk involved in the investment of Stock-A and Stock-B (See Subsection 3.3.2)

9. Explain the process of estimating the mutual dependence of the two or more random variables. (See Subsection 3.3.3)

10. (a) What is portfolio mean ?
(b) A portfolio consists of shares of 5 companies with a mix of 1 : 2 : 3 : 4 : 5. Now, estimate the portfolio mean on the basis of the following information :

Stocks	Rate of return (%) under		
	Good economic environment	Average economic environment	Bad economic environment
1	20	18	14
2	30	28	18
3	28	27	– 10
4	14	16	12
5	38	30	20

(See Subsection 3.4.1)

11. Estimate the portfolio risk on the basis of the following information :

State of the economy :	Good	Average	Bad
Probability of occurrence of the state of the economy :	25%	50%	25%
Rate of return from portfolio (%) :	32·35%	28·25%	15·75%

(See Subsection 3.4.2)

12. Calculate the portfolio mean and variance on the basis of the following information :

Expected return from Stock-1	15%
Expected return from Stock-2	12%
Standard deviation of Stock-1	7%
Standard deviation of Stock-2	15%
Weightage of Stock-1 in portfolio :	40%
Weightage of Stock-2 in portfolio :	60%
Covariance of Stock-1 & 2	1%

(See Subsection 3.4.2)

13. Distinguish between systematic and unsystematic risks. (See Subsection 3.4.3)

14. Explain how and to what extent the portfolio risk can be minimised through diversification. (See Subsection 3.4.4)

15. "Portfolio risk cannot be brought down to zero through diversification if the rates of returns on stocks are correlated" – Explain. (See subsection 3.4.4)

16. Estimate portfolio risk and return of a two-asset portfolio on the basis of the following information for each portfolio :

	Different proportions of two stocks						
Portfolio :	A	B	C	D	E	F	G
Weightage of Stock - 1 :	100	80	60	50	40	20	0
Weightage of Stock - 2 :	0	20	40	50	60	80	100

Assumptions : Rate of return on Stock - 1 = 15%
Rate of return on Stock - 2 = 30%
S.D. of rate of return of Stock - 1 = 10%
S.D. of rate of return of Stock - 2 = 20%
Correlation coefficient of the prospects of Stock -1 & 2 = + 1

(See Subsection 3.4.5)

17. Consider the Q. No. 16 with one extra portfolio mix (B*) where weightage of Stock - 1 is 66·6% and weightage of Stock-2 is 33·3%. It is also assumed that the correlation coefficient of the prospects of Stock -1 and 2 is (–) 1. Now calculate the portfolio risk and return for each portfolio based on all other information given in Q. No. 16. (See Subsection 3.4.5)

18. Consider Q. No. 16 and assume that the prospects of Stock -1 and 2 are not correlated. Now, calculate the risk-return mix for each portfolio and plot them on a graph. (See Subsection 3.4.5)

19. (a) What is a feasible set of portfolio ?
(b) Explain the properties of a feasible set of portfolios. (See Subsections 3.4.6 - 3.4.7)

20. (a) Explain the meaning of an efficient portfolio.
(b) Show the minimum variance portfolio with the help of a diagram. (See Subsection 3.4.8)

21. Discuss basic proposition of Markowitz model and the assumptions of this model. (See Section 3.5)

22. What is a minimum variance portfolio as suggested by the Markotiz model ? Explain the process of finding out a minimum variance portfolio. (See Section 3.5)

23. Let us consider three uncorrelated assets with variance of the rates of return $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$. The expected rate of return for these three assets are $\bar{r}_1 = 1$, $\bar{r}_2 = 2$ and $\bar{r}_3 = 3$ respectively. Estimate the values of weights of these assets for an efficient portfolio. Also estimate the minimum variance for a given value of portfolio return of $\bar{r}_P = 2$. (See Section 3.5)

24. (a) State the basic proposition of two-fund theorem.
(b) Conside the following vector of expected rate of return on three stocks (Stock -1, 2 and 3) and the variance - covariance matrix (VC) of these stocks :

$$E(\bar{r}) = \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \end{bmatrix} = \begin{bmatrix} 16\cdot2 \\ 24\cdot6 \\ 22\cdot8 \end{bmatrix};$$

$$VC(\sigma) = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix} = \begin{bmatrix} 146 & 187 & 145 \\ 187 & 854 & 104 \\ 145 & 104 & 289 \end{bmatrix}$$

(i) Prepare corner portfolios A and B with the following weight vectors :

$$w(A) = \begin{bmatrix} \cdot 00 \\ 1\cdot 00 \\ 0\cdot 00 \end{bmatrix}, \; w(B) = \begin{bmatrix} \cdot 00 \\ \cdot 22 \\ \cdot 78 \end{bmatrix}$$

(ii) Also prepare another portfolio (AB) with the combination of those two corner portfolios taking 50% from w(A) and 50% from w(B).
(See Subsection 3.5.)

25. (i) What is a risk-free asset ? What will be the variance of the rate of return on such an asset ?

(ii) 'Government bonds may not always be considered as risk-free assets' – Do you agree with the statement ? Give reasons.
(See Subsection 3.5.)

26. Explain the nature of expected rate of return and the variance of the rate of return on a portfolio that consists of both risky assets and risk-free asset.
(See Subsection 3.5.)

27. An investor prepares a portfolio with one risky asset and one risk-free asset where the risk-free rate of return is 4% and the expected rate of return on risky asset is 15·5%, and the variance of the rate of return on risky asset is 144. The weightages of these two assets in the portfolio are as follows :

Portfolio :	A	B	C	D	E
Weightage of risk-free asset :	1·0	0·75	0·5	0·25	0
Weightage of risky asset :	0	0·25	0·5	0·75	1·0

Estimate the corresponding risk-return profiles for these portfolios. *(See Subsection 3.5.)*

28. Explain the process where the investor wants to combine a risk-free asset with the efficient portfolios in the 'Markowitz feasible set' with the assumption that risk-free lending is possible but not risk-free borrowing.
(See Subsection 3.5.)

29. (a) Based on the following information estimate the portfolio risk and return for a risky portfolio assuming 80% of Stock-1 and 20% of Stock - 2.

Risky stocks	Expected rate of return	Variance of the rate of return
Stock - 1	15·5	184
Stock - 2	25·2	265

(b) Also estimate the portfolio risk and return for a portfolio formed out of a combination of the risky portfolio and a risk-free asset with the following weights :

	Weightage	
Risky portfolio :	0·25	0·50
Risk-free asset :	0·75	0·50

(See Subsection 3.5.3)

30. Explain the impact upon the efficient portfolio in Markowitz model when risky portfolios can be combined with the risk-free asset assuming only risk-free lending. *(See Subsection 3.5.4)*

31. Explain the process of deriving optimal portfolio on the basis of one-fund theorem. *(See Subsection 3.5.5)*

32. Consider the following utility function of an investor : $U = f(\bar{r}_P, \sigma_p)$ where $\bar{r}_P$ = Expected rate of return on a portfolio, and σ_p= Standard deviation of the rates of return on a portfolio.
Now, given the efficient portfolio frontier in Markowitz sense, find out the optimal portfolio for such an investor who is assumed to be risk-averter. *(See Subsection 3.5.6)*

33. An investor is allowed to lend and borrow at risk-free rate. Now, using the utility function of this investor in the risk-return plane, the efficient portfolio frontier in Markowitz sense and the capital allocation line

$$\bar{r}_P = r_f + \frac{(\bar{r}_s - r_f)}{\sigma_s} \cdot \sigma_p$$

[where r_f = risk-free rate of return,
$\bar{r}_P$ = expected rate of return on portfolio,
$\bar{r}_s$ = expected rate of return on risky asset,
σ_s = Standard deviation of the rate of return on risky asset,
σ_p = Standard deviation of the rate of return on portfolio]

find out the optimal portfolio. *(See Subsection 3.5.7)*

34. Consider an investor who invests both in risky and risk-free assets, and his utility function is

$$U = \bar{r}_P - 0\cdot 005 A\sigma_p^2$$

Where A = coefficient of risk aversion,
$\bar{r}_P$ = expected rate of return on risky portfolio, σ_p^2 = Variance of the rates of return on risky portfolio.

Let the risk-free rate of return be 8%, and the optimal portfolio corresponds to $\bar{r}_P$ = 21.84% and σ_p = 19·83% (as determined by the tangency point of capital market line and efficient frontier of risky portfolios).
Now, determine the optimal portfolio that maximises the utility of this investor when (i) the investor is highly averse to risk with A = 5, and (ii) the investor has a comparatively low aversion to risk with A = 3.
(See Subsection 3.5.7)

35. (a) State the basic assumptions of CAPM.

(b) Explain the process of deriving the security market line as suggested in CAPM.
(See Subsections 3.6.1, and 3.6.5)

36. Give a short note on capital market line. *(See Subsection 3.6.2)*

37. Explain the relationship between capital market line and security market line. *(See Subsection 3.6.4)*

38. Give a short note on portfolio beta (β). *(See Subsection 3.6.6)*

39. Explain the concepts of systematic risks and non-systematic risks on the basis of the security market line as suggested by the CAPM. *(See Subsection 3.6.5)*

40. If the market portfolio's expected rate of return is 14% with a standard deviation of 25% then estimate the expected rate of return on the portfolio of an investor where the risk-free rate of return is 7% and the standard deviation of the rate of return of *i*-th security is 20%. *(See Subsection 3.6.6)*

41. An investor manages a portfolio of 4 stocks with their following market values and *beta* (β) :

Stock	Market Value (₹)	Beta (β)
1	2,50,000	1·17
2	1,50,000	1·25
3	80,000	0·75
4	60,000	0·50

Let the risk-free rate of return be 4%, and the market rate of return be 12%. Calculate the expected rate of return on this portfolio based on CAPM. *(See Subsection 3.6.6)*

42. If the following assets are correctly priced on the security market line then (i) estimate the expected rate of return on market portfolio, and (ii) the risk-free rate of return.
Here, for Stock -1 : $\bar{r}_1$ = 9·50%, for Stock - 2 : $\bar{r}_2$ = 14·8% ; and β_1 = 0·6, β_2 = 1·2. *(See Subsection 3.6.6)*

[illegible]. Mr. Basu is planning to purchase the stock of ITC Ltd. He expects that the share of ITC would earn a return of [illegible]% in the next year. If the risk-free rate of return is [illegible]%, beta of ITC's share is [illegible], and the expected rate of return on market portfolio is [illegible]% then should Mr. Basu invest in ITC's stock? Explain.

(See Subsection [illegible])

[illegible]. The beta (β) of Stock-*x* is [illegible], the expected rate of return on market portfolio is [illegible]% and the risk-free rate of return is [illegible]%.

(i) Estimate the expected rate of return on Stock-*x* based on CAPM.

(ii) If the risk premium on market portfolio goes up by [illegible]% then what would be the revised expected rate of return on Stock-*x*?

(See Subsection [illegible])

[illegible]. The Treasury Bills give a rate of return of [illegible]% and the expected rate of return on market portfolio is [illegible]%.

(i) Estimate the risk premium over market rate of return.

(ii) Also calculate the beta (β) value and required rates of return for the following portfolio mix:

Asset	Weightage				
Treasury Bills :	100	80	70	30	0
Risky assets :	0	20	30	70	100

(See Subsection [illegible])

[illegible]. Discuss how CAPM can be used for investment decision-making. (See Subsection [illegible])

[illegible]. How can CAPM be used to evaluate the performance of a portfolio? Explain. (See Subsection [illegible])

[illegible]. Write short notes on :

(a) Jensen Index, and

(b) Sharpe index.

(See Subsection [illegible])

[illegible]. How can an investor use CAPM to estimate the price of a security? Explain. (See Subsection [illegible])

[illegible]. An investor has the following information regarding the expected value of a security after 1 year, the risk-free rate of return, the expected rate of return on market portfolio and the beta (β) value of the security.

P_1	r_f	r_M	β
₹ [illegible]	[illegible]%	[illegible]%	[illegible]

(i) Estimate the present price of that security based on CAPM pricing formula.

(ii) If beta value rises to [illegible] then other things remaining same, what would be its impact on the present price of the security?

(See Subsection [illegible])

[illegible]. Consider the following information for two assets :

Asset	$\bar{r}$	σ	σ_{AB}	w_i
A B	[illegible]% [illegible]%	[illegible]% [illegible]%	[illegible]	[illegible] [illegible]

(w_i = weight of *i*-th asset)

(i) Calculate the mean and variance of the portfolio. (Ans. $\bar{r}_p$ = [illegible]%, σ^2_p = [illegible])

(ii) Show the feasible set of two assets in a diagram.

[C.U., B.Sc(H), Sem-V, [illegible]]
(See Subsection [illegible])

[illegible]. State and prove the portfolio diagram lemma.

[C.U., B.Sc(H), Sem-V, [illegible]]
(See Subsection [illegible])

[illegible]. Assume that the expected rate of return on the market portfolio is [illegible]% and the risk-free return is [illegible]%. The standard deviation of the market portfolio return is [illegible]%. Assuming that the market portfolio is efficient,

(a) Derive the equation of the capital market line. Interpret the slope of the line.

(b) What will be the standard deviation of the position if an expected return of [illegible]% is desired?

(c) If anyone invests ₹ [illegible] in the risk-free asset and ₹ [illegible] in the market portfolio, how much money should that person expect to have at the end of the year?

(d) Consider an asset with expected pay-off ₹ [illegible] and covariance of [illegible] with the market. Determine the current value of the asset.

[C.U., B.Sc(H), Sem-V, 2020]
(See Subsection [illegible])

[illegible]. "The CAPM is derived directly from the condition that the market portfolio is a point on the edge of the feasible region that is tangent to the capital market line." — Discuss. [C.U., B.Sc(H), Sem-V, 2020]
(See Subsection [illegible])

[illegible]. Show that the points on the efficient frontier can be characterised by an optimisation problem formulated by Markowitz.
(See Section [illegible] and Subsection [illegible])

4

Derivatives and Options

4.1. Introduction

The objective of an investment decision is to get required rate of return with minimum risk. To achieve this objective, various instruments, practices and strategies have been devised and developed in the recent past. With the opening of boundaries for international trade and business, the world trade gained momentum in the last decade, the world has entered into a new phase of global integration and liberalisation. The integration of capital markets world-wide has given rise to increased financial risk with the frequent changes in the interest rates, currency exchange rate and stock prices. To overcome the risk arising out of these fluctuating variables and increased dependence of capital markets of one set of countries to the others, risk management practices have also been reshaped by inventing such instruments as can mitigate the risk element. These new popular instruments are known as financial derivatives which, not only reduce financial risk but also open us new opportunity for high risk takers. The current chapter covers the topics like derivatives, forward and futures contracts, options and other type of derivatives, the use of futures for hedging and various hedging strategies, option markets, call and put options, factors affecting option prices, put-call parity theorems, option trading strategies, e.g., spreads, straddles, strips and straps, strangles, the principle of arbitrage, binomial model and risk-neutral valuation.

After going through this chapter the reader will be able to :

(a) Understand meaning and evolution of derivatives ;

(b) Describe the features and types of financial derivatives ;

(c) Understand uses and functions of derivative securities ;

(d) Distinguish between futures and forward contracts ;

(e) Conceptualise Option and Pricing mechanism of Option ;

(f) Various trading strategies for risk mitigation.

4.2. Derivatives : Basic Concept

In this era of globalisation, we are witnessing innovations in Financial Engineering and Financial Economics, which result in the evolution of a new set of products in the banking and financial sector named 'Derivative'. The growth of these products in the last 25 years has been one of the extraordinary and important features of the financial market place and derivatives have an emphatic role in the economy.

4.2.1. Meaning of Derivatives

Before explaining the term 'financial derivative', let us see the dictionary meaning of 'derivative'. It means :

(a) A word formed by derivation ;

(b) Something derived e.g., value of an asset is derived from the present value of it's future benefits ;

(c) The limit of the ratio of the change is a function to the corresponding change in its independent variable (e.g., dy / dx) ;

(d) A substance that can be made or derived from other substances in one or more steps.

"The word Derivative originates from Mathematics and refers to a variable which has been derived from another variable. Derivatives are so called because they have no value of their own". Derivatives are financial instruments whose value is derived from the value of something else. They derive their value from the value of some other assets such as commodities, bonds, equities, currencies, etc. which is known as the underlying, and are used to either hedge those assets or improve the returns on those assets.

Hence a financial derivative means a contract or an agreement for exchange of payments, whose value derives from the value of an underlying asset or underlying reference rates or indices. Derivative is a financial product which has been derived from another financial asset. Financial asset can be any financial product, interest rate, commodity, foreign exchange, security, index, currency or any other financial products. Without the underlying product or market, the derivative would have no independent existence.

As per L.C. Gupta Committee, 'Derivative' means forward, futures or options contract of predetermined fixed duration, linked for the purpose of contract fulfilment to the value of specified real or financial asset or to an index security.

4.2.2. History of Derivatives Trading

Towards the end of the Second World War, representatives of 44 nations gathered in 1944 in Bretton Woods town in New Hampshire, USA and agreed on a fixed exchange rate system which lasted till the early 1970s. Under that system, the exchange rates of all currencies were fixed against the US dollar. As the US dollar was then convertible to gold at $35 per ounce, all currencies were indirectly fixed in terms of gold. In 1973, the Bretton Woods agreement, the pact that instituted a fixed exchange rate regime for the world's major nations, effectively collapsed when the US suspended the dollar's convertibility into gold.

In 1972 the Chicago Mercantile Exchange (CME) launched the world's first exchange-traded currency futures. In 1975 they introduced interest rate futures and in 1982 the innovative stock index future. In India, the future market is started and regulated for castor and black pepper.

Following are some example of using derivative instruments in the early age :

(a) **Japanese rice traders :** In 1730, Japanese merchants petitioned shogun Tokugawa Yoshimune to officially authorise trade in rice futures at the Dojima Exchange, the world's first organised (but unsanctioned) futures market.

(b) **Venetian spice traders :** In the year 1173 a bankrupt Venetian merchant by the name of Romano Mairano went looking for a way out of financial ruin. Over a trading career spanning several decades, Mairano had decided to orchestrate a risky trade that could help him pay off his loans and restore his wealth, a trade for one of the most valuable commodities of the day: pepper.

(c) **American ranchers :** The historic American ranchers of the late 19th century arose from the vaquero traditions of northern Mexico and became a figure of special significance and legend. They tend the horses used to work cattle. In addition to ranch work, some of them work for or participate in rodeos.

Followings are some derivative market in the globe :

(i) Chicago Mercantile Exchange(CME) : exchange-traded currency futures in 1973 ;

(b) Chicago Mercantile Exchange interest rate futures in 1975 ;

(c) Philadelphia Stock Exchange currency options in 1982 ;

(d) New York Futures Exchange (NYFE) : 1980 ;

(e) London International Financial Futures Exchange (LIFFE) : 1982 ;

(f) Singapore Monetary Exchange (SIMEX) : 1983.

4.2.3. Characteristics of Derivatives

The important characteristics of derivatives are as follows :

(a) Derivatives possess a combination of novel characteristics not found in any other forms of assets.

(b) Derivatives are traded globally having strong popularity in financial markets.

(c) There is strong relation between the values of derivatives and their underlying assets. Without the underlying assets, derivative does not have any existence.

(d) As all transactions in all major derivatives take place in future specific dates, it is easier to short position in derivatives than any other assets.

(e) Derivatives traded on exchanges are liquid and involves the lowest transaction costs due to high volume of trade and competition. Margin requirement in the exchange traded derivatives is relatively low, which reflects that the risk associated with this instrument is low.

(f) It is possible to construct portfolio which is exactly needed, without having the underlying assets. For example, suppose a firm with a floating rate loan needs to limit its exposure to sharp increases in the interest rate. The firm can purchase a derivative called an interest rate cap (explained later). This derivative pays the firm the difference between the floating rate of interest and a predetermined maximum called the cap rate whenever the floating rate exceeds the cap.

(g) An important feature in the evolution of derivatives has been the evolution of Over-the-Counter (OTC) market. "Financial engineers" using off-the-shelf futures and options products to satisfy special needs can develop custom-made solutions. Such products with unique risk / reward profiles are called hybrids.

(h) *Major market participants of derivative market: Arbitrageurs, Traders, Hedgers and Speculators.*

- *Arbitrageurs seek to earn risk free profits by taking advantage of differences in interest and currency rates and transfer funds from one country to another.*
- *Traders cover the risk of loss on export or import transactions denominated in foreign currencies.*
- *Hedgers, mostly firms, engage in foreign contracts to protect the home currency value of various foreign exchange denominated transactions.*
- *Speculators actively expose themselves to currency risk in order to profit from exchange rate fluctuation.*

4.3. Types of Derivatives

Due to complexity in nature, it is very difficult to classify the financial derivatives, so in the present context, the basic financial derivatives which are popular in the market have been described in brief. The details of their operations, mechanism and trading, will be discussed in the forthcoming sub-sections of this chapter. In simple form, the derivatives can be classified into different categories which are shown in the Fig.-4.1.

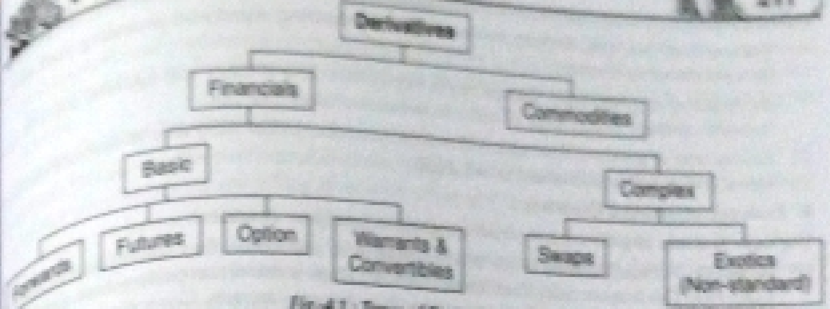


Fig.-4.1 : Types of Derivatives

4.3.1. Forward Contracts

A forward contract is a transaction in which buyer/seller agree upon delivery of a specified quality and quantity of asset at a future specified date. A price may be agreed upon in advance or at the time of delivery. For example, in the oil industry, entering into a forward contract to sell a specific number of barrels of oil can be used to protect against potential downward swings in oil prices.

A forward contract is a simple customised contract between two parties to buy or sell an asset at a certain time in the future for a certain price. Unlike future contracts, they are not traded on an exchange, rather traded in the over-the-counter market, usually between two financial institutions or between a financial institution and one of its clients. In brief, a forward contract is an agreement between the counter parties to buy or sell a specified quantity of an asset at a specified price, with 17 delivery at a specified time (future) and place. These contracts are not standardised, each one is usually customised to its owner's specifications.

Forward contracts exist for a variety of underlying assets like as follows :

(a) Currency (forward Foreign Exchange transactions) ;

(b) Metals (contracts for base metals on LME) ;

(c) Energy Products (crude oil and oil products) ;

(d) Interest Rates (Forward Rate Agreements - FRAs) (Explained in details later on).

A company make forward contract for the foreign currencies which are receivable or payable in a future date. Examples of forward contract can be as follows :

(i) ABC Factory in Edinburgh is looking to buy motorbikes from Taiwan. The business meets with the supplier, and agrees to pay USD $ 500,000 in 3 months from now. The current GBP/USD exchange rate at the time of the deal is GBP £ 1·00 = USD $ 1·32. ABC Factory therefore expects to pay GBP £ 378,788 for the equipment. (GBP : Great Britain Pound = £, USD = US Dollar = $) In 3 months' time, GBP £ 1·00 = USD $ 1·25. Here is what could happen :

Scenario A : If ABC Factory doesn't use a Forward contract

In 3 months' time, when the business is ready to pay for the goods from Taiwan, the exchange rate has moved adversely for ABC Factory, GBP £ 1·00 = USD $ 1·25. This means that the goods would cost £ 400,000. ABC Factory would pay £ 21,212 more than anticipated originally.

Scenario B : ABC Factory does use a Forward contract

After 3 months, ABC Factory is ready to purchase the equipment from Taiwan. The exchange rate has moved adversely, however, as GBP £ 1·00 = USD $ 1·25, ABC Factory negotiated a forward contract with a currency provider.

The result is that ABC Factory saves £ 21,212 by thinking ahead and protecting itself with forward currency contract.

An importer of goods from Europe needs to pay Euro after 90 days, and can enter into a forward contract to buy Euro in order to reduce exchange rate risk.

(2) An exporter of goods to Australia will receive Australian Dollar AUD after 30 days and can enter into a 30-day forward contract to sell AUD.

■ Features of Forward Contracts :

The basic features of a forward contract are given in brief here as under :

(a) **Bilateral :** Forward contracts are bilateral contracts, and hence, they are exposed to counter party risk. This is more risky than futures contract. There is risk of non-performance of obligation by either of the parties, so these are riskier than futures contracts. Generally here bank is counter party. This is not traded in secondary market and there is no single location for trading. It is dealt by telephone / telex. No collateral or margin is usually required. Forward contracts are normally available for 1, 2, 3, 6 or 12 month delivery. Banks also tailor forward contracts for odd maturities : when exact time of delivery not known. Difference between buy and sell rate of currency is known as Spread. Bid-ask (buy-sell) spreads in forwards tend to be lower for shorter maturities and widely traded currencies.

(b) **Customised contracts :** Each contract is custom designed or tailor made contract and hence, is unique in terms of contract size, expiration date, the asset type, quality, etc.

(c) **Long and short positions :** In forward contract, one of the parties takes a long position by agreeing to buy the asset at a certain specified future date. The other party assumes a short position by agreeing to sell the same asset at the same date for the same specified price. A party with no obligation offsetting the forward contract is said to have an open position. A party with a closed position is, sometimes, called a hedger.

(d) **Delivery price :** The specified price in a forward contract is referred to as the delivery price. The forward price for a particular forward contract at a particular time is the delivery price that would apply if the contract were entered into at that time. It is important to differentiate between the forward price and the delivery price. Both are equal at the time the contract is entered into. However, as time passes, the forward price is likely to change whereas the delivery price remains the same.

(e) **Synthetic assets :** In the forward contract, derivative assets can often be contracted from the combination of underlying assets, such assets are known as synthetic assets in the forward market. The forward contract has to be settled by delivery of the asset on expiration date. In case the party wishes to reverse the contract, it has to compulsorily go to the same counter party, which may dominate and command the price it wants as being in a monopoly situation.

(f) **Pricing of arbitrage based forward prices :** In the forward contract, covered parity or cost-of-carry relations are relation between the prices of forward and underlying assets. Such relations further assist in determining the arbitrage-based forward asset prices.

(g) **Popular in forex (Foreign Exchange) market :** Forward contracts are very popular in foreign exchange market as well as interest rate bearing instruments. Most of the large and international banks quote the forward rate through their 'forward desk' lying within their foreign exchange trading room. Forward foreign exchange quotes by these banks are displayed with the spot rates.

(h) **Different types of forward :** As per the Indian Forward Contract Act 1952, different kinds of forward contracts can be done like hedge contracts, transferable specific delivery (TSD) contracts and non-transferable specific delivery (NTSD) contracts. Hedge contracts are freely transferable and do not specify, any particular lot, consignment or variety for delivery. Transferable specific delivery contracts are though freely transferable from one party to another, but are concerned with a specific and predetermined consignment. Delivery is mandatory. Non-transferable specific delivery contracts, as the name indicates, are not transferable at all, and as such, they are highly specific.

■ Forward Contract Terminologies :

(1) **Spot Rate :** This refers to the purchase of the underlying asset for immediate delivery. In other words, it is quoted price by buying and selling of an asset at the spot or immediate delivery.

(2) **Forward Rate :** In forward contract, an agreement is entered into between buyer and seller today to exchange the commodity or instrument for cash at a predetermined future date at a price agreed upon today. Such predetermined price or rate is Forward Rate.

(3) **Forward Premium :** It occurs when forward rate is higher than spot rate.

(4) **Forward Discount :** It occurs when forward rate is lower than spot rate.

(5) **Forward Cover** $= \dfrac{\text{Forward rate - Spot rate}}{\text{Spot rate}} \times [12 / \text{Forward contract in months}]$

If forward rate is higher than spot rate, its premium and if forward rate is lesser than spot rate, its discount.

(6) **Long Position :** The party who agrees to buy in future is said to hold long position.

(7) **Short Position :** The party who agrees to sell in future is said to hold short position.

■ Merits and Demerits of Forward Contract :

Forward contract is a non-standardised contract between two parties to buy or sell an asset at a specified future at an agreed price. **The merits of forward contracts are as follows :**

(1) They can be matched against the time period of exposure as well as for the cash size of the exposure.

(2) Forwards are tailor made and can be written for any amount and term.

(3) It offers a complete hedge.

(4) Forwards are over-the-counter products.

(5) The use of forwards provides price protection.

(6) They are easy to understand.

(7) Allows the business to lock in an exchange rate for a trade that will occur at a future pre-agreed date.

(8) It will choose a rate which suits the business that will allow you to buy and sell in the future at a known rate.

(9) Manage and budget cash flow without worrying about Foreign Exchange volatility. Forward exchange contracts can be used as hedging mechanisms for a business.

The Demerits of Forward Contracts are as follows :

(1) It requires tying up capital. There are no intermediate cash flows before settlement.

(2) It is subject to default risk.

(3) Contracts may be difficult to cancel.

(4) There may be difficult to find counter-party.

(5) High Risk. If the rate moves unfavourably in the future, a forward contract could be loss making. There is a contractual obligation to fulfill a forward exchange rate contract.

(6) A deposit is often required on the commencement of the transaction.

(7) The forward rate that is quoted is often given as a premium to the spot rate. The evaluation of a forward contract is also based on demand.

■ Forward Price :

The forward price refers to the predetermined price of an underlying asset which is agreed upon in a forward contract.

The forward price formula (which assumes zero dividends) is expressed as :

$$F = S_0 \times e^{rT}$$

Where F = The forward price of a contract

S_0 = The spot price of the underlying asset

r = The risk-free rate of return which applies to the life of the forward contract.

T = The delivery date in years.

Example :

Mr. Saha is intended to make a forward contract for an underlying asset which is currently traded at a price of ₹ 1,000. The risk-free rate of return in the market is given as 4% p.a. The forward price of the asset will be :

$$F = ₹\ 1{,}000 \times e^{(0.04 \times 1)}$$

$$= ₹\ 1040.81.$$

If there arises a carrying cost (viz., the cost of holding the asset till the futures contract matures) then the forward price formula will be : $F = S_0 \times e^{(r+q)T}$

where q = *carrying costs.*

Similarly, if the underlying asset generates dividend then the forward price formula is given as :

$$F = (S_0 - D) \times e^{rT}$$

where D = *the present value of the flow of dividends.*

Illustration 4.1

An Indian Importer has purchased capital goods worth $ 6,50,000 from US which is payable in 3 months' time. The Importer expects that Rupee will weaken over a period against Dollar. He has asked his banker for forward exchange cover. The rates existing at that time are :

(a) *Spot 1 US $: ₹ 70·36*

(b) *Forward Premium for 3 months : ₹ 0·37*

Solution :

The Rupee exposure to US $ purchased in forward market for delivery in 3 months as follows :

= $ 6, 50,000 × (₹ 70·36 + 0·37) = ₹ 4, 59,74,500

That means if Indian Rupee will weaken further i.e. 1 US $ will be more than ₹ (70·36 + ·37) = ₹ 70·73 after 3 months, Indian Importer will have to accept the loss due to change in foreign exchange rate. As he has made a forward contract, his dollar price is fixed irrespective of current market price after 3 months.

Illustration 4.2

Greenwood Ltd. imports a sophisticated machine from US on 1st April, 2019 and has to pay $ 8,00,000 after 3 months, on 1st July, 2019. The current spot rate of exchange is $ 1 = ₹ 69·89. It is expected that the exchange rate prevails at ₹ 70·54. In order to protect from foreign exchange rate fluctuation, the company arranges for a forward exchange contract with his banker undertaking to by $ 8,00,000 at a fix rate of ₹ 70. If the spot rate prevails at ₹ 70·89 on 1st July 2019, what would be the cash flow of the company ?

Solution :

(i) If forward cover is taken

Amount payable to the bank = $ 8,00,000 × ₹ 70 = ₹ 5,60,00,000

Saving in Cash flow = $ 8,00,000 × (₹ 70·89 – ₹ 70) = ₹ 7,12,000

(ii) Amount payable if exposure is not covered = $ 8,00,000 × ₹ 70·89 = ₹ 5,67,12,000

(iii) If the spot rate prevails at ₹ 69·89, the cash flow would be as follows :

Amount payable to the bank = $ 8,00,000 × ₹ 69·89 = ₹ 5,59,12,000

Extra cost incurred due to forward cover

= $ 8,00,000 × (₹ 70 – ₹ 69·89) = ₹ 88,000.

4.3.2. Future Contracts

Futures are a form of Forward Contract in which one agrees to take delivery at an agreed price, quantity and time in the future in a specific market. Future contracts differ from Forward Contracts by the fact that they are traded on a recognised public exchange. In future contract guarantee will be given by the clearing house, whereas in forward it will be the counter party risk.

Hedging in interest rates, currency rates and share prices by taking a position that is equal and opposite of an existing exposure.

A futures contract obligates the buyer to purchase the underlying contract and the seller to sell it, unless the contract is sold to another before settlement date, which may happen in order to take a profit or limit a loss. In practice, only a very small percentage of futures contracts result in delivery of the underlying commodity or security.

■ **Features of Future Contract :**

Financial futures, like commodity futures are contracts to buy or sell financial aspects at a future date at a specified price. The following features are there for future contracts :

(a) Future contracts are traded on organised future exchanges. These are forward contracts traded on organised futures exchanges.

(b) Future contracts are standardised contracts in terms of quantity, quality and amount.

(c) Margin money is required to be deposited by the buyers or sellers in form of cash or securities. This practice ensures honor of the deal. Margins are deposits which hedgers and speculators offer as collateral for their futures position. As the value of a position may change daily, the margin is adjusted to ensure adequate collateral. The initial margin is based upon the value of the position and its inherent risk as measured by its volatility.

(d) In case of future contracts, there is a daily of opening and closing of position, known as marked to market. The price differences every day are settled through the exchange clearing house. The clearing house pays to the buyer if the price of a futures contract increases on a particular day and similarly seller pays the money to the clearing house. The reverse may happen in case of decrease in price.

(e) Period of contract normally trade in a cycle of four times annually – say four delivery dates in a year. (e.g. 2nd Wednesday of March, June, September and December on the LIFFE)

(f) It is an exchange traded instruments. Hence credit worthiness of the exchange is important. Settlement is done through clearing house.

(g) Credit worthiness of the exchange is maintained by imposition of margins - 'marked to market' on daily basis.

Table - 4.1

Difference between Future and Forward Contract

Basis	Future Contract	Forward Contract
Location	Futures Exchange	No single location
Trading medium	Open Outcry	Telephone/ telex
Contract size	Standardised	As reqd. by customer
Maturity/ Deliver Date	Standardised	As reqd. by customer
Counterparty	Clearing House	Known bank/ trader
Credit Risk	Clearing House	Individual counterparty
Commissions	Always Payable	Negotiable
Security	Margin required	Counterparty risk
Liquidity	Provided by margins	Provided by credit risk
Leverage	Very high	No formal gearing
Settlement	Via Clearing House	Via bank arrangements

■ **Future Contract Terminologies :**

A futures market is an auction market in which participants buy and sell commodity and futures contracts for delivery on a specified future date. Futures are exchange-traded derivatives contracts that lock in future delivery of a commodity or security at a price set today.

Examples of futures markets are the New York Mercantile Exchange (NYMEX), the Kansas City Board of Trade, the Chicago Mercantile Exchange (CME), the Chicago Board of Trade (CBoT), Chicago Board Options Exchange (CBOE) and the Minneapolis Grain Exchange.

Originally, such trading was carried on through open outcry and the use of hand signals in trading pits, located in financial hubs such as New York, Chicago, and London. Throughout the 21st century, like most other markets, futures exchanges have become mostly electronic.

■ ***Difference between a forward contract and a futures contract :***

Both forward and futures contracts are contracts to buy or sell a specified asset at a specified future time at a specified price determined when the contract is entered into.

Forward contracts are custom-made according to the needs of the two parties to the contract, and are used mainly to hedge the exposure that the party has. These are private contracts entered into over-the-counter markets, and are non-negotiable. The forward contracts can lead to counterparty risk, where one of the parties may not fulfill their obligations under the contract.

Futures contracts are standardised and are traded in futures exchanges according to the rules and regulations of the exchange. These are negotiable contracts and are used by hedgers, arbitrageurs and speculators. Since they are traded on exchanges, counterparty risk is eliminated.

• **In Future market trading includes following terminologies :**

Market order : This order has to be executed immediately at the best possible rate after it reaches the trading floor. Ex-Buy 1 Dec 2012 1000 corn — i.e. 1000 bushels of corn are to be bought immediately at the market rate of the current rate.

Market-if-touched (MIT) : Ex : Sell 1 Dec 2012 1000 corn 22·00 MIT

Time Orders :

Day orders : Order expires at the close of the market

Good till Cancelled (GTC) : This contract remains in effect until executed or cancelled by the customer

Good this week (GTW) : valid till last day of trading this week

Good this month (GTM) : valid till last day of trading this month

Good through date (GTD) : last trading hour of the date

Market on Close (MOC) : Ex : BUY 1 Dec. 2012 1,000 corn MOC– i.e. 1000 bushels of corn are to be purchased at the closing time in the market irrespective of the price.

Stop-loss order : Ex : Buy 1 Dec 2012 1000 corn 21·50 Stop– i.e. 1000 bushels of corn are to be bought at any price below 21·5.

Discretionary order : Buy 1 Dec. 2012 1000 corn 21·50 with 1 point disc– i.e. 1,000 bushels of corn are to be bought at any price at 21·5 but the broker can pay 1 point more if he feels that the order will otherwise not be fulfilled as the prices are racing first.

Not held order : Buy 1 Dec 2012 1000 corn 21·50 not held – i.e. the broker can execute the order to buy at 21·5 or wait if he feels that the price may fall further.

Spread order : Spread Buy 1 Dec 2012 1000 corn & sell 1 Dec. 2012 1,000 corn, 1 premium – i.e. 1,000 bushels of corn are to be bought & sold at a price differential to yield at a minimum of 1 of profit between the purchase & sell price.

Basis : The difference between cash price and future price. When future price is below cash price.

Convenience yield : When there is shortage in a commodity, there is an implied yield by holding the commodity.

Contango : Basis is negative, future price > cash price (home currency depreciates or vice versa)

Backwardation : Basis is positive, future price < cash price.

Margin : Since the clearing member is not obliged to take any position in a contract, but simply clears them, he or she will collect margins from brokers and traders. Each broker will collect the margin from the trader and maintain their margin account.

Margin Account : This margin account will be marked-to-market. Marking-to-market means that the margin account of the traders is adjusted every day using the daily settlement price. Profit or loss has to be determined on every day basis. If marking-to-market results in gain for the trader, the margin balance will increase by the amount of gain. If it results in a loss for the trader, the margin balance will decrease by the amount of loss. By marking-to-market, the contract can be considered closed out every day; at the beginning of the next day, a new contract is effectively entered into.

■ **Motives behind using future :**

Initially futures were devised as instruments to fight against the risk of future price movements and volatility. Apart from the various features of different futures contracts and trading, futures markets play a significant role in managing the financial risk of the corporate business world. The important

motives behind using futures contract are described as follows :

Price Discovery : The Future Market is not primarily responsible for price rise. The commodity futures market is a mechanism for price discovery and price risk management. The price of any commodity is determined by actual demand and supply position in the market. The futures market thus discovers the likely prices of a given commodity at future points of time depending on the expectations of supply and demand.

The most important motive behind using futures market is the price discovery which reveals information about futures cash market prices through the futures market. Further, price discovery function of the futures market also leads to the inter-temporal inventory allocation function. According to this, the traders can compare the spot and futures prices and will be able to decide the optimum allocation of their quantity of underlying asset between the immediate sale and futures sale. The price discovery function can be explained by an example.

Supposing, a copper miner is trying to take a decision whether to reopen a marginally profitable copper mine or not. Assuming that the copper ore in the mine is not of the best quality and so the yield from the mine will be relatively low. The decision will depend upon the cost incurred on mining and refining of copper and the price of the copper to be obtained in futures. Hence, the crucial element in this decision is the futures price of copper. The miner can analyse the copper prices quoted in the futures market today for determining the estimate of the futures price of copper at a specified future period. In this calculation, the miner has used the futures market as a vehicle of price discovery.

Hedging : The primary motive of the futures market is the hedging function which is also known as price insurance, risk shifting or risk transference function. Futures markets provide a vehicle through which the traders or participants can hedge their risks or protect themselves from the adverse price movements in the underlying assets in which they deal.

For example, a farmer bears the risk at the planting time associated with the uncertain harvest price his crop will command. He may use the futures market to hedge this risk by selling a futures contract. For instance, if he is expected to produce 500 tons of cotton in next six months, he could set a price for that quantity (harvest) by selling 5 cotton futures contracts, each being of 100 tons. In this way, by selling these futures contracts, the farmer tends to establish a price today that will be harvested in the futures. Further, the futures transactions will protect the farmer from the fluctuations of the cotton price, which might occur between present and futures period. Here two prices come into picture : future price and spot price. The difference between the two is the profit or loss for the farmer.

Long hedging : When futures are bought against the price increase is known as long hedge or buying hedge.

Suppose a farmer produces rice and he expects to have an excellent yield on rice; but he is worried about the future price fall of that commodity. How can he protect himself from falling price of rice in future? He may enter into a contract on today with some party who wants to buy rice at a specified future date on a price determined today itself. In the whole process the farmer will deliver rice to the party and receive the agreed price and the other party will take delivery of rice and pay to the farmer. In this illustration, there is no exchange of money and the contract is binding on both the parties. Hence future contracts are forward contracts traded only on organised exchanges and are in standardised contract-size. The farmer has protected himself against the risk by selling rice futures and this action is called **short hedge** while on the other hand, the other party also protects against risk by buying rice futures is called **long hedge.**

Short hedging : *When futures are sold against the price decrease is known as short hedge or selling hedge (Explained in above example).*

Optimal hedging ratio : *It will determine how many future contracts should be bought or sold to minimise the risk.*

Pricing of Future or Forward :

Future prices are changing continuously. Hence future price can be an estimate of the expected future spot price. There are several theories which have made efforts to explain the relationship between spot and futures prices. Out of them Cost of carry model is the most significant one.

Cost-of-Carry model :

Some economists like Keynes and Hicks, have argued that futures prices essentially reflect the carrying cost of the underlying assets. In other words, the inter-relationship between spot and futures prices reflects the carrying costs, i.e., the amount to be paid to store the asset from the present time to the futures maturity time (date). For example, food grains on hand in June can be carried forward to, or stored until, December. Cost of carry which includes storage cost plus the interest paid to finance the asset less the income earned on assets. It will establish the relationship between future prices and spot prices. It measures the storage cost (in the commodity market) plus the interest that is paid to finance or 'carry' the asset till delivery less the income earned on the asset during the holding period. For equity derivatives, carrying cost is the interest paid to finance the purchasing of equity less (minus) dividend earned. For more understanding of the concept, let's take the following case :

If C_t = cash price at time t

$S_{t,T}$ = annualised interest rate on borrowings

$G_{t,T}$ = storage cost

$(T-t)$ = time period

$F_{t,T}$ = future price at time t, which will be delivered at time T

$D_{t,T}$ = Dividend Income Rate

Therefore, as per cost of carry model,

$$\text{Future Price} = F_{t,T} = C_t + C_t \times S_{t,T} \times (T-t)/365 + G_{t,T} - C_t \times D_{t,T} \times (T-t)/365$$

For example :

ICICI Bank shares are selling at INR 1250 on January 1

Feb futures expiring on February 27 are available

Risk-free rate = 6%/year

Futures price = 1250 + 1250 × 0·06 × (58/365) = INR 1261·9

If ICICI pays a dividend of INR 10 on Jan 21, then

Futures price = 1250 + 1250 × 0·06 × (58/365) – 10[1+ 0·06 × (37/365)]

= INR 1251·84

Illustration 4.3

The stock of Aptech Ltd. (FV ₹ 10) quotes ₹ 920 today on NSE and the 3 month futures price quotes at ₹ 950. The borrowing rate is given as 24% p.a. and the expected annual dividend yield is 15% p.a. payable before expiry. Calculate the price of 3 month Aptech Futures.

Solution :

$$\text{Future Price} = F_{t,T} = C_t + C_t \times S_{t,T} \times (T-t)/365 + G_{t,T} - C_t \times D_{t,T} \times (T-t)/365$$

$$= 920 + 920 \times (0{\cdot}24) \times (90/365) + 0 - 920 \times 0{\cdot}15 \times (90/365)$$

$$= ₹\ 940{\cdot}41$$

4.3.3. Options Contract

The buyer of the option has the right but not the obligation to buy or sell a specific quantity of a particular asset, at a specified price at or before a specific date in the future. On account of price movements, the option may increase, decrease or remain unchanged in value.

Option may be defined as a contract between two parties where one gives the other the right (not the obligation) to buy or sell an underlying asset as a specified price within or on a specific time. The underlying may be commodity, index, currency or any other asset. As an example, suppose that a party has 1000 shares of Satyam Computer whose current price is ₹ 4000 per share and other party agrees to buy these 1000 shares on or before a fixed date (i.e. suppose after 4 month) at a particular price say it is become ₹ 4100 per share. In future within that specific time period he will definitely purchase the shares because by exercising the option, he gets ₹ 100 profit from purchase of a single share. In the reverse case suppose that the price goes below ₹ 4000 and declines to ₹ 3900 per share, he will not exercise at all the option to purchase a share already available at a lower rate. Thus option gives the holder the right to exercise or not to exercise a particular deal. In present time options are of different varieties like — foreign exchange, bank term deposits, treasury securities, stock indices, commodity, metal etc. Similarly the example can be explained in case of selling right of an underlying asset.

■ **Features of option contract :**

The following features are common in all types of options.

(a) **Contract :** Option is an agreement to buy or sell an asset obligatory on the parties.

(b) **Premium :** In case of option a premium in cash is to be paid by one party (buyer) to the other party (seller).

(c) **Payoff :** From an option in case of buyer is the loss in option price and the maximum profit a seller can have in the options price.

The optionality characteristic of options results in a non-linear payoff for options. In simple words, it means that the losses for the buyer of an option are limited; however the profits are potentially unlimited. The writer of an option gets paid the premium. The payoff from the option writer is exactly opposite to that of the option buyer. His profits are limited to the option premium; however his losses are potentially unlimited. These nonlinear payoffs are fascinating as they lend themselves to be used for generating various complex payoffs using combinations of options and the underlying asset. We look here at the four basic payoffs.

(d) **Holder and writer :** Holder of an option is the buyer while the writer is known as seller of the option. The writer grants the holder a right to buy or sell a particular underlying asset in exchange for certain money for the obligation taken by him in the option contract.

(e) **Exercise price :** There is call strike price or exercise price at which the option holder buys (call) or sells (put) an underlying asset.

(f) **Variety of underlying asset :** The underlying asset traded as option may be variety of instruments such as commodities, metals, stocks, stock indices, currencies etc.

(g) **Tool for risk management :** Options is a versatile and flexible risk management tools which can mitigate the risk arising from interest rate, hedging of commodity price risk. Hence options provide custom-tailored strategies to fight against risks.

(h) **Call Option :** Gives the buyer the right to buy as per the option contract.

(i) **Put Option :** Gives the buyer the right to sale as per the option contract.

(j) **In-the-Money :** The option has an exercisable value, i.e. in the case of a Call Option the exercise price is below the spot price; and in the case of a Put Option, the exercise price is above the prevailing spot price.

(k) **At-the-Money :** The Option exercise price equals the prevailing price of the underlying asset.

(l) **Out-of-the-Money :** The Option price lies above the prevailing price of the underlying asset in the case of a Call or below in the case of a Put.

(m) **Intrinsic Value :** It implies the positive difference between exercise price and market price. An option has intrinsic value if it is in-the-money. For a call option the strike price has to be under the price of the underlying; for a put option the strike price has to be over the price of the underlying. Options will be exercised only when a trader can benefit from this and if exercising an option leads to losses, the trader will not do so. If asset prices move in such a way that the option is in the money, i.e. exercising the option will benefit the trader, the option will have a higher value. This is known as the intrinsic value, which is the benefit for the trader if they exercise the option. If options are out-of-money, i.e. if the exercise will lead to losses, the intrinsic value will be zero.

(n) **Time value :** The difference between the option premium & the intrinsic value.

(o) **Maturity or Expiration Date :** Final day on which an option may be exercised.

■ **Types of Option Contract :**

There are various types of options depending upon the time, nature and exchange of trading. The following is a brief description of different types of options :

1. Put and call option
2. American and European option
3. Exchange traded and OTC (over the counter) options
4. Currency and Interest rate options.

Put option is an option which confers the buyer the right to sell an underlying asset against another underlying at a specified time on or before a predetermined date. The writer of a put must take delivery if this option is exercised. In other words put is an option contract where the buyer has the right to sell the underlying to the writer of the option at a specified time on or before the option's maturity date.

When a put option would be exercised ?

When a trader buys a put option he gets the right to sell the underlying asset at exercise price at maturity date (in case of European option) and anytime including maturity date (in case of American option). Since he will be receiving the exercise price while selling the underlying asset if he exercises the put option, he would exercise only when exercising the option will be beneficial. Benefit occurs only when the market price of the underlying asset is less than the exercise price as he will be able to sell the underlying asset at a price higher than the market price through exercise. Thus, a trader will exercise a put option only when the market price of the underlying asset is less than the exercise price or $S_T < S_X$ in case of European option. However, American options can be exercised early and will exercise the American option early only when the time value of the call option is negative so that in-the-money value of option is greater than the option price in the market.

Call option is an option which grants the buyer (holder) the right to buy an underlying asset at a specific date from the writer (seller) a particular quantity of underlying asset on a specified price within a specified expiration/maturity date. The call option holder pays premium to the writer for the right taken in the option.

When a call option would be exercised ?

When a trader buys a call option he gets the right to buy the underlying asset at exercise price at maturity date (in case of European option) and anytime including maturity date (in case of American option). Since he will be paying the exercise price to buy the underlying asset if he exercises the call option, he would exercise only when exercising the option will be beneficial. Benefit occurs only when the market price of the underlying asset is greater than the exercise price as he will be able to buy the underlying asset at a price lower than the market price through exercise. Thus, a trader will exercise a call option only when the market price of the underlying asset is greater than the exercise price or $S_T > S_X$ in case of European option. However, American options can be exercised early and will exercise the American option early only when the time value of the call option is negative so that in-the-money value of option is greater than the option price in the market. (Where, S_T = Spot price and S_X = Exercise price)

American option provides the holder or writer to buy or sell an underlying asset, which can be exercised at any time before or on the date of expiry of the option.

On the other hand a **European option** can be exercised only on the date of expiry or maturity. This is clear that American options are more popular because there is timing flexibility to exercise the same. But in India, European options are prevalent and permitted. Exchange traded options can be traded on recognised exchanges like the futures contracts.

Over the counter options (OTC) are custom tailored agreement traded directly by the dealer without the involvement of any organised exchange. Generally large commercial bankers and investment banks trade in OTC options.

Exchange traded options have specific expiration date, quantity of underlying asset but in OTC traded option trading there is no such specification and terms are subjective and mutually agreed upon by the parties. Hence OTC traded options are not bound by strict expiration date, specific limited strike price and uniform underlying asset. Since exchange traded options are guaranteed by the exchanges, hence they have less risk of default because the deals are cleared by clearing houses. On the other side OTC options have higher risk element of default due to non-involvement of any third party like clearing houses. Offsetting the position by buyer or seller in exchange traded Option is quite possible because the buyer sells or the seller buys another option with identical terms and conditions. Hence the rights are transferred to another option holder. But due to unstandardised nature of OTC traded options the OTC options cannot be offset. Margin money is required by the writer of option but there is no such requirement for margin funds in OTC optioning. In exchange traded option contracts, there is low cost of transactions because the credit-worthiness of the buyer of options is influencing factor in OTC-traded options.

■ **Concept of Option Contract :**

Let's understand the cases when to exercise an option and when not exercise it. In case of a call option the buyer of call will exercise the option if the strike/exercise price (X) is less than the current market (spot price) while a seller will do differently. Similar case is with writer of an option. The seller (writer) will exercise the option if the strike price (X) is higher than the current (spot) price. The following table shows those cases :

Call Option	Exercise / Not Exercise	Put Option	Exercise / Not Exercise
If X > S	Not Exercise	If X > S	Not Exercise
If X < S	Exercise the option	If X < S	Exercise the option

Suppose X is exercise price and S is spot current market price.

The intrinsic value of an option is called fundamental or underlying value. It is the difference between the market/spot/current price and the strike price of the underlying asset.

For a call option, it can be calculated as follows : Max [(S-X), 0] where S is the current/spot price and X is the exercise/strike price of the underlying asset and as clear from the above table, the option holder will exercise the option if the exercise price is less than the current market price i.e. if $S > X$ or $X < S$. The difference between S and X will be positive and this is known a positive intrinsic value and in case if $S = X$ then the intrinsic value is zero. In any case it cannot be negative because then the holder will not exercise the option.

Similarly the intrinsic value of a put option is the difference as shown : Max [(X-S), 0]. If $X > S$ or $S > X$ then the writer will exercise the option. In case of equal values of X and S the intrinsic value will be zero. There is no negative value of a put because the writer will not exercise his right to sell an underlying if the exercise price is less than the market price.

Further an option is said to be in-the-money if the holder (writer) gets the profit if the option is immediately exercised. The option is said to be out of the money if it gives loss when exercised immediately. If the current/spot price is equal to the strike price the option becomes at-the-money.

As you know that an American option can be exercised any time before the expiration date, there lies a probability that the stock price will fluctuate during this period. It is the time at which the option holder should exercise the option.

Suppose an option holder wants to exercise his option right at a particular time (t), because at that time he thinks that it is profitable to exercise the option. Hence, the difference between the value of option at time (t) and the intrinsic value of the option is known as time value of the option. Now there are various factors which affect the time value as follows :

1. Stock price volatility
2. The time remaining to the expiration date
3. The degree to which the option is in-the-money or out of the money.

In other words, the **time value of an option** is the difference between its premium and its intrinsic value. The maximum time value exists when the option is **At the Money (ATM)**. The longer the time to expiry, the greater is an option's time value. At expiration date of an option, it has no (zero) time value.

For better understanding let's assume that X is the exercise price and S is the stock current price. Suppose this is a case of a call, where the holder will exercise only when $S > X$.

Before expiration, the time value of a call will be

Time value of a call = C_t – [Max [0, S-X]].

C_t is the premium of a call.

Similarly, for a put the time value will be

Time value of put = P_t – [Max [0, X-S]] where P_t is the premium of a put option.

The similar concept can be understood by the following graphs.

Fig-4.2 exhibits the pay-off versus spot price relation for a call option buyer's and seller's perspective.

The vertical axis represents pay-off and the horizontal axis represents spot price of USD. Let us assume that the exercise price for the contract = ₹ 46·50 per $ and premium = ₹ 0·50 per $.

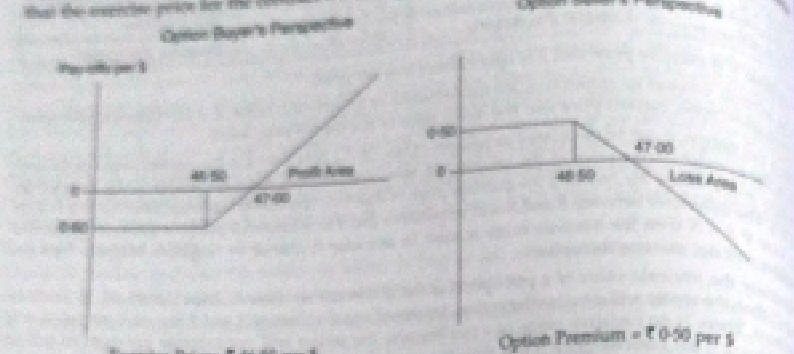


Fig.-4.2 : Call Option Perspectives

Fig. 4.3 exhibits the pay-off versus spot price relation for a put option buyer's and seller's perspective. The vertical axis represents pay-off and the horizontal axis represents spot price of USD. Let us assume that the exercise price for the contract = ₹ 46·50 per $ and premium = ₹ 0·50 per $.

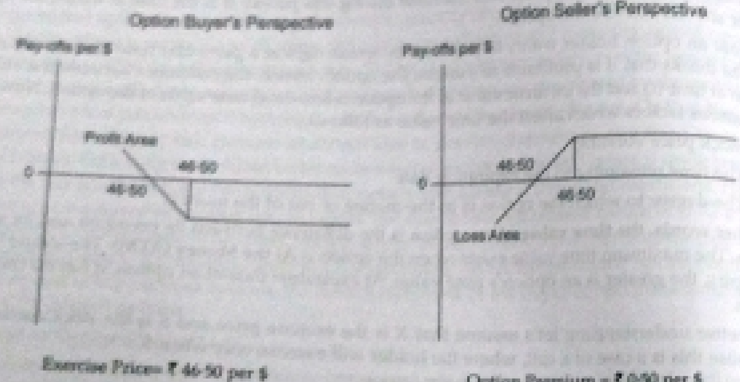


Fig.-4.3 : Put Option Perspectives

■ **Currency Option :** Option contracts are used for foreign currency trading. With the opening and integration of capital markets world-wide, the free flow of foreign currency from one country to another has increased at a faster pace. Foreign currency options are used by different market participants e.g. exporters, importers, speculators, arbitrageurs, bankers, traders and financial institutions. Currency options are devised to protect the investors against unfavorable movements/fluctuations in foreign exchange rates. Like other option instruments, currency options are also financial instruments which give the option holder the right not the obligation to buy or sell a particular currency

at a specific exchange rate (price) on or before an expiration date. Here the underlying asset is the foreign currency.

For example, a US manufacturer imports his raw materials from UK. In six months' time (i.e. say June ...) he will need to import goods worth STG 1 million and has to pay for the imports. He wants to protect himself from exchange fluctuations, yet wants to take advantage of favourable ER movement.

... 2018 : Market conditions : STG/USD = 1·50

... Calls @ Strike Price of STG/USD = $ 1·51, premium 4 US cents

Result : In June 2018, the USD does weaken and the new spot rate is STG/USD = 1·60. Hence the Call Option now has an intrinsic value of 9 US cents: the exercise of the Call Option Contract will net $ 90,000 ($ 1·6 million minus $ 1·51 million.)

The premium price was $ 40,000 (1million × 4/100). Net gain by US importer

= $ 90,000 – $ 40,000 = $ 50,000.

■ **Interest Rate Option :**

In case of interest rate option, option holder has the right to exercise his or her contract as per the agreed rate.

They enable a co. to take advantage of favourable movements of interest rates by providing the right but not the obligation to fix a rate of interest, on a notional loan or deposit, for an agreed amount, for a fixed term, on a specified forward date. The seller of the option guarantees an interest rate if the option is exercised. The seller receives a fee, the premium, for providing this guarantee.

LIBOR/MIBOR : LIBOR is the benchmark interest rate at which major global banks lend to one another. LIBOR is administered by the Intercontinental Exchange, which asks major global banks how much they would charge other banks for short-term loans.

LIBID is the London Interbank Bid Rate, the "bid" rate at which banks are willing to borrow eurocurrency deposits. The "offer" rate at which banks are willing to lend to each other is the more popular LIBOR.

The Mumbai Interbank Offer Rate (MIBOR) is the India's interbank rate, which is the rate of interest charged by a bank on a short-term loan to another bank. As India's financial markets have continued to develop, India felt it needed a reference rate for its debt market, which led to the development and introduction of the MIBOR. MIBOR is used in conjunction with the Mumbai interbank bid and forward rates (MIBID and MIFOR) by the central bank of India to set short-term monetary policy.

The most common type of interest rate option is available to borrowers as a hedge against rising interest rates. This is known as the interest rate cap.

Cap : Major international banks offer, for a fee, a kind of insurance cover for fluctuations in interest rates like the LIBOR. In such cases, the bank agrees to reimburse to the borrower the cost of LIBOR exceeding a particular level during the currency of the loan. This is known as a 'cap'.

The fee to be paid by the borrower would depend upon the difference between the cap and the current rate, the period for which the contract is to run, the anticipated interest rate volatility, etc. The higher the cap, the lower the fee; the longer the period, the higher the fee, etc.

Collar : Interest rate floors protect investors against falling interest rates. When a contract specifies both the cap and floor, it is known as a 'collar' or 'band'.

Dealing with an interest rate collar is cheaper than buying the straight interest rate cap or floor since the buyer is giving up some of his upside benefit if rates move in his favour. Effectively, the simultaneous purchase and sale of a cap and a floor is known as a 'collar'. For example, if the actual LIBOR is lower than the band, the buyer of the collar will pay the difference to the insurer.

■ **Difference between a futures contract and an options contract :**

In a futures contract, the price at which an exchange that will take place at the specified future is determined. Both parties to the contract have obligations to fulfill. These contracts are traded in exchanges.

In an options contract, the buyer gets the right to either buy or sell the asset at a future time at a specified price, and has no obligation to exercise the right. On the other hand the seller of an option has the obligation to either buy or sell the asset if the buyer exercises their right. Options are traded in exchanges. Options can also be entered into as private contracts in over-the-counter markets.

While futures fix the price of the underlying asset, this is useful only when asset prices move against the trader (known as downside risk), so that their position is hedged. If prices move in the trader's favour, this is known as upside risk, and they cannot take advantage of the favourable price movement. Options protect the trader against downside risk when prices move against them. At the same time they allow the trader to benefit from upside risk when prices move in favour.

4.3.4. Warrants and Convertibles

Convertibles and warrants are securities offered by companies to attract investors and raise finance.

Convertibles are long-term securities which can be changed into another type of security, such as common stock. Convertibles include bonds and preferred shares, but most commonly take the form of bonds. Convertibles are attractive to investors who are looking for an investment with greater growth potential than that offered by a traditional bond. By purchasing a convertible bond, the investor can still receive returns as if it were a traditional bond, but has the additional option of converting that bond into shares if the share price increases enough to make it worthwhile.

Warrants are also long-term securities but are generally shorter-term than convertibles. They give investors the right to purchase shares at a fixed price (known as the "exercise price") for a predetermined amount of time, often several years. Warrants are often tied to bonds or preferred stock, but can also be issued independently. The exercise price is usually higher than the price at which the shares for the company are currently trading, but if those shares then increase in value, the investor will still be able to purchase at the exercise price. Warrants are more valuable in volatile markets when chances of the price swinging above the exercise price are good. They become less valuable as the warrant expiration date approaches because the chances of a favourable price swing are greatly reduced.

■ **Difference between warrant and convertibles :**

Two common types of attractive investments are warrants and convertible securities.

A stock warrant gives investors the right to purchase the underlying security for a particular price. Convertible securities give investors the ability to convert the security into the company's common stock. Warrants and convertibles possess many variables. Investors deciding whether to invest in warrants or convertibles should understand the difference in features, advantages and disadvantages of both types of securities before making an investment decision.

Warrants are call options that give the holder the right, but not the obligation, to buy shares of common stock directly from a company at a fixed price for a given period of time. They tend to have longer maturity periods than exchange traded options. They are generally issued with privately placed bonds as an "equity kicker". They are also combined with new issues of common stock and preferred stock, given to investment bankers as compensation for underwriting services. In this case, they are often referred to as a Green Shoe Option.

Convertible bonds and warrants are like call options. However, there are important differences :

(1) Warrants are issued by the firm.

(2) Warrants and convertible bonds have different effects on corporate cash flow and capital structure.

(3) Warrants and convertibles cause dilution to existing shareholder's claims.

Many arguments, both plausible and implausible, are given for issuing convertible securities. Convertible bonds give lends the chance to benefit from risks and reduces the conflicts between bondholders and stockholders concerning risk.

4.3.5. Swap

The dictionary meaning of 'swap' is to exchange something for another. Swap, a popular financing tool, is a contract between two parties (counter parties) to exchange two streams of payment for an agreed period of time. Variants of swaps are interest rate, currency, commodities and equity. Financial swaps are a funding technique, which permit a borrower to access one market and then exchange the liability for another type of liability.

The global financial markets present borrowers and investors with a wide variety of financing and investment vehicles in terms of currency and type of coupon — fixed or floating. It must be noted that swaps by themselves are not a funding instrument. They are a device to obtain the desired form of financing indirectly. The borrower might otherwise have found this too expensive or even inaccessible. The various terminologies used in swap transactions are as follows :

Swap : An agreement to exchange cash flows over a fixed period of time.

Counterparties : The two parties in a swap contract.

Notional principal : A monetary figure used as a part of the calculation to determine payment amounts.

Tenor : The length of time for which payments will be exchanged, also known as term, maturity, or expiration of swap.

Swap facilitators : Specialists who help clients to design swaps, e.g., Bank.

Swap brokers : Bring counterparties together for a swap transaction.

Swap dealers : Can enter into swap agreements as one of the counterparties, e.g., Bank. Bank will act as both swap dealer and swap facilitator in swap transaction.

Cash flows : The present values of future cash flows are estimated by the counterparties before entering into a contract. Both the parties want to get assurance of exchanging same financial liabilities before the swap deal.

Less documentation : Less documentation is required in case of swap deals because the deals are based on the needs of parties, therefore, less complex and less risk consuming.

Transaction costs : Generating very less percentage is involved in swap agreement.

Benefit to both parties : The swap agreement will be attractive only when parties get benefits of these agreements.

Default-risk : It is higher in swaps than the option and futures because the parties may default the payment.

■ **Swap Mechanism :**

Like other financial derivatives, swap is also agreement between two parties to exchange cash flows. The cash flows may arise due to change in interest rate or currency or equity etc. In other words, swap denotes an agreement to exchange payments of two different kinds in the future. The parties that agree to exchange cash flows are called 'counter parties'. In case of interest rate swap, the exchange may be of cash flows arising from fixed or floating interest rates, equity swaps involve the exchange of cash flows from returns of stocks index portfolio. Currency swaps have basis cash flow exchange of foreign currencies and their fluctuating prices, because of varying rates of interest, pricing of currencies and stock return among different markets of the world.

- **Types of Swaps :**

The two most widely prevalent types of swaps are interest rate swaps and currency swaps.

- **Interest Rate Swaps :**

An interest rate or coupon swap involves an exchange of different payment streams which are fixed and floating in nature. Swap coupon is the fixed interest rate in Swap transaction.

The life of the swap can range from two years to over 15 years. This type of a standard fixed floating rate swap is also called a plain vanilla swap in the market jargon. London Inter-bank Offer Rate (LIBOR) is often the floating interest rate in many of the interest rate swaps.

Example of Interest Rate Swap/ Coupon Swap :

Interest rate swaps are calculated based on the underlying notional using applicable rates. Example of Coupon Swap.

Cost of Funds	*Fixed Rate*	*Floating Rate*
Good Credit/Corporate	6%	Libor
Poor Credit /Bank	8%	Libor + 1%
Spread	2%	1%

(Net spread = 2 − 1 = 1%, benefit for each = 0·5%, Assumed equal distribution)

(L : Libor)

Net Position	*Good Credit*	*Poor Credit*
Cost of Funds	6	L+1
Less Receipts on Swap	(6·5)	(L)
Plus Payments on Swap	L	6·5
Overall Cost	L-0·5	7·5

Good Credit Company is having comparative advantage in fixed rate loan (6% interest rate : 2% benefit) but looking for floating rate loan. Whereas Poor Credit Company is having less comparative disadvantage in floating rate loan (L+1% : 1% less) but looking for fixed interest loan.

Hence Good Credit Company will take fixed rate loan and Poor Credit Company will take floating interest loan and after that they will swap each other.

After swapping, finally Good Credit Company will avail floating loan (as per own choice) at a overall cost of (L-0·5) %[(L+1) − (L-0·5) = 0·5% benefit]. Similarly, Poor Credit Company will enjoy fixed rate loan (as per own choice) at a overall cost of 7·5%[8% − 7·5% = 0·5% benefit]. In both the cases, they can avail their own preference type of loan with a 0.5% benefit from their own eligibility.

It is possible because of Swap arrangement.

- *Currency Swaps :*

It involves exchanging principal and fixed rate interest payments on a loan in one currency for principal and fixed rate interest payments on an approximately equivalent loan in another currency. What is important to the trader who structures the swap deal is that difference in the rates offered to the companies on both currencies is not the same. Though one company has a better deal in both the currency markets, another company does enjoy a comparatively lower disadvantage in one of the markets. This creates an ideal situation for a currency swap.

Examples of Currency Swap :

1. *HDFC raises floating rate dollar debt in the US market, backed by the guarantee of US aid - leading to borrowings being at very fine rates. However, HDFC's requirement is for long term*

fixed rate rupees. Hence, HDFC swaps floating rate dollar loans with Indian banks for fixed rate rupees.

Result : The counter parties (the Indian banks) have access to floating rate dollars at a rate they would not have been able to raise on their own, while HDFC has access to fixed rate rupees at less than market rates.

Two firms A & B have the following interest rates. A wants to borrow in sterling & B prefers to borrow in $. Assume that the exchange rate is $ 1·5/ Sterling. Explain how a currency swap can be structured.

Firm	Dollars	Sterling
A	8·0%	10·6%
B	10·0%	11·0%

(Note : Equally benefitted by two parties.)

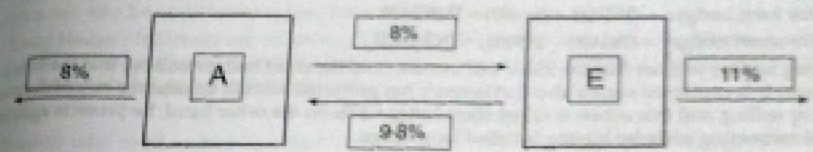
Solution :

Spread in $ = (10 − 8 = 2%), Spread in sterling = (11 − 10·6 = 0·4), Net spread = 2 − 0·4 = 1·6%, benefit equally divided , i.e. @ 0·8% each.

During the tenure of the loan, A will pay B, interest in sterling @ 9·8% (10·6 − 0·8 = 9·8%). Further, A will pay $ interest @ 8% to its lender. A will also receive $ interest from B @ 8%.

B will pay sterling interest @ 11% to its borrower. It will pay $ interest @ 8% to A. B will receive sterling interest @ 9·8% from A.

At the end of the swap, A will give B 1 million sterling to enable it to repay the sterling loan. B will give A 1 million USD so that A can repay its $ loan.



4.3.6. Forward Rate Agreement (FRA)

FRAs allow borrowers to lock-in today an interest rate (say, LIBOR) accruing from a forward start date for a given period, for e.g. for month 6 in the future to month 9. It is very popular in 2 - 3 years range.

The FRA is a contract between two parties to agree on an interest rate on a notional loan or deposit of a specified amount and maturity at a specified future date and to make payments between counter parties computed by reference to changes in the interest rate. FRAs involve no exchange of principal amount.

Example of an FRA :

The agreed 6 month LIBOR under an FRA is 3·5% per annum on a given future date. If the actual LIBOR rate happens to be 4%, the bank will reimburse to the buyer of the FRA the difference of 0·5% p.a.

On the other hand, if the actual LIBOR rate happens to be 3% p.a., the borrower will have to pay the difference to the bank.

It is not necessary that the bank be a lender in the transaction.

4.3.7. More of Future Contract

We can now discuss some other varieties of future contracts.

Stock Index Future :

Index captures the overall behaviour of a group of stocks. Index is created by selecting a group of stocks that are representative of the whole market, or a specified sector. Index backs the changes in the value of portfolio of stocks. Based on index, index funds, index futures and options are created. Index futures use a particular stock index as the underlying asset. Index futures are used to hedge equity portfolio risk. Index futures are also used for speculative purposes. Portfolio managers use index futures to hedge their equity positions against a loss in stocks. Speculators can also use index futures to bet on the market's direction. Some of the most popular index futures are based on equities including the E-mini S&P 500, E-mini Nasdaq-100 and E-mini Dow. These can also be used for portfolio insurance. Portfolio insurance is a hedging technique frequently used by institutional investors when the market direction is uncertain or volatile. Short selling index futures can offset any downturns, but it also hinders any gains. The workings of this portfolio insurance strategy is driven by buying index put options. A multiple factor is attached in Index future. For example CNX Nifty Index '50' will be taken as the multiplier value because of 50 stocks in Nifty.

Example of Index Futures :

CNX Nifty Index is at 5200 on December 1

CNX Nifty Index futures with expiry on December 28 is at 5232

If CNX Nifty Index is at 5300 on December 28, the long hedger will gain, and the short hedger will lose

Value of futures on December 1 = 5232 × 50 = INR 261,600

Value of futures on December 28 = 5300 × 50 = INR 265,000

Gain for long hedger = (265000 – 261600) = INR 3400

Loss for short hedger = (261600 – 265000) = INR 3400

The long hedger will bet that the index will increase, and the short hedger will bet that the index will decrease. It is explained earlier also that investor has protected himself against the risk of decreasing price by selling and this action is called short hedge while on the other hand, he protects against the risk of increasing price by buying is called long hedge.

- **Interest Rate Future :**

If interest rates are expected to increase, one will take a short position in futures. If interest rates are expected to decrease, one will take a long position in futures. This is risky, as huge losses can result if interest rates move in opposition to expectations.

Interest Rate Futures are the financial derivatives where the underlying asset is interest rate bonds. In this type the futures securities traded are interest bearing instruments like T-bills, bonds, debentures, euro dollar deposits and municipal bonds, notional gilt contracts, short term deposit futures and Treasury note futures.

4.4. Headging Strategies

A party faces a loss when the price of some asset changes—they want to reduce this loss by trading futures contracts. Hedging is done to lock in a price at the current time for a future transaction. Hedging helps in forecasting future cash flows with some certainty. A perfect hedge is achieved when price uncertainty is fully eliminated and the hedger knows for certain what the future cash flow will be (Hedging Effectiveness) whereas an imperfect hedge is a partial hedge in which the price uncertainty is reduced but not fully eliminated.

Hedging strategies using futures ? When is a short futures position appropriate ? When is a long futures position appropriate ? Which futures contract should be used ? What is the [illegible] size of the

futures position for reducing risk ? At this stage, we restrict our attention to what might be termed hedge-and-forget strategies. The objective is usually to take a position that neutralizes the risk as far as possible. Consider a company that knows it will gain $ 10,000 for each 1 cent increase in the price of a commodity over the next three months and lose $ 10,000 for each 1 cent decrease in the price during the same period. To hedge, the company's treasurer should take a short futures position that is designed to offset this risk. The futures position should lead to a loss of $ 10,000 for each 1 cent increase in the price of the commodity over the three months and a gain of $ 10,000 for each 1 cent decrease in the price during this period. If the price of the commodity goes down, the gain on the futures position offsets the loss on the rest of the company's business. If the price of the commodity goes up, the loss on the futures position is offset by the gain on the rest of the company's business.

Suppose an oil importer knows in advance on July 10, that it will need to buy 30,000 barrels of crude oil at some time in October or November and the contract size is 1000 barrel.

The company therefore decides to use December futures contract for hedging and takes a long position in 18 December contracts.

The future price on July 10 is $ 50 per barrel. The company finds itself in a position to purchase crude oil on November 12 closes its futures position on that date.

The spot price on November 12 is $ 52 per barrel and $ 51·20 per barrel.

The gain on the future contract is 51·20 – 50 = $ 1·20 per barrel.

The basis on the date when the contract is closed is 52·00 – 51·20 = $ 0·80 per barrel.

The effective price is the final spot price less the gain on the futures or 52·00 – 1·20 = 50·80

This can also be calculated as the initial futures price plus the final basis. 50·00 + 0·80 = 50·80

The total price received is 50·80 × 30000 = $ 1,52,4000.

4.4.1. Types of Hedging

Hedging can also be of different types. These are discussed below.

(a) **Long Hedge :** If futures are on interest rates themselves, like in Eurodollar futures, a borrower would take a long position in these futures locking in a known interest at the current time—this is a long hedge (interest rate increases).

Example :

(1) Traders who need to buy the asset at a future time will use long hedge through commodity futures.

(2) Importers who need to buy foreign currency at a future time will use long hedge through currency futures.

(3) Borrowers who need to borrow at a future time will use long hedge through interest rate futures.

(4) Investors who need to invest at a future time will use long hedge through government bond futures.

(b) **Short hedge :** An investor would take a short position in these futures, locking in a known interest rate, which is a short hedge (interest rate decreases).

Example :

(1) Traders who own the asset now and need to sell at a future time will use short hedge through commodity futures.

(2) Exporters who will receive foreign currency cash flows at a future time will use short hedge through currency futures.

(3) Investors who need to invest at a future time will use short hedge through interest rate futures.

(4) Borrowers who need to borrow at a future time will use short hedge through government bond futures.

Some more Examples :

1. A rice merchant estimates that he will require 50 MT of rice on July 31. Since he needs to buy at a future time, he will enter into a long hedge in rice futures for a total value of 50 MT. If futures price on May 1 is INR 50,000/MT, he will pay 50,000*50 = INR 2,500,000 on July 31 and receive 50 MT of rice.
2. On January 1, a producer of steel ingots estimates that he will need to sell 30 MT of steel on March 31. Since he must sell steel at a future time, he will enter into a short hedge for a total value of 30 MT. If the steel ingot futures price on Jan 1 is INR 45,000/MT, he will need to provide 30 MT of steel ingots and receive INR 13,500,000 on March 31.
3. On Jan. 1, an importer buys goods from the USA for USD 1 million, to be paid on March 31. Since he needs to buy USD on March 31, he will enter into a long hedge to buy USD futures. If futures price is INR 45.30, the importer will pay INR 45.3 million and receive USD 1 million.
4. On Jan. 1, the spot price of rice is INR 30/kg. The March futures price of rice is INR 33/kg. A merchant enters into a long hedge, taking a long position in futures, agreeing to buy the rice at INR 33/kg on 31 March. On 31 March, the spot price of the rice is INR 35/kg, and the futures price is INR 35/kg **(Perfect Long Hedge)**.
5. On April 1, the spot price of dal is INR 2000/quintal. The April futures price of dal is INR 2000/quintal. The producer of dal enters into a short hedge to sell dal on April 30. On April 30, the spot and futures price are INR 1950/quintal **(Perfect Short Hedge)**.

4.5. Option Markets in India

Introduction of Foreign Currency — Rupee Options has introduced in India with effect from July 7, 2008. Initially only OTC contracts introduced – plain vanilla products, i.e. European exercise call and put options. Only customers with genuine foreign currency exposures are eligible to enter into contracts. At present options cannot be used to hedge contingent or derived exposures except exposures arising out of tender bids in foreign exchange. Customers can also enter into packaged products involving cost reduction structures and does not involve customers receiving premium.

4.5.1. Factors influencing option prices

The factors which are normally responsible for influencing the option prices are as follows :

1. **Option Type :** The option value depends on its type. There are basically two types of options : Put or Call. The difference clearly hinges on which side the investor would exactly stand. This is probably the simplest variable comprehensible to the average trader.
2. **Underlying Price & Strike Price :** The value of calls and puts are affected by changes in the underlying stock price in a relatively straightforward manner. When the stock price goes up, "calls" should gain in value because the investor would be able to buy the underlying asset at a lower price than where the market is, and "puts" should decrease. Similarly, when the stock price goes down the put options should increase in value and calls should drop since the put holder gives the right to sell stock at prices above the falling market price.

 The strike price may be defined as the price payable by the call owner to purchase stock, while a put owner decides to sell his stock. The average investor would prefer such rights that enable him to purchase stocks at lower prices at any moment of the day. This makes calls more expensive with the strike price moving downwards. Similarly, puts become more expensive when the strike price spirals up.

3. **Time to Expiration :** It is important to note that all options come with a definite lifespan and tend to expire on or after a certain date. Therefore, the value of an option increases with additional time. The more time available until expiry, the greater are the chances of making profitable moves.

 Time works in favour of the stock trader because the stocks of good companies tend to rise over long periods of time. But time is the enemy of the buyer of the option because, if days pass without any significant change in the price of the underlying asset, the value of the option will decline. In addition, the value of an option will decline more rapidly as it approaches the expiration date. However, that is a good news for the option seller who tries to benefit from 'time decay', especially during the final month when it occurs most rapidly.
4. **Interest Rates :** Like most other financial assets, options prices are also influenced by the prevailing interest rate. They are also impacted by interest rate changes. Call option and put option premiums are affected inversely as interest rates change : calls benefit from rising interest rates while puts lose value. The opposite is true when interest rates fall.
5. **Dividends :** When the stock trades and its holder receive no dividend, the situation is termed as ex-dividend and the price of the stock gets diminished by the amount of dividend payable. With rising dividends, put values increase while call values decrease.
6. **Volatility :** In simpler terms, volatility is the difference recorded in day-to-day stock prices. It is also referred to as swings that affect the price of a stock. The more volatile stocks are more frequently subject to a varying strike price level as compared to their non-volatile counterparts. With big moves, the chances are higher to make money and the investor shifts out of the Blue sphere. Thus options on volatile stocks are definitely more expensive than the less or non-volatile ones. For any prudent investor it is important to remember, therefore, that even the minutest changes in volatility estimates impact options prices substantially.

Option pricing models require the trader to enter future volatility during the life of the option. Naturally, option traders don't really know what it will be and have to guess by working the pricing model. Normally a trader already knows the price at which the option is trading and can examine other variables including interest rates, dividends, and time left with a bit of research. As a result, the only missing number will be future volatility, which can be estimated from other inputs and try to figure out an implied volatility (a probability). Traders use this implied volatility (IV) to gauge whether options are cheap or expensive. When the option traders say that premium levels are high or that premium levels are low then what they really mean is that the current IV is high or low. Once understood, the trader can determine when it is a good time to buy options (because premiums are cheap) and when it is a good time to sell options (since the premiums are high).

4.5.2. Black & Scholes Option Pricing Formula

Mathematical model developed by Fischer Black and Myron Scholes (1973). Adapted by Garman and Kohlhagen (1983) for currency options. The Black-Scholes model determines a fair value price based on current price (spot), exercise price, time remaining before expiration of the option, the compounded risk free rate of interest and the value of the cumulative normal density function. The fair value price as well as supply and demand then determines market price of the option. Nowadays various menu driven packages are available for arriving at the values of call and put options after feeding the primary data.

Assumptions :

1. Stocks pay no dividends during life of option.
2. All asset markets are perfectly efficient. Market are efficient i.e. information are costless.
3. No transaction cost or tax.
4. Risk-free interest rate will remains constant.
5. Returns are log-normally distributed.
6. European exercise terms are used.
7. The option being valued cannot be exercised any time before the expiration date.

As per the Black and Scholes Option pricing model,

Call Option Price = $P_c = [P_s][N(d_1)] - [P_e][e^{(-R_f)t}][N(d_2)]$

Where

P_c = market price of the call option

P_s = price of the stock

P_e = striking price of the option

R_f = annualised interest rate

t = time to expiration (in years)

$N(d_1)$ and $N(d_2)$ are the values of the cumulative normal distribution defined by

$d_1 = [\ln(P_s/P_e) + (R_f + 0.5\,\sigma^2)t] \,/\, \sigma\sqrt{t}$

$d_2 = d_1 - (\sigma\sqrt{t})$

Where $\ln(P_s/P_e)$ = the natural logarithm of (P_s/P_e)

σ^2 is the variance of continuously compounded rate of return on the stock per time period

Put option Price = $P_p = [P_e]\,[e^{(-R_f)t}]\,[N(-d_2)] - [P_s]\,[N(-d_1)]$

Illustration 4.4

An investor intends to have a price for a December 2000 European Call option and a Put option on a particular stock. This option is due to expire on Dec. 25th 2000. The option is bought on Nov. 16th 2000. The stock does not pay dividends. The following information is available about the call and the put option.

(a) Stock sells at ₹ 145

(b) Strike price of the Call and Put option = ₹ 140

(c) Interest rate is 10%

(d) The SD of stock returns is 20%

(e) Compute the value of Call and Put Option.

Solution :

P0 = 145, P0 = 140 (for call), P_s = 140 (Put), R0 = 10%, σ = 20%

- No. of days to expire = Nov. 16 to Dec. 25 = 39 days , t = 39/365
- Value of Call Option = $P_c = [P_s]\,[N(d_1)] - [P_e]\,[e^{(-R_f)t}]\,[N(d_2)]$
- $d_1 = [\ln(P_s/P_e) + (R_f + 0.5\,\sigma^2)t] \,/\, \sigma\sqrt{t} = 0.7329$
- $d_2 = d_1 - (\sigma\sqrt{t}) = 0.6675$
- N(01) = N(0.7329) = 0.7682

- N(02) = N(0.6675) = 0.7478
- C = Call value = 145(0.7682) − 140 e − 0.139365 (0.7478) = ₹ 7.81

Illustration 4.5

Use the Black-Scholes model to value the following Call Option – Stock price : ₹ 210, Strike price : ₹ 220, Time to expiration : 167 days, Risk-free interest rate : 10%, Variance of annual stock returns : 20%. $N(d_1)$ = 0.6189 & $N(d_2)$ = 0.5.

Solution :

S = 210, X = 220, t = 167/365 years, R_f = 10%, SD = sq.rt.(0.2).

- $d_1 = [\ln(P_s/P_e) + (R_f + 0.5\,\sigma^2)t] \,/\, \sigma\sqrt{t} = 0.3025$
- $d_2 = d_1 - (\sigma\sqrt{t}) = 0$
- $N(d_1)$ = 0.6189 (Given)
- $N(d_2)$ = N(0) = 0.5.
- C = Call Value $P_c = [P_s]\,[N(d_1)] - [P_e]\,[e^{(-R_f)t}]\,[N(d_2)]$ = ₹ 24.89.

4.5.3. Put-Call Parity Theorem

Put-Call-Parity is the relationship between the market price of a put and a call that have the same exercise price, ex. date & underlying stock. Following table describes the put-call parity theorem. There is a fixed relationship between put and call on the same share with similar strike price and maturity period which is called put-call parity. Assume that there exists a European put and a European call written on the same underlying stock, which currently has a value equal to X. Both options expire at time T and the riskless return rate is r_f. The basic put-call parity formula is as follows:

Value of call + PV of exercise price = Value of put + value of share.

Table - 4.2

Put-Call Parity Theorem

(S = Spot price X = Exercise price)	Exercise decision	S > X	Exercise decision	S < X
Call + Cash (C = call value)	Yes	(S − X) + X = S	No	0 + X = X
Put + Stock (P = put value)	No	0 + S = S	Yes	(X − S) + S = X
Present values of these 2 portfolios which have equal future values will be also equal : $c + Xe^{-r_fT} = p + S$				

That is, a portfolio consisting of one call with an exercise price equal to X and a pure discount risk-free note (zero coupon riskless bond) with a face value equal to X must have the same value as a second portfolio consisting of a put with exercise price equal to X and one share of the stock underlying both options. This relation is proven by first assuming the existence of a portfolio A consisting of one call with an exercise price equal to X and a pure discount risk-free note with a face value equal to X. It is also assumed that portfolio B, which consists of a put with exercise price equal to X and one share of the stock underlying both options. Irrespective of the final stock price, portfolio A will have the same terminal value as portfolio B at time 1. Therefore, at time 0, the two portfolios must have equal value. This is put-call parity.

Illustration 4.8. (Contd.) Using Put-Call parity (Since both Call and Put have the same exercise ...

$c - p = S - PV(X)$ [Where $PV(X)$ = Present Value of $X = Xe^{-R_f t}$]

Value of Put = P

$p = -S + PV(X) + C$

$= -145 + 138.51 + 7.81$

$= ₹\ 1.32$

As per the Black and Scholes Option pricing model, we know the call and put option prices. They are as follows :

Call Option Price = $P_c = [P_s]\,[N(d_1)] - [P_e]\,[e^{-R_f t}]\,[N(d_2)]$

Put option Price = $P_p = [P_e]\,[e^{-R_f t}]\,[N(-d_2)] - [P_s]\,[N(-d_1)]$

Therefore, the impact of variables of affecting call and put option prices are described in the following table :

Table - 4.3
Impact of variable on call and put prices

Factors	Effect on	
	Call option	Put option
Increase in underline asset value	Increases	Decreases
Increase in strike price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase to time to expiration	Increases	Increases
Increases in interest rates	Increases	Decreases
Increases in dividend paid	Decreases	Increases

4.5.4. Sensitivity Analysis

The above Table-4.3 has described how a wide variety of factors can affect an option's value. The sensitivity analysis of option premium deals with the measurement of changes in option price due to the change in the underlying parameters that determine the option prices. These parameters include stock price, time period, interest rate and volatility. There are five measures of sensitivities. They are Delta, Gamma, Theta, Rho and Vega.

Let us discuss each one of them in more detail.

Delta (Spot price) : Δ

Delta is the ratio of change in option price to change in price of underlying asset w.r.t. stock price.

Delta call = $dP_c/dP_s = N(d_1)$ (value ranges from 0 to 1)

Delta Put = $dP_p/dP_s = N(-d_1)$ (value ranges from -1 to 0)

Where P_c = Call option price, P_p = Put option price, P_s = Stock Price, σ = Volatility, t = time.

R_f = Risk free interest rate, P_e = Exercise or strike price

$N(d_1)$ & $N(d_2)$ are already defined in section 2.5.2.

Gamma : γ

Gamma is the rate of change of option's delta (Δ) w.r.t. price of underlying stock.

It is 2nd order derivative of option w.r.t. to price $\left(\dfrac{d^2 P_c}{dP_s^2}\right)$

Gamma (γ) of put/call is always equal and higher gamma (γ) means higher delta (Δ) w.r.t. stock price.

G_c or $G_p = N'(d_1) / P_s \sigma\sqrt{t}$

Theta (Time) : θ

Theta is a measure of option sensitivity w.r.t. expiration time (t). As time passes (maturity approaches), option value for call & put both loses.

$dP_c/dt = -[P_s][N'(d_1)]\ \sigma/2\sqrt{t} - [R_f][P_e][\text{anti ln}(-R_f t)][N(d_2)]$

$$N'(d_1) = \frac{e^{-(d_1 \times d_1)/2}}{\sqrt{2\pi}}$$

$dP_p/dt = -[P_s][N'(d_1)]\ \sigma/2\sqrt{t} + [R_f][P_e][\text{anti ln}(-R_f t)][N(d_2)]$

Rho (Risk free interest rate) : ρ

It is the 1st derivative of an option price w.r.t. risk free rate of return of underlying stock.

It is positively related in case of call option & negatively related in put option.

Rho (call) = $[P_e]\ t\ [\text{anti ln}(-R_f t)][N(d_2)]$

Rho (put) = $-[P_e]\ t\ [\text{anti ln}(-R_f t)][N(d_2)]$

Vega (Volatility) : σ

It is the 1st derivative of an option price w.r.t. volatility of underlying stock.

$dP_c/d\sigma = P_s \times \sqrt{t} \times N(d_1)$

$dP_p/d\sigma = P_p \times (1/\sqrt{t}) \times N(d_2)$

4.6. Trading Strategies in Option

Option traders often trade in options in combination to benefit from unpredictable behaviour in the prices of underlying assets. Option prices are determined as a function of the price of the underlying asset, the time until expiration, risk free interest rate, volatility of the underlying asset and the exercise price. We will discuss the strategies in this section like Covered Call, Protective Put, Straddle and Strangle, Strip and Strap and Spread. Steps for each strategy will be organised in terms of :

(1) Identify elementary strategies,
(2) Define expiration date & Profit and Loss associated in each strategy,
(3) Construct the Profit Diagram, and
(4) Evaluate the Strategy.

Covered Call :

It is happened when buying the underlying asset & writing a call on that asset with a belief that there is a scope of small price appreciation. It enjoys price rise upto strike rate or exercise price. But it regrets if spot price is higher than strike price or if price falls sharply.

Protective Put :

It is happened when buying an underlying asset & buying a Put on that asset. It earns protection against downside fluctuation of stock price.

Straddle and Strangle :

At first we shall discuss the concept of straddle.

Straddle :

It is a combination of one call & one put option with same strike price & date. A straddle buyer buys a call & a put & sellers sells a call & a put at the same exercise Price & date. Maximum loss will be premium paid for buying 2 options. In this strategy, profit increases when asset prices rises sharply & limited when it falls significantly.

Strangle :

It is a combination of a call & a put with same exercise date but different exercise price for call (x_1) & put (x_2) (Where $x_1 > x_2$).

Strip and Strap :

The concepts of strip and strap are discussed below :

Strip :

It is a combination of long position in 1 call & 2 put with same exercise price & date with a belief that there will be huge stock price movement, but chances of price fall will be higher than rise. Strip buyers can make profit in both rise & fall. But amt. of profit is higher when stock price decreases sharply. This strategy will be useful when stock is volatile and likely to fall.

Strap :

It is a combination of long position in 2 calls & 1 put with same exercise price & date with a belief that there will be huge stock price movement, but it will rise than fall. This strategy will be useful when stock is volatile and likely to rises sharply.

Spreads :

It is a simultaneous buying & selling with moderately bullish/bearish belief about the market. Following are various types of spread strategies in option trading.

Vertical/Price spread : Buying an option & selling another of same type & time but with different exercise price.

Bull Spread : Combination of option created to make profit from a rise in stock price buy a bull spread using buy a call with lower strike & sell a call with higher strike.

Bear Spread : Combination of option created to make profit from fall in stock price.

Time/ Horizontal Spread : Options are of same kinds (put/call– buying & selling) with same exercise price but different date. Generally these are long term options and considered time value of option. It is also known as **Calendar Spread.**

Diagonal Spreads : Similar to time spread, only options have different exercise price and different exercise Date.

Box Spread : Combination of bull & bear spread with call & put respectively with same set of exercise price.

Butterfly Spread : It can be executed by using 4 identical options (either all calls or Puts buying & selling simultaneously) with same exercise date but different exercise price. A trader who is long buys 1 call with low exercise price (x1) & buys 1 call with high exercise price (x3) & sells 2 calls with an intermediate exercise price(x2) so that x1 < x2 < x3.

The above all the strategies are explained in detailed with numerical examples in the following case study.

Case Study :

On November 1, 2008, Akhil, the manager of Bharat Funds, is contemplating how he can provide positive returns to the shareholders of the fund.

Bharat Fund was started on January 1, 2005, with a total capital of INR 300 million. This capital was mainly invested in the equity of stocks traded on the Indian market. Since the Indian market was doing very well from 2005 to 2008, this fund also did very well during this period. For example, the CNX Nifty index started at 2,115 on January 3, 2005. The return on the fund and the benchmark Nifty Index are shown here :

Period	Return on Benchmark CNX Nifty Index	Return on Bharat Fund
2005	34%	40%
2006	40%	48%
2007	55%	62%

Akhil was happy that he was able to beat the benchmark index by a big margin during this period. The net asset value increased from INR 300 million on January 1, 2005, to INR 1,007 million by December 2007. However, he started facing problems when the Indian stock market dropped considerably, in line with all the other markets during the financial crisis. He calculated the return on the benchmark index and his fund for every quarter from January 2008 to September 2008 and for October 2008, as shown below :

Period	Return on Benchmark CNX Nifty Index	Return on Bharat Fund
Q1, 2008	-23.0%	-18%
Q2, 2008	-14.7%	-10%
Q3, 2008	-2.9%	-2%
October 2008	-26.4%	-22%

From January 1, 2008, to October 31, 2008, the benchmark index dropped from 6,138 to 2,885, a drop of 53% over 10 months. The net asset value had decreased to INR 568 million, a decrease of 44% from January 1, 2008, to October 31, 2008.

All the global markets had been going down considerably since January 1, 2008, and there was no consensus about how long the effect of the financial crisis will last. All the governments in the world were using stimulus plans to spur the economic growth and many analysts believed that the economy as well as the stock market will recover and start an increasing trend from January 1, 2009.

On November 10, Akhil wants to follow some strategies that will protect the shareholders of Bharat Fund from a further drop in the net asset value of the fund. Since the market has been highly volatile over the last 10 months, he decides to concentrate on his portfolio on a month-to-month basis. He wants to use options to protect the net asset value from dropping and to provide additional gains.

He has collected the following data about various options available on the CNX Nifty Index as of November 1, 2008.

There were 49 call options and 49 put options available with exercise prices ranging from INR 2,300 to INR 4,750 and with an exercise date of November 27, 2008. He has also estimated that the Index is likely to be in the range of 2,600 to 3,300 on November 27, 2008. The following table shows the call and put prices for various exercise prices with the expiry date of November 27, 2008.

Exercise Price (INR)	Call Price (INR)	Put Price (INR)
2,600	403·85	124·70
2,650	301·05	141·55
2,700	338·40	151·90
2,750	299·60	167·00
2,800	268·35	189·30
2,850	242·55	208·35
2,900	218·30	229·80
2,950	181·90	306·40
3,000	169·05	279·35
3,050	147·75	264·00
3,100	127·35	337·10
3,150	110·75	429·95
3,200	88·95	425·85
3,250	72·50	498·80
3,300	60·55	495·80

Akhil has heard about covered call writing and portfolio insurance using options but is not sure which of these strategies will be better.

Question 1.

If he wants to enter into covered call writing, which of these options should he choose? If the value of the index on November 27, 2008, is 2,752, what will be the value of the portfolio on November 27, 2008 ?

Solution :

Covered call writing is used when the market is expected to drop and the losses due to decrease in the market price will be offset by the cash received from writing the calls. However, the risk is that the market price may go beyond the exercise price which can cause losses to the call writer. Therefore, the call that is used for covered call writing should be chosen such that the chances of price going beyond the exercise price are small. The expectation is that the market index is likely to be between 2,600 and 3,300. Thus, the appropriate call to use would be the call with exercise price of 3200. Exercise price of 3200 is chosen so that the loss would occur only when the index value is above (3200 + option price of 88·95) 3288·95 which is close to 3300.

Portfolio value as of November 1, 2001 is INR 568 million.

Index value on October 31 is 2885.

The contract multiplier is 50

If he enters into covered call writing using the call with option exercise price of 3200, the cash flow from each call will be 88·95 × 50 = INR 4447·50

Number of calls written = Portfolio value/exercise price = 568,000,000/(50 × 3200) = 3550

Total cash inflow = 3550 × 4447·50 = 15,788,625

If the index value on November 27 is 2752, drop in the index = (2885 − 2752)/2885 = 4·61% from

November 1 to November 27. Assuming that Bharat fund also loses 4·61%, portfolio value on November 27 will be 568,000,000 × (1 − 0·0461) = INR 541,815,200

Net portfolio value = INR 541,815,200 + 15,798,625 = INR 557,603,825

Question 2.

If Akhil enters into a portfolio insurance strategy using puts, which of these options should he choose? If the value of the index on November 27, 2008, is 2,752, what will be the value of the portfolio on November 27, 2008 ?

Solution :

Portfolio insurance is undertaken to have a minimum value for the portfolio even when the index is falling. Portfolio insurance involves buying put options on the index and if one wants to have a minimum value of portfolio, it is appropriate to use put option that has the highest exercise price. Therefore, put option with exercise price of 3,300 will be chosen.

Number of put options to buy = 568,000,000/(50 × 3300) = 3442

Cash outflow for 3442 options = 3442 × 495·80 × 50 = INR 85,327,180

As calculated above, portfolio value without options = INR 541,815,200

Gain from put option = (3300 − 2752) × 50 × 3442 = INR 94,310,800

Net portfolio value adjusted for put option price paid = 541,815,200 + 94,310,800 − 85,327,180 = INR 550,798,820.

Since covered call writing results in higher portfolio value compared to portfolio insurance, it is better to go for covered call writing.

Question 3.

Since the market is expected to be bearish, Akhil wants to enter into a bearish money spread. How can this be accomplished using call options and what would be the gain from this money spread transaction if the index is at 2,752 on November 27 ?

Solution :

Bearish money spread using call options :

Write a call with low exercise price of 2600 and buy call with high exercise price of 3300.

Net cash inflow = (403·85 − 60·55) × 50 = INR 17,165

If the index value is 2752, low exercise price call will be exercised and high exercise price call will not be exercised.

Loss from exercise of low exercise price call = (2752 − 2600) × 50 = INR 7600

Net gain for each money spread = 17,165 − 7600 = INR 9,565.

Question 4.

Since the market is expected to be bearish, Akhil wants to enter into a bearish money spread. How can this be accomplished using put options and what would be the gain from this money spread transaction if the index is at 2,752 on November 27 ?

Solution :

Bearish money spread using put options :

Buy put with high exercise price of 3300 and sell put with low exercise price of 2600.

Cash outflow = (495·80 − 124·70) × 50 = INR 18555

If the index value on November 28 is 2752, high exercise price put will be exercised and low exercise price put will not be exercised.

Gain from put = (3300 – 2752) × 50 = INR 27,400

Gain from bearish money spread using puts = 27,400 – 18555 = INR 8845

Question 5.

How can Akhil use a butterfly spread using calls and what would be the gain if the index is at 2,752 on November 27 ?

Solution :

Butterfly spread using calls :

Buy one call with high exercise price of 3300, buy one call with low exercise price of 2600, and write two calls with exercise price of 2950.

Cash inflow = (2 × 181·90 – 60·55 – 403·85) × 50 = – 5030

On November 27, index is at 2752.

Low exercise price call will be exercised and the other two will not be exercised. Gain from exercised call = (2752 – 2600) × 50 = 7600

Net gain from butterfly spread = 7600 – 5030 = INR 2570

Question 6.

How can Akhil use a straddle strategy and what would be the gain if the index is at 2,752 on November 27 ?

Solution :

Straddle :

Since the price is expected to be within the range of 2600 and 3300, one should go for written straddle. Written straddle involves writing a call as well as writing a put with the same exercise price. The option would be chosen such that its exercise price is in the middle of the range which is 2950.

Cash inflow = 50 × (181·90 + 306·40) = INR 24,415

At index value of 2752, call will not be exercised but put will be exercised. Loss from written put = (2950 – 2752) × 50 = INR 9900

Gain from straddle = 24415 – 9900 = INR 14,515

Question 7.

How can Akhil use a strip strategy and what would be the gain if the index is at 2,752 on November 27 ?

Solution :

Strip :

Since the price is expected to drop, a written strip is more appropriate. Write 2 puts with exercise price of 2950 and write one call with exercise price of 2950.

Net cash flow = 50 × (2 × 306·40 + 181·90) = INR 39,745

Loss from exercised put = 50 × 2 × (2950 – 2752) = INR 19,800

Gain from strip = INR 19,935

Question 8.

How can Akhil use a strap strategy and what would be the gain if the index is at 2,752 on November 27 ?

Solution :

Strap :

Since the strap is used when the index is likely to increase, will write two calls with exercise price of 2950 and write one put with exercise price of 2950.

Net cash outflow = 50 × (2 × 181·90 + 306·40) = INR 33,510

Loss from written put option = 50 × (2950 – 2752) = INR 9,900

Gain from strap = 33,510 – 9,900 = INR 23,610

Question 9.

Akhil wants to use a calendar spread using a 2,700 call with expiry on November 27 and December 28. The price of the 2,700 call with expiry on November 27 is INR 338·40 and the price of the 2,700 call with expiry on December 28 is INR 402 on November 1, 2008. The 2,700 December call is priced at INR 185·50 on November 27, when the index value is 2,752.

Solution :

Calendar or Time Spread :

Calendar spread involves writing a call option with shorter maturity and buying a call option with longer maturity. Here, strategy is to write call with maturity on November 27 and buy a call with maturity on December 28. Exercise price is 2700.

Net cash outflow = (402 – 338·40) × 50 = INR 3,180

On November 27, written call will be exercised and loss from written call = 50 × (2752 – 2700) = INR 2600

On November 27, sell the call with maturity on December 28 at 185·50. Cash flow from sale = 185·50 × 50 = INR 9275

Gain from calendar or time spread = 9275 – 2600 – 3180 = INR 3,495.

4.7. Principles of Arbitrage

'Arbitrage profits' are riskless profits. You take simultaneous but opposite positions in two markets to reap gains from pricing disparities. Acting on this belief, your friend tried to find the arbitrage profit by trading simultaneously in futures and stock index. He has collected the following information :

1. Present level of stock index : 3000
2. Index future priced at : 2000
3. Risk-free rate of return : 10% p.a.
4. 50% stocks are to pay dividends at 6%
5. The index futures has a multiple of 100
6. The future has six months to expiration

Calculate the fair price of the index future and investor's gain or losses if any and discuss the risk associated in it.

Fair price of Future = 3000 + (3000 × 0·1 × 0·5) – (3000 × 0·06 × 0·5) = 3060 (cost of carry model) = Theoretical or expected value of Future

Index future is under-priced. (Index future priced at 2000)

Go long on future and short on stocks

Multiple =100

Risk free rate of return = 0.1 × 0.5 × 3000 × 100 = 15000

	Future	Stock	Risk-Free Return	Net Pay[illegible]
Index future closes at 6-m 4000	200000	(100000)	15000	1150[illegible]
Index future closes at 6-m 1000	(100000)	200000	15000	1150[illegible]

4.8. Discrete Process

In finance, the binomial options pricing model (BOPM) provides a generalisable numerical meth[illegible] the valuation of options. Essentially, the model uses a "discrete-time" (lattice based) model o[illegible] varying price over time of the underlying financial instrument, addressing cases where the cl[illegible] form Black-Scholes formula is wanting.

The binomial model was first proposed by William Sharpe in the 1978 and formal[illegible] by Cox, Ross and Rubinstein in 1979. For binomial trees as applied to fixed income and intere[illegible] derivatives. BOPM is described in detail in the next section.

4.9. The Binomial Tree Model

The Binomial Model for pricing stock options is a discrete time model. It clearly explai[illegible] fundamental economic principle of option valuation by the risk-less arbitrage method. The bi[illegible] model provides a good analytical approximation for the movement of the stochastic variable an[illegible] be used to value derivative securities when exact formulas for the stochastic process are not re[illegible] available. In this, a single period binomial model will be presented to price a call option and illu[illegible] the risk-free arbitrage principle of valuation. The basic idea is to develop an appropriate hedge por[illegible] to replicate the future returns on the call. The binomial framework is useful for modeling and pri[illegible] real options.

In the single-period model, an investor assumes that the stock price S at the end of the period [illegible] take one of two values : S_u with probability p or S_d with probability $1 - p$. Let K be the current val[illegible] the call option ; C_u and C_d the value of the call at the end of period one if the stock price goes to S_u an[illegible] S_d respectively. In the single period model, the call expires one period away, and hence the payoff [illegible] the call at the expiration date is

$C_u = max\ (0, S_u - K)$ with probability p .... (4.1)

$C_d = max\ (0, S_d - K)$ with probability $1 - p$. .... (4.2)

Whereas, $S_u = uS$ and $S_d = dS$.

Assume that an investor can construct a hedge portfolio of stocks and risk-free bonds. For instan[illegible] one can buy stocks and borrow against them in a proportion that replicates the future payoff of th[illegible] call option. Suppose n is the number of stocks which the investor needs to buy at price S, A is th[illegible] amount of funds that can be borrowed at the risk-free rate r_f (where, $r = 1 + r_f$).

Please note that for an investor not to make any arbitrage profits, it should be $u > r > d$.

If $u, d > r$, the investor could make a profit by borrowing and investing in the stock. On the other hand, if $u, d < r$, the investor would make profit by investing in bonds.

The cost of constructing the hedge at the current time is $nS + A$. The value of this portfolio at the end of one period would be either $(nS_u + Ar)$ with probability p or $(nS_d + Ar)$ with probability $1 - p$. Since the hedge was selected to replicate the call value at the end of one period : $C_u = nS_u + Ar$ and $C_d = nS_d + Ar$. From these two equations, the following expressions for the values of n and A :

$n = (C_u - C_d)/S(u - d)$

$A = (uC_d - dC_u)/(u - d)\ r$ .... (4.3)

Where $C_u = nS_u + Ar$ and $C_d = nS_d + Ar$ .... (4.4)

[illegible] n is called the hedge ratio because it is the number of shares required to balance the portfolio to [illegible] replicate the future payoff of the call. The current value of the call cannot be less than the [illegible] under the no-arbitrage principle. If $C < nS + A$, then the investor can profit by buying the [illegible] and selling the portfolio. Similarly, the current value cannot be greater that the portfolio. The [illegible] is that if $C > nS + A$, the investor sell the call and buy the portfolio. Thus, in equilibrium, the [illegible] value of the option should be exactly equal to the portfolio (i.e., $C = nS + A$). Substituting the [illegible] for n and A yields the following exact formula for the price of the call :

$C = S\ (C_u - C_d)/S(u - d) + (uC_d - dC_u)/(u - d)\ r = nS + A$ .... (4.5)

[illegible] note that the formula is independent of the probability (p) and this value is never used in the [illegible] arbitrage pricing method to value the call. Therefore, it does not really matter what risk [illegible] an investor has. In the binomial approach, the investor can always construct a hedge [illegible] and use it with the replication argument to price a call under equilibrium conditions. Another [illegible] feature is that the model does not indicate how to value the stock, only how to value the [illegible] given the value of the stock. Rearranging the formula for the value of the call option, we [illegible]:

$C = [qC_u + (1 - q)C_d] / r$, where $q = (r - d)/(u - d)$ .... (4.6)

[illegible] $0 < q < 1$, q can be viewed as a probability, and call value (C) can be interpreted as the [illegible] taken with respect to risk-neutral probabilities. When the binomial model is used to derive [illegible] value for a call option on a stock, the time to maturity is divided into small time intervals Δt to get [illegible] better approximation to the Black and Scholes Model. The following values are used to develop the [illegible] period binomial lattice:

$u = e^{\sigma\sqrt{\Delta t}}$, $d = e^{-\sigma\sqrt{\Delta t}}$, $q = (e^{r\Delta t} - d) / (u - d)$ .... (4.7)

Similarly, one can obtain the pricing formula for a put option using the risk-free arbitrage principle.

For Put Option, Value of Put $= P = qP_u + (1 - q)\ P_d$ and $q = (e^{rt} - d) / (u - d)$ .... (4.8)

One of the key properties of an option value is that it can never be negative.

The prior discussion can be generalised as the binomial model and shown as follows :

Probability up $= p = \frac{(e - d)}{(u - d)}$

Probability down $= 1 - p$

$a = e^{rh}$

$d = e^{-\sigma\sqrt{h}}$

$u = e^{\sigma\sqrt{h}}$

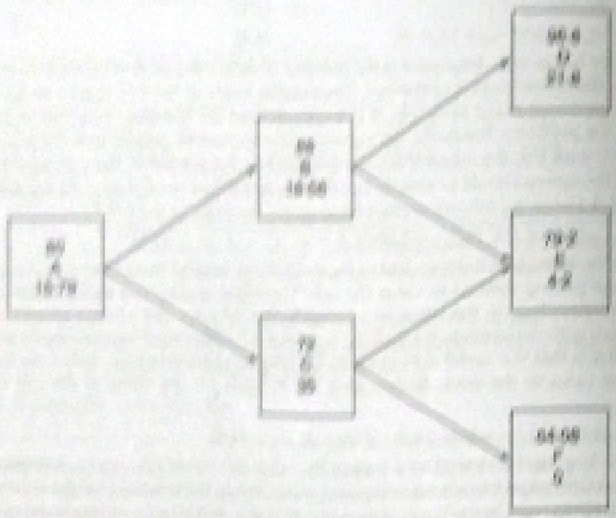
$h = \Delta t$ = time interval as % of year

Example :

A company's stock is currently traded at the market at ₹ 80. A two year American call option on the company's stock with strike price of ₹ 75 is available at the market. The price of the stock in the two years time either moved up or down by 10% in each year. The risk-free interest rate is 8%.

You are required to use two-step Binomial Model to find out the price of the two year American call option on the company's stock.

The situation can be represented in the following way :



Using single period model, the probability of price increase,

$$p = \frac{R - d}{u - d} = \frac{1.08 - 0.90}{1.10 - 0.90} = 0.90$$

Therefore, probability of price decrease = 1 − 0.90 = 0.10

The value of American call option at node D, E and F will be equal to the value of European call option on these nodes.

Value at node D : 96.8 − 75 = 21.8

Value at node E : 79.2 − 75 = 4.2

Value at node F : As stock price is less than strike price, so call has zero value.

Using single period model, the value of call option at node B is

$$C = \frac{C_u \times p + C_d (1 - p)}{R} = \frac{21.8 \times 0.9 + 4.2 \times 0.1}{1.08} = 18.56$$

At node B pay-off from early exercise is ₹ 13, which is less than the value calculated using single period model. Hence, at node B early exercise is not advisable and value of American call option will be ₹ 18.56.

Value at node C is

$$C = \frac{4.2 \times 0.9 + 0 \times 0.1}{1.08} = 3.50$$

At node C, value of early exercise is zero, hence at node C value of call is ₹ 3.50.

Value of American call option at node A is

$$C = \frac{18.56 \times 0.9 + 3.5 \times 0.1}{1.08} = 15.79$$

The value of early exercise at node A is ₹ 5, which is less than the value arrived through single-period binomial model.

Therefore, the value of two-year American call option as per Binomial Model = ₹ 15.79

4.10. Risk-Neutral Valuation

The 'risk-neutral' technique is frequently used to value derivative securities. It was developed by John Cox and Stephen Ross in 1976. The name of the article is "The Valuation of Options for Alternative Stochastic Processes", published in Journal of Financial Economics. Risk-neutral valuation means that you can value options in terms of their expected payoffs, discounted from expiration to the present, assuming that they grow on average at the risk-free rate.

Option value = Expected present value of payoff (under a risk-neutral random walk).

Therefore the real rate at which the underlying grows on average doesn't affect the value. Of course, the volatility, related to the standard deviation of the return of underlying asset, does matter. In practice, it is usually harder to estimate this average growth than the volatility, so we are rather spoiled in derivatives, that we only need to estimate the relatively stable parameter, volatility. The reason that this is true can be ascertained by hedging an option with the underlying asset. So we remove any exposure to the direction of the stock, whether it goes up or down ceases to matter. By eliminating risk in this way we also remove any dependence on the value of risk. End result is that we may as well imagine that we are in a world in which no one values risk at all, and all tradable assets grow at the risk-free rate on average.

4.11. Additional Numerical Problems

1. **Company A can borrow at a fixed rate of 8% or at a floating rate of MIBOR + 150 basis points. Company B can borrow at a fixed rate of 9% or at a floating rate of MIBOR + 50 basis points. Show that these two companies can improve their position through an interest rate swap. What would be the gain to the two parties ?**

Solution :

	Fixed	Floating
Company A	8%	MIBOR + 150
Company B	9%	MIBOR + 50

Since A can borrow cheaply at the fixed rate compared to B, and B can borrow cheaply at the floating rate compared to A, A will borrow at the fixed rate, B will borrow at the floating rate, and the two will swap.

Assume swap rates are 8.5% fixed, and MIBOR + 100 floating.

Position of A :

Borrow at a fixed rate of 8%

Receive fixed rate of 8.5% from B through the swap

Pay floating rate of MIBOR + 100 to B through the swap

Net cost : MIBOR + 1% + 8.5% − 8% = MIBOR + 50, or 1% savings

Position of B :

Borrow at floating rate of MIBOR + 50

Receive floating rate of MIBOR + 100 from A through the swap

Pay fixed rate of 8.5% to A through the swap

Net cost : MIBOR + 50 − (MIBOR + 100) + 8.5% = 8%, or 1% savings.

2. ABC Corporation can borrow at 6% fixed rate or at a floating rate of LIBOR + 50 basis points. GH Corporation can borrow at 8% fixed rate or at a floating rate of LIBOR + 100 basis points. Show that these two corporations can be better off by entering into an interest rate swap. Assume that the comparative advantage is equally shared by the two parties.

Solution :

	Fixed	Floating
ABC	6%	LIBOR + 50
GH	8%	LIBOR + 100

Comparative advantage : ABC = 2% in fixed and ABC = 0.5% in floating.

Net advantage = 1.5%

Split equally, savings for each party will be 0.75%

Let the swap rates are 7% fixed and LIBOR + 75 floating.

ABC will borrow fixed at 6% ; GH will borrow floating at LIBOR + 100, and the two will swap their commitments.

Net cost

ABC :

Pay fixed at 6%

Pay floating at LIBOR + 75

Receive fixed at 7%

Net cost = 6% − 7% + LIBOR + 0.75% = LIBOR − 0.25% or LIBOR − 25, which is 0.75% lower

GH :

Pay floating rate at LIBOR + 100

Receive floating at LIBOR + 75

Pay fixed at 7%

Net cost = LIBOR + 100 − (LIBOR + 75) − 7% = 7.25% or 0.75% lower

3. ***BHP, Australia, can borrow at 8% fixed rate in Australia and at 9% fixed rate in India. Tata Steel can borrow at a fixed rate of 7% in India and at a fixed rate of 11% in Australia. The current exchange rate is AUD 1 = INR 36. Explain how the two companies can engage in a five year currency swap with payments every six months.***

Solution :

	Australia	India
BHP	8%	9%
Tata Steel	11%	7%

Since BHP has advantage in Australia, and Tata Steel has an advantage in India, BHP will borrow at 8% in Australia and Tata Steel will borrow at 7% in India and the two companies will swap the commitments.

Assume swap rates as : BHP pays 8.5% in INR, and Tata Steel pays 9% in AUD.

Net cost for BHP :

Pay 8% AUD in Australia

Pay 8.5% INR under swap

Receive 9% AUD under swap

Net cost = 8.5% INR − 1% AUD

Net cost for Tata Steel

Pay 7% INR in India

Receive 8.5% INR under swap

Pay 9% under swap

Net cost = 9% AUD − 1.5% in INR

Swap payments at initiation

BHP pays AUD 1M and receives INR 36M

On each coupon payment date every 6 months

BHP will pay $\left(\frac{36M\,(8.5\%)}{2}\right)$ = INR 1.53M

Tata Steel will pay $\left(\frac{1M\,(9\%)}{2}\right)$ = AUD 0.045M

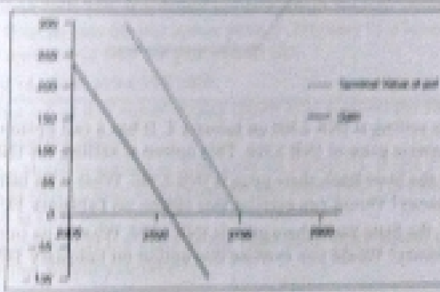
At the end of swap

BHP pays INR 36M, and receives AUD 1M and Tata Steel pays AUD 1M and receives INR 36M.

4. ***A State Bank share is selling for INR 2,500 on January 1. It has a call option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 85. Draw a diagram showing the terminal value of this option as well as the gains from buying this option for possible stock prices of INR 2,300 to INR 3,000.***

Solution :

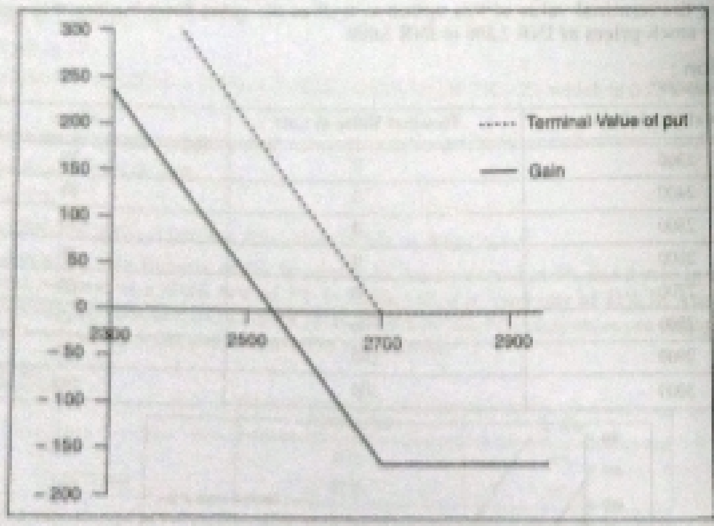
Terminal Stock Price	Terminal Value of Call	Gain
2300	0	−85
2400	0	−85
2500	0	−85
2600	0	−85
2700	0	−85
2800	100	15
2900	200	115
3000	300	215



5. A State Bank share is selling for INR 2,500 on January 1. It has a put option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 160. Draw a diagram showing the terminal value of this option as well as the gains from buying this option for possible stock prices of INR 2,300 to INR 3,000.

Solution :

Terminal Stock Price	Terminal Value of Put	Gain
2300	400	240
2400	300	140
2500	200	40
2540	160	0
2600	100	-60
2700	0	-160
2800	0	-160
2900	0	-160
3000	0	-160



6. **A State Bank share is selling at INR 2,500 on January 1. It has a call option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 85.**

(i) **On February 14, the State Bank share price is INR 2,540. What is its intrinsic value? Is the option in-the-money? Would you exercise this option on February 14? Explain.**

(ii) **On February 14, the State Bank share price is INR 2,620. What is its intrinsic value? Is the option in-the-money? Would you exercise this option on February 14? Explain.**

Solution :

Share price on January 1	INR 2500
Call exercise price	INR 2700
Exercise date	March 31
Call price	INR 85

(i) February 14 Share price INR 2540

Intrinsic value = 0 as it is out-of-money (Stock price is below the exercise price) will not exercise the option.

(ii) On February 14, share price INR 2820

Intrinsic value = (2820 – 2700) = INR 120

The call option is in-the-money as stock price is above exercise price.

Option exercise will depend on the price of the call on February 14. If the call price on February 14 is more than INR 120, it is better not to exercise because selling the option will provide a higher cash flow. In case the call option price on February 14 is lower than INR 120, it is better to exercise the option and take the gain of INR 120.

7. **A State Bank share is selling at INR 2,500 on January 1. It has a put option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 160.**

(i) **On February 14, the State Bank share price is INR 2,540. What is its intrinsic value ? Is the option in-the-money ? Would you exercise this option on February 14 ? Explain.**

(ii) **On February 14, the State Bank share price is INR 2,620. What is its intrinsic value ? Is the option in-the-money ? Would you exercise this option on February 14 ? Explain.**

Solution :

Share price on January 1	INR 2500
Call exercise price	INR 2700
Exercise date	March 31
Put price	INR 160

(i) February 14 Share price INR 2540

Intrinsic value = (2700 – 2540) = INR 160

The put option is in-the-money as stock price is below exercise price.

Option exercise will depend on the price of the put on February 14. If the put price on February 14 is more than INR 160, it is better not to exercise because selling the option will provide a higher cash flow. In case the put option price on February 14 is lower than INR 160, it is better to exercise the option and take the gain of INR 120.

(ii) On February 14, share price INR 2820

Intrinsic value = 0 as it is out-of-money (Stock price is above the exercise price)

Will not exercise the option

8. **A State Bank share is selling at INR 2,500 on January 1. It has a call and a put option with maturity on March 31 with an exercise price of INR 2,700. The call is priced at INR 85 and the put is priced at INR 160.**

(i) **If you believe that the price of the State Bank share would be INR 2,750 on March 31, what action would you take ?**

(ii) If you believe that the price of the State Bank share would be INR 2,650 on March 31, what action would you take?

(iii) If you believe that the price of the State Bank share would be INR 2,530 on March 31, what action would you take?

(iv) If you believe that the price of the State Bank share would be INR 2,400 on March 31, what action would you take?

Solution

Share price on January 1	INR 2500
Call exercise price	INR 2700
Exercise date	March 31
Call price	INR 85
Put price	INR 160

(i) If share price is expected to be INR 2750 on March 31, one can take either of the following three actions :

(a) Buy call which will provide a gain of 2750 – 2700 – 85 = – 35

(b) Write a call which will provide a gain of 85 – (2750 – 2700) = 35

(c) Write a put which will provide a gain of 160.

Best strategy is to write a put.

(ii) If share price is expected to be INR 2650 on March 31, one can take either of the following three actions :

(a) Write call which will provide a gain of 85

(b) Write a put which will provide a gain of 160 – (2700 – 2650) = 110

(c) Buy a put which will provide a gain of (2700 – 2650) – 160 = –110

Best strategy is to write a put.

(iii) If share price is expected to be INR 2530 on March 31, one can take either of the following three actions :

(a) Write call which will provide a gain of 85

(b) Write a put which will provide a gain of 160 – (2700 – 2530) = – 10

(c) Buy a put which will provide a gain of (2700 – 2530) – 160 = 10

Best strategy is to write a call.

(iv) If share price is expected to be INR 2400 on March 31, one can take either of the following three actions :

(a) Write call which will provide a gain of 85

(b) Write a put which will provide a gain of 160 – (2700 – 2400) = – 140

(c) Buy a put which will provide a gain of (2700 – 2400) – 160 = 140

Best strategy is to buy a put.

9. On July 1, call and put options are available on the CNX Nifty index with expiry on September 30. The exercise price of this option is 4,200. The call option is priced at INR 120 and the put option is priced at INR 220. On July 1, the CNX Nifty index is at 4,080. The contract multiplier is 50.

(i) If on September 30, the value of the CNX Nifty index is 4,260, what will be the gain or loss for the call option buyer ?

(ii) If on September 30, the value of the CNX Nifty index is 4,260, what will be the gain or loss for the put option buyer ?

(iii) If on September 30, the value of the CNX Nifty index is 4,260, what will be the gain or loss for the call option writer ?

(iv) If on September 30, the value of the CNX Nifty index is 4,260, what will be the gain or loss for the put option writer ?

(v) On September 12, the CNX Nifty index is at 4,220 and the call option is selling at INR 135. What is the intrinsic value of the call option and the time value of the call option ?

(vi) Can you exercise the call option on the CNX Nifty index on September 12 when the index is at 4,220 ?

Solution :

Nifty Index on July 1	4080
Call and put exercise price	4200
Exercise date	September 30
Call price	INR 120
Put price	INR 220
Contract multiplier	50

(i) On September 30, value of index = 4260

Gain for call buyer = (4260 – 4200 –120) *50 = -3000

(ii) On September 30, value of index = 4260

Gain for put buyer = – 220 *50 = – 11,000

(iii) On September 30, value of index = 4260

Gain for call writer= (4200 – 4260 + 120) *50 = 3000

(iv) On September 30, value of index = 4260

Gain for put writer = 220 *50 = 11,000

(v) On September 12, value of index = 4220

Call price = 135

Intrinsic value = 4220 – 4200 = 20

Time value = Call price – intrinsic value = 135 – 20 = 115

(vi) Do not exercise the call as the time value is positive.

10. On September 1, call and put options are available on the Bank Nifty index with expiry on September 30. The exercise price of these options is 7,480. On September 1, the Bank Nifty index is at 7,350. The call is priced at INR 100, and the put option is priced at INR 240. The contract multiplier for the Bank Nifty index is 50.

(i) If on September 30, the value of the Bank Nifty index is 7,450, what will be the gain or loss for the call option buyer?

(ii) If on September 30, the value of the Bank Nifty index is 7,450, what will be the gain or loss for the put option buyer?

(iii) If on September 30, the value of the Bank Nifty index is 7,450, what will be the gain or loss for the call option writer?

(iv) If on September 30, the value of the Bank Nifty index is 7,450, what will be the gain or loss for the put option writer?

(v) On September 12, the Bank Nifty index is at 7,320 and the put option is selling at INR 250. What is the intrinsic value of the call option and the time value of the call option?

(vi) Can you exercise the put option on the Bank Nifty index on September 12 when the index is at 7,320?

Solution :

Bank Nifty Index on September 1	7350
Call and put exercise price	7480
Exercise date	September 30
Call price	INR 100
Put price	INR 240
Contract multiplier	50

(i) On September 30, value of index = 7450
Gain for call buyer = – 100 *50 = – 5000

(ii) On September 30, value of index = 7450
Gain for put buyer = (7480 – 7450 – 240)*50 = –10,500

(iii) On September 30, value of index = 7450
Gain for call writer = 100 *50 = 5000

(iv) On September 30, value of index = 4260
Gain for put writer = (240 – (7480 – 7450)) *50 = 10,500

(v) On September 12, value of index = 7320
put price = 250
Intrinsic value = 7450 – 7320 = 160
Time value = Call price – intrinsic value = 250 – 160 = 90

(vi) Do not exercise the call as the time value is positive.

11. On September 1, call options are selling at INR 70 on ICICI Bank shares with an exercise price of INR 800 and an exercise date of October 31. ICICI Bank shares are selling at INR 750 on September 1. The ICICI option contract size is 350 shares.

(i) If the share price of ICICI Bank is INR 860 on October 31, what will be the gain or loss for the call option buyer ?

(ii) If the share price of ICICI Bank is INR 860 on October 31, what will be the gain or loss for the call option writer ?

(iii) On September 30, the share price of ICICI Bank is INR 840 and the call option is selling at INR 135. What is the intrinsic value of the call option and the time value of the call option ?

(iv) Can you exercise the call option on ICICI stock on September 30 when the shares of ICICI Bank are selling at INR 840 ?

Solution :

ICICI share price on September 1	750
Call and put exercise price	800
Exercise date	October 31
Call price	INR 70
Contract size	350

(i) On October 31, share price = 860
Gain for call buyer = (860 – 800 – 70)*350 = – 3500

(ii) On October 31, share price = 860
Gain for call writer = (– 860 + 800 + 70)*350 = 3500

(iii) On September 30, share price = 840
Call price = 135
Intrinsic value = (840 – 800) = 40
Time value = 135 – 40 = 95

(iv) Do not exercise the call as the time value is positive.

12. The contract size of Allahabad Bank options is 2,450. Allahabad Bank shares are selling at INR 95 on March 1. Call options and put options are available with expiry on April 29 and an exercise price of INR 100. The volatility of the stock price is 18%, and the risk-free rate is 8%. Using the Black-Scholes options pricing model, calculate the call option price on March 1.

Solution :

$S_t = 95$; $S_X = 100$; $r = 8\%$; $T = \frac{60}{365}$; $\sigma = 8\%$. So,

$$d_1 = \frac{\left(\ln\left(\frac{S_t}{S_x}\right) + \left(\frac{\sigma^2}{2} + r\right)T\right)}{\sigma\sqrt{T}} = -0.48616$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.51859$$

$N(d_1) = 0.3134$; $N(d_2) = 0.2880$

$C = S_t\,N(d_1) - S_X e^{-rt}\,N(d_2) = 1.3485$

13. The contract size of Allahabad Bank options is 2,450. Allahabad Bank shares are selling at INR 95 on March 1. Call options and put options are available with expiry on April 29 and an exercise price of INR 100. The volatility of the stock price is 18%, and the risk-free rate is 8%. Using the Black-Scholes options pricing model, calculate the put option price on March 1.

Solution :

$S_t = 95$; $S_X = 100$; $r = 8\%$; $T = \frac{60}{365}$; $\sigma = 8\%$. So,

$$d_1 = \frac{\left(\ln\left(\frac{S_t}{S_x}\right) + \left(\frac{\sigma^2}{2} + r\right)T\right)}{\sigma\sqrt{T}} = -0.48616$$

$d_2 = d_1 - \sigma\sqrt{T} = -0.55914$

$N(-d_1) = 0.6866 ; N(-d_2) = 0.7120$

$C = S_X e^{-rT} N(-d_2) - S_0 N(-d_1) = 5.042$

14. Assume that Asian Paints stock is currently selling for INR 1,750. There is a put option on Asian Paints with a maturity of 90 days and an exercise price of INR 1,800. The volatility of the stock price is 15%, and the risk-free rate is 9%. Form a risk-less hedge and calculate the price of a call option and a put option on the stock using Black-Scholes model.

Solution :

$S_0 = 1750 ; S_X = 1800 ; r = 9\% ; T = \frac{90}{365} ; \sigma = 15\%$. So,

Riskless hedge involves writing one call and buying $N(d_1)$ shares, or buying one put and buying $N(-d_1)$ shares.

$$d_1 = \frac{\left(\ln\left(\frac{S_0}{S_X}\right) + \left(\frac{\sigma^2}{2} + r\right)T\right)}{\sigma\sqrt{T}} = -0.043$$

$d_2 = d_1 - \sigma\sqrt{T} = -0.1175$

$N(d_1) = 0.4828$

$N(d_2) = 0.4532$

$N(-d_1) = 0.5172$

$N(-d_2) = 0.5468$

$C = N(d_1) S_0 - S_X e^{-rT} N(d_2) = 47.06$

$P = S_X e^{-rT} N(-d_2) - S_0 N(-d_1) = 57.56$

Riskless hedge involves writing one call and buying 0.4828 shares, or buying one put and buying 0.5172 shares.

15. (a) *Assume that a security is selling at INR 400 and call and put options are available on the stock with a maturity of 90 days and an exercise price of INR 420. The call is selling at INR 6, and the risk-free rate is 8% per annum. According to put–call parity, what should the put sell for ? Assume that the stock does not pay any dividends during the life of the option.*

Solution :

Share price	INR 400
Call and put maturity	90 days
Call and put exercise price	INR 420
Call price	INR 6
Risk-free rate	8%

Put price according to put-call parity :

$P = C - [S_0 - S_X e^{-rT}] = 6 - [400 - 420\, e^{-0.08*(90/365)}] = \text{INR } 17.60$

(b) Assume that a security is selling at INR 400 and call and put options are available on the stock with a maturity of 90 days and an exercise price of INR 420. The call is selling at INR 6, and the risk-free rate is 8% per annum. According to put–call parity, what should the put sell for? Assume that the stock will pay a dividend of INR 5 per share after 30 days.

Solution :

Share price	INR 400
Call and put maturity	90 days
Call and put exercise price	INR 420
Call price	INR 6
Risk-free rate	8%
Dividend to be paid after 30 days	INR 5

Put price according to put-call parity :

$P = C - [S_0 - De^{-rt} - S_X e^{-rT}] = 6 - [400 - 5e^{-0.08*30/365} - 420e^{-0.08*(90/365)}] = \text{INR } 22.76$

(c) Assume that the BSE Sensex Index is at 16,500 and call and put options are available on the index with a maturity of 90 days and an exercise price 17,250. The index multiplier is 10. The call is selling at INR 25, and the risk-free rate is 8% per annum. According to put-call parity, what should the put sell for? Assume that the index has a dividend yield of 2%.

Solution :

Sensex value	16500
Call and put maturity	90 days
Call and put exercise value	17,250
Contract multiplier	10
Call price	INR 25
Risk-free rate	8%
Dividend yield	2%

Put price according to put-call parity :

$P = C - [S_0 - S_X e^{-(r-d)T}] = 25 - [16500 - 17250 e^{-(0.08-0.02)*(90/365)}] = \text{INR } 521.67$

16. Jet Airways requires 2,000,000 barrels of aviation fuel every month. Since the price of aviation fuel depends on the price of crude oil, Jet Airways faces price risk. At the beginning of each month, Jet Airways goes for a long hedge in crude oil futures contract for 2,000,000 barrels, with expiry by the end of that month.

(i) What is meant by a long hedge ?

(ii) What is the purpose of the long hedge undertaken by Jet Airways ?

(iii) Would Jet Airways be able to completely eliminate the price risk of aviation fuel ? Explain.

Solution :

(i) A long hedge means that the hedger needs to purchase a commodity or asset at a future time, and is using futures contracts to hedge the risk of price increase.

(ii) Jet Airways requires aviation fuel every month, and fuel prices are highly volatile. In order to forecast future cash flows more efficiently, Jet Airways will undertake a long hedge using futures

Solution :

$S = 210$, $X = 220$, $t = 167/365$ years, $R_f = 10\%$, $SD = \text{sqrt}(0.2)$, $d_1 = 0.3025$, $d_2 = 0$, $N(d_1) = 0.61$, $N(d_2) = 0.5$, C = Call Value = ₹ 24.89

20. **A spectacular expects stiff movements in the Stock price of XYZ Ltd. from the current ₹ 44.50 in the next 3 months. He is not sure of the direction in which the price change will take place. He wants to adopt a strategy suitable for his view. The following is the information relating to 3 month Options on the Stock of XYZ Ltd. In Call Options, for a Strike Price of ₹ 44.25, ₹ 44.50 and ₹ 44.75, the premiums are 55 paisa, 25 paisa and 5 paisa respectively. In Put Options, for a Strike Price of ₹ 44.25, ₹ 44.50 and ₹ 44.75, the premiums are 5 paisa, 10 paisa and 40 paisa respectively. The spectacular believes that increase in the Stock price as likely as decrease. You are required to —**
   - (i) **State the strategies the spectacular may adopt.**
   - (ii) **Calculate the maximum loss the spectacular may incur if his expectations do not come true, for each of the strategies mentioned above.**
   - (iii) **Calculate the break-even price for each of the strategies mentioned above.**

Solution :

(i) The strategies are Straddle and Strangle.

(ii) Straddle : Total premium paid = 0.25 + 0.10 = ₹ 0.35

Strangle : Total premium paid = 0.05 + 0.05 = ₹ 0.10

(iii) Straddle : Break-even price

Strike price – Premium = 44.50 – 0.35 = ₹ 44.15

Strike price + Premium = 44.50 + 0.35 = ₹ 44.85

Strangle : Break-even price

Higher Strike price + Premium = 44.75 + 0.10 = ₹ 44.85

Lower Strike price - Premium = 44.25 – 0.10 = ₹ 44.15

4.12. Some Relevant Concepts & their narrations

1. **Why does commodity price risk need to be hedged by a firm ?**

   One of the major issues for any firm is forecasting future cash flow so that it can arrange appropriate financing. Since commodity prices can be highly volatile, a firm's future cash flows can also be volatile. The firm will therefore have difficulty finding appropriate financing if cash flows turn out to be too low. Hedging is done in this case so that the firm can fix a price for a future transaction; this will help to forecast future cash flows.

2. **How does interest rate risk affect a firm ?**

   Interest rate risk can affect a firm especially when it is planning to borrow at a future time, and the rate at which it can borrow is uncertain. If the firm has issued floating-rate bonds, these floating bonds can also lead to interest rate risk. This is because the firm will not know the interest that needs to be paid at the next reset period.

3. **Impact of exchange rate risk on the value of a firm.**

   Since the economic value of a business is the present value of all future net cash flows, exposure to foreign exchange can have an impact on the current as well as future cash flows. If a company hedges its currency exposure, its future cash flows can be made more certain. Hence value can be calculated more easily.

4. ***What is meant by hedging ? How does hedging improve the effectiveness of the operations of a business ?***

   Hedging means reduction of risk. A business faces risk of changing commodity prices, changing interest rates, and fluctuating exchange rates. Because of these risks, future cash flows become uncertain. Through hedging, a company can reduce the uncertainty in its future cash flows. This will enable the business to plan its operations more efficiently.

5. **What factors determine the need to hedge ?**

   Some factors that determine the need to hedge are :

   (a) Amount of exposure. *If exposure is too small, hedging may involve higher costs in relation to hedging benefits ;*

   (b) Price volatility. *If price volatility is very low, hedging may not be necessary ;*

   (c) Liquidity of instrument used to hedge. *If liquidity is very low, it may be difficult to trade in the market at fair value, which will reduce hedging effectiveness ;*

   (d) Ability to forecast price movement. *If one can forecast price movement, it will be easy to formulate appropriate hedging strategies that will provide benefits.*

6. **What is risk-free rate of interest ?**

   Generally the Government Treasury Bills provide a risk-free interest rate since there remains no default risk in it. Risk-free interest rate provides a known terminal value at the time when an investment is made.

7. **What is meant by LIBOR ? Why can it be used as a proxy for risk-free rate while taking loans from a bank ?**

   LIBOR stands for London Interbank Offer Rate, which is the rate charged by a bank when another bank borrows money in the Euro Market. It is often used as risk-free rate for bank loans, because loans taken by banks are inherently risk-free.

8. **Forward contracts are used to hedge future uncertainty. With respect to commodities, when would a party enter into a long forward contract to buy and when would a party enter into a short forward contract, i.e., a contract to sell ?**

   Since producers of commodities are concerned about possible price decreases which would provide lower cash flows, producers enter into short forward contracts in order to sell at a future time at a known forward price. Since users of commodities are concerned about possible price increases, which would require high cash flows, users enter into long forward contracts to buy at a future time at a known forward price.

9. **What are the Meanings of FRA and cash settlement ?**

   In an FRA, the agreement is made between the party who plans to borrow at a future time, and another counterparty in which the interest rate at which the borrowing will take place is fixed. Note that counterparty need not be the lender in this case.

   The borrower will actually borrow at the market rate. If the agreed rate is below the market rate, the counterparty must compensate the borrower for the increased interest payment based on the market rate instead of at the agreed rate. If the agreed rate is above the market rate, the borrower has to compensate the counterparty for the ensuing difference. This compensation will be done in cash. This process is known as a cash settlement in FRA.

11. **An Indian vegetable merchant exports fruits and vegetables to Singapore, pricing them in Singapore dollars. What price risks does he face and how can he reduce the risks ?**

   Since fruits and vegetables are priced in Singapore dollars, the amount of INR that the exporter receives will depend on the variability in exchange rate. If INR appreciates against SGD, the

Indian exporter will receive less INR. On the other hand if INR depreciates against SGD, it would receive more INR. Thus, depending on his expectations about exchange rate movements, an exporter might decide to use currency forwards to hedge. Hedging is beneficial only if INR is expected to appreciate against SGD, and an exporter can enter into a forward contract to sell SGD at a future time. If it is expected to depreciate further than the forward rate, hedging will not be necessary.

11. What is meant by basis and basis risk ?

Basis is the difference between the spot price and the futures price at any given time.

In order to avoid arbitrage, basis on the maturity date of futures should be zero. If it is zero, the hedger will achieve a perfect hedge, and the final price will be the same as the futures price originally contracted.

If the basis on maturity date of futures is not zero, then there will be basis risk, the hedge will not be perfect, and the final price will not be known with certainty at the time the contract is entered into.

12. Under what conditions would a hedger not be able to get a perfect hedge using futures ?

Perfect hedge is not attained when there is basis risk or when the final basis is not zero. This may happen when :

(a) The asset underlying the futures is different to the asset exposed to.

(b) The maturity date of futures is different from the date of end of exposure to the asset.

(c) The quantity of asset exposure is not an integer multiple of the contract size of the futures.

13. What type of hedging would be undertaken under the following circumstances ?

(i) An Indian company has exported products to the USA and expects to *receive* USD 10 million from the importer in the USA in three months' time. Indian rupee futures are available through banks in India.

(ii) A tyre manufacturer wants to reduce the price risk of rubber, which they use in the manufacture of rubber, and rubber futures are available in MCX India.

(iii) An oil producer would like to reduce the unknown price risk of crude oil. Crude oil futures are available in the NCDEX.

(i) At the end of 3 months, Indian Company needs to sell USD 10 million, and hence will take a short position in USD futures.

(ii) Since the manufacturer will be using rubber, they need to purchase at a future time. Hence they will take a long position in rubber futures.

(iii) Oil producers need to sell oil at a future time, and will therefore take a short position in oil futures.

14. Explain what happens to the position of a short hedger if the basis strengthens and if the basis worsens.

If basis strengthens, the difference between spot price and futures price decreases over time and because of that a short hedger's position will improve—this will lead to gains. On the other hand, if basis weakens, the short hedger's position will worsen.

15. If the minimum variance hedge ratio is 1, does it mean that you can completely eliminate price risk ?

Minimum variance hedge ratio is 1. This means that for each rupee of exposure, one rupee of futures should be used. However, this does not mean that a perfect hedge will arise, because there will still be basis risk.

16. What is the rationale for introducing currency futures ?

With the increasing exchange rate volatility of the Indian rupee, hedging needs for Indian businesses have increased. Even though forward contracts are available, there are a number of restrictions imposed by the RBI in entering into currency forward contracts with banks. With forward contracts, only known exposures and not anticipatory exposures can be hedged. Moreover, banks will enter into forward contracts only when the exposure amount is substantial. If exposure is small, a bank may not enter into a forward contract as it would not be economical to do so for the banks. Furthermore, speculators who are likely to provide stability to the currency market through price discovery are not allowed in the forward market. Since futures can alleviate all these problems, currency futures are introduced.

17. Under what conditions would you make arbitrage profits ?

In order for arbitrage profits to be made, it is necessary that one should execute the trade in both the spot market and the futures market simultaneously. Unfortunately, it may not be possible, because transacting a large amount in spot currency market takes time, whereas the transaction in futures market can be executed comparatively faster.

18. What is the motivation behind an interest rate swap ?

The major motivation behind interest rate swaps is that both parties to the swap will be able to reduce funding costs.

19. What is Currency swap ?

A currency swap is an agreement between two parties, whereby one agrees to exchange a specified cash flow in one currency for a specified cash flow in another.

20. What is a swaption & What are the uses of swaptions ?

A swaption is an option to enter into a swap at a future time. Swaptions can be used to bring in a swap when hedging becomes necessary, or to remove an existing swap when it becomes unattractive. It can also be used to enhance the yield on underlying positions by selling a swaption.

21. State major differences between an interest rate swap and a currency swap.

In interest rate swap, there is only a notional principal. Principal is never exchanged between parties. Only periodic interest payments are exchanged, and both parties deal in a single currency.

In a currency swap, principal amount is exchanged at the start as well as at the end of the swap, at the exchange rate which prevails at the swap's beginning. In addition, periodic interest payments in two different currencies are also swapped.

22. Under what circumstances would you enter into a forward swap ?

A forward swap is one that commences at a future date. This can be used by companies which are planning to enter into swap at a future time.

23. A swap contract can be considered as a series of forward contracts. Explain why.

A swap contract is a long-term contract which periodic reset dates whenever there is a fixed-rate floating-rate swap. This can be considered as a series of forward contracts starting at each reset date and terminating at the end of the reset period. Even in a fixed-rate swap, it can be considered as a series of forward contracts starting on the day from the last swap payment, and ending on the day of the next swap payment.

24. Since exchange of payments takes place in different currencies in a currency swap, a currency swap involves currency risk. Then why would anyone enter into currency swaps ?

Currency swaps are useful when a company has cash inflow in the currency to which it has swapped, so that interest payments in the currency that is to be paid can be paid from the cash inflow in that currency. If there is no cash inflow in that currency, there will be currency risk.

25. **What is the rationale behind using a commodity swap ?**

A commodity swap involves payment exchange between two parties at set time periods. One leg of a swap is determined by the price of the commodity; the other one usually involves a fixed rate. This swap provides a known price of the commodity, avoiding price volatility. Commodity price risk is transferred to the counterparty.

26. **What is the rationale behind using an equity swap ?**

In an equity swap, one party agrees to make a series of payments determined by return on equity to another party in return for a cash flow based on either fixed or floating rate, or on equity. Through equity swap, one can either obtain exposure to equity without owning it, or can avoid exposure to changing prices of equity securities.

27. **How do banks can manage their gap using interest rate swaps ?**

For banks, the gap arises because it owns assets, which are usually long-term loans with a fixed rate and its liabilities and deposits are usually short-term. When interest rate increases, the value of assets will fall relatively more than the value of liabilities. In order to avoid this, a bank can enter into interest rate swaps, converting fixed-rate assets into floating-rate ones in order to match short-term liabilities.

28. **Some relevant terms :**

(a) **Call option :** Call option provides the right to buy the underlying asset at the exercise price on or before the exercise date.

(b) **Put option :** Put option provides the right to sell the underlying asset at the exercise price on or before the exercise date.

(c) **European options :** European options can be exercised only on the exercise date.

(d) **American options :** American options can be exercised at any time before and including the exercise date.

(e) **Exercise price :** Exercise price is the price at which the asset can be bought in the case of call and sold in the case of put, if the option is exercised.

(f) **Exercise date :** Exercise date is the last date by which the option can be exercised, and indicates the life of the option.

(g) **Option premium :** Option premium is the price that is paid by the buyer of the option to the seller of the option.

29. **Mention the difference between the positions of an option buyer and an option writer.**

An option buyer gets the right to exercise the option, and will exercise only when it is beneficial for him. If it is not beneficial, he will let the option expire without exercise. The maximum loss for the option buyer is the price paid for the option, while gain from options can be high. An option writer has an obligation to fulfill if the buyer decides to exercise. The maximum gain for an option writer is the option price received and losses can be high.

30. **Mention the circumstances under which an option would be exercised.**

An option will be exercised only when it is beneficial for the buyer. A call option will be exercised if the market price of an asset is greater than the exercise price, while a put option will be exercised if the exercise price is greater than the market price of the asset.

31. **Under what circumstances one would buy a call option ?**

A call option provides the right to buy the underlying asset at the exercise price and a trader will exercise the option only when the market price of the underlying asset is greater than the exercise price. However, the trader needs to pay a premium (C) to buy this call option. If the market price moves beyond the sum of the exercise price and the premium paid for the call, that is, $S_X + C$, the

trader will make money. Thus, one would buy a call only when he expects that the market price of the underlying asset will be greater than $S_X + C$.

32. **Under what circumstances one would buy a put option ?**

A put option provides the right to sell the underlying asset at the exercise price and a trader will exercise the option only when the market price of the underlying asset is less than the exercise price. However, the trader needs to pay a premium (*P*) to buy this put option. If the market price moves below the difference between the exercise price and the premium paid for the put, that is, $S_X - P$, the trader will make money. Thus, one would buy a put only when he expects that the market price of the underlying asset will be less than $S_X - P$.

33. **Under what circumstances one would write a call option ?**

The writer of a call option needs to sell the underlying asset at the exercise price if the call option is exercised and will receive the call premium (C) when he writes the option. The call buyer will exercise the option only when the market price of the underlying asset is greater than the exercise price. However, the call writer will lose only when the market price moves beyond the sum of the exercise price and the premium paid for the call, that is, $S_X + C$. Thus, one would write a call only when he expects that the market price of the underlying asset will not be greater than $S_X + C$.

34. **Under what circumstances one would write a put option ?**

The writer of a put option needs to buy the underlying asset at the exercise price if the put option is exercised and will receive the put premium (*P*) when he writes the option. The put buyer will exercise the option only when the market price of the underlying asset is less than the exercise price. However, the put writer will lose only when the market price moves below the difference between the exercise price and the premium paid for the put, that is, $S_X - P$. Thus, one would write a put only when he expects that the market price of the underlying asset will not be less than $S_X - P$.

35. **State minimum and maximum values of a call option and a put option.**

For a call option, minimum value is the intrinsic value of the call which will be zero if it is out-of-money and will equal the difference between the market price and the exercise price if it is in-the-money. The maximum value of a call will be the market price of the underlying asset as no one will pay more than the market price of the underlying asset to buy the call.

For a put option, minimum value is the intrinsic value of the put which will be zero if it is out-of-money and will equal the difference between the exercise price and the market price if it is in-the-money. The maximum value of the put will be the exercise price as no one would be willing to pay more than the exercise price for buying the underlying asset.

36. **State the advantage of writing a covered call over writing a naked call.**

Writing a naked call means that the trader does not own the underlying asset, and this strategy will provide a constant profit of the option premium received when the market price of the asset is below the exercise price. This will lead to losses if the market price goes above (exercise price + option premium) and losses can be high if asset prices increase by a large amount. In covered call writing, the trader owns the underlying asset and writes a call option when he believes that the market price will not go above the exercise price. Covered call writing will reduce losses from owning the asset by the call premium received, and can lose if the market price exceeds (exercise price + option premium).

37. **How can one achieve portfolio insurance using put options ?**

Portfolio insurance strategy provides a known minimum value for the portfolio in case stock prices fall. By buying put options on indices whose return has the highest correlation with the

portfolio return, any loss on the index will be exactly offset by gains from the put option, res[illegible] in a minimum value of the portfolio which will be based on the exercise price of the option. If [illegible] market price increases, the portfolio value will increase.

38. What is the concept behind calendar spread transactions ?

In a calendar spread transaction, a trader will take positions in two options on the same [illegible] with the same exercise price but different exercise dates. Usually, calls are written with [illegible] maturity, and calls with longer maturity are bought. The value of the calendar spread at the expiry date of a shorter-term option will be the time value of the longer-term option at that [illegible]. A calendar spread will provide profits for a range of stock prices that are close to the exe[illegible] price of a long-term option. If the stock price moves away from this range, a calendar spread [illegible] lead to losses.

39. When one would enter into a butterfly spread transaction ?

A butterfly spread involves positions in options with three different exercise prices, but with the same exercise date. Options with high exercise price and low exercise price are bought and two options with the medium exercise price are sold to create a butterfly spread. A butterfly spread is used when the market price is expected to be close to the medium exercise price. Medium exercise price is usually the current stock price and the market is not expected to move substantially from the current stock price in either direction.

40. When one would enter into a bought straddle and a written straddle transaction ?

Bought straddle involves buying one call and buying one put on the same underlying asset with the same exercise price and exercise date. Bought straddle is useful when the market price is expected to move substantially in either direction, in which case it will provide profits.

Written straddle involves writing one call and writing one put on the same underlying asset with the same exercise price and exercise date. Written straddle is useful when the market price is not expected to move substantially and is likely to be within a small range.

41. When one would enter into a bought strip and a written strip transaction ?

A bought strip consists of a long position in one call and two puts, with the same exercise price and exercise date. This is used when the probability of price increase is smaller than the probability of price decrease, and the price is expected to move substantially in either direction.

Written strip consists of a short position in one call and two puts with the same exercise price and exercise date. This is used when the probability of a price increase is smaller than the probability of a price decrease, and the price is expected to stay within a given range.

42. When one would enter into a bought strap and a written strap transaction ?

When a trader takes a long position in two calls and one put with the same exercise price and exercise date, he is engaging in a bought strap transaction. It is used when the probability of price increase is larger than the probability of decrease, and the price is expected to move substantially in either direction.

Written strap is created when a trader takes a short position in two calls and one put with the same exercise price and exercise date. It is used when the probability of a price increase is higher than the probability of a decrease, and the price is expected to stay within a given range.

43. When one would enter into a strangle transaction ?

Strangle involves the purchase of a call and a put with the same exercise date, but different exercise prices. The call usually has higher exercise price. As long as the market price moves substantially away, Strangle will provide gain. Downside risk may be lower, but upside gain is possible only if the price moves to a high value.

44. State major advantage of binomial options pricing models as compared to the Black - Scholes Model.

The major advantage of the binomial model is that it makes no assumptions about how the returns on underlying assets are generated, and can account for any kind of price movement as long as the price can move either up or down by a known percentage at any given interval. On the other hand, the Black-Scholes model assumes that prices follow a log-normal distribution.

45. Explain the concepts of interest rate caps, interest rate floors, and interest rate collars and their uses.

Interest rate cap is an option in which an interest rate is specified as the exercise rate. It is useful for borrowers who plan to borrow at a future time, or those who have undertaken floating rate loans. These fix the highest interest rate that a borrower will pay.

Interest rate floor is an option in which an interest rate is specified as the exercise rate. It is useful for investors who plan to invest at a future time, or those who have lent at floating rates, as this fixes the lowest interest rates that can be received.

Interest rate collar is a combination of a long interest rate cap and a short interest rate floor, such that net investment is zero. Interest rate collar is used by borrowers who expect interest rates to increase. The cost of an interest rate collar is lower than that of a cap.

46. Explain the meanings of the terms : (i) Delta hedging, (ii) Gamma hedging, and (iii) Vega hedging.

(i) **Delta hedging :** Delta hedging means that the number of stocks that need to be bought for each call written equals the delta of the option, where delta refers to the rate of change in option price with respect to changes in the underlying asset price . Delta hedging requires a position in the underlying asset and options simultaneously. As delta changes with changes in the price of the underlying asset, delta hedging needs to be dynamic. It provides protection against small changes in the underlying asset price.

(ii) **Gamma Hedging :** Gamma refers to changes in delta with respect to small changes in the prices of the underlying asset. Since delta changes along with changes in asset price, one can enter into gamma hedging to make the portfolio gamma-neutral. When portfolios are gamma-neutral, the gamma will be zero and will keep the delta to be the same when there is a small price change in the underlying asset. This requires additional options and additional underlying assets. Gamma hedging provides protection where there are large price changes in the underlying asset price.

(iii) **Vega Hedging :** Vega refers to change in the value of portfolio with respect to change in the volatility of the asset price. If vega is large, sensitivity of portfolio value to small changes in volatility could be high. Vega hedging means that additional options are used to make the vega of the portfolio be zero, or to render the portfolio vega-neutral.

47. Explain the meaning of the terms : delta, gamma, theta, vega, and rho of options.

Delta refers to the rate of change in the option price with respect to the price of the underlying asset, or : $\Delta = \frac{\delta C}{\delta S}$

Gamma is the rate of change in the value of the option portfolio with respect to its delta, or : $\gamma = \frac{\delta C}{\delta \Delta}$

Vega is the rate of change in the value of the option portfolio with respect to the volatility of the underlying asset, $\Lambda = \frac{\delta C}{\delta \sigma}$

Theta refers to the rate of change in the value of the option portfolio with respect to the time to maturity, $\theta = \frac{\delta C}{\delta T}$

Rho refers to the rate of change in the value of the option portfolio with respect to the risk free interest rate, $\rho = \frac{\delta}{\delta r}$

48. State the use of index futures for speculation.

If one expects the market to do well in the next few days, then one can buy or go long in index futures. If the index value increases after a few days, the futures price at that time will be higher than the contracted futures price and the speculator can close out the position and can make money. If the expectation is that the market will fall in the near future, a short position in index futures should be taken. If the market falls, the futures price would also fall and when the speculator closes out the position at the lower futures price, he will make money.

49. Explain in brief the difference between using stock futures and stock index futures for speculative purposes.

Stock futures are used to speculate on the direction of price movement of an individual stock. Index futures are used to speculate on the direction of movement of market as a whole measured in terms of the index value.

50. State the meaning of index arbitrage.

Index arbitrage is undertaken when index futures are priced in the market which is different from the theoretical price. Index futures are priced theoretically as $F = S.e^{(r-d)T}$.

If the actual futures price in the market is higher than the theoretical value, a short position in futures and a long position in the index would provide arbitrage profit when the futures price in the market converges to the theoretical price. This is the same in reverse — a long position in futures and a short position in the index should be taken when the actual futures price is lower than the theoretical value.

Summary

During the last decade, derivatives have emerged as innovative financial instruments for their risk aversion capabilities. There are two types of derivatives: commodity and financial. Basically derivatives are designed for hedging, speculation or arbitrage purpose. Derivative securities are the outcome of future and forward market, where buying and selling of securities take place in advance but on future dates. This is done to mitigate the risk arising out of the future price movements. Future contracts are standardised having more liquidity and less margin payment requirements while vice versa is the case of forward contracts. Based on the nature of complexity, these are of two types: basic and complex. In basic financial derivatives, the focus is only on the simplicity of operation i.e. forward, future, option, warrants and convertibles. A forward contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price, whereas a futures contract is an agreement between two parties to buy or sell a specified quantity of assets at a predetermined price at a specified time and place. Futures contracts are standardised and are traded on an exchange. Option is a contract between two parties which gives the right (not obligations) to buy or sell a particular asset, at a specified price, on or before, a specified date. Option holder is the person who acquires the right to buy (hold), while option seller/writer is the person who confers the right. In a call option, the holder has the right to buy an asset at a specified price and time, while in case of a put option, the holder has the right to sell an asset at specified time and price. The price at which an option is exercised is known as exercise price or strike price and the date is known as expiration date. In case of an American option, option can be exercised on or before the expiration date but European option can be exercised only on date of expiration, warrants are also options which give the holders right to purchase a specified number of shares at a fixed price in a fixed time period. On the other hand, convertibles are hybrid securities which are also called equity derivative securities with features of fixed as well as variable return attributes. Swaps are latest derivatives which can be exchanged for something. There are two types of swaps: interest rate swaps and currency swaps. In interest rate swap, one party agrees to pay the other party interest at a fixed rate on a notional principal amount and in which receives interest as a floating rate on the same principal notional amount for a specified period. Currency swap involves an exchange of cash payment in one currency for cash payments in another currency. Future value of cash flows is required for calculation purposes.

The various factors which influence the price of any option are exercise/strike price, expiry date of the option, expected price volatility, risk-free rate of return, expected cash payments of the stock etc. The option price is directly proportional to the current price of the underlying. In case of a call lower the strike price, higher will be the value of option and vice versa in put case. Longer the expiry period of the option, the higher will be the option price. There are two models to express pricing of option. One time binomial model is based on assumptions of no market friction, transaction costs, no bid/ask spread, no margin requirement, no restriction on short-selling etc. It predicts the value of an option with the help of two possible outcomes either upward or downward movements of prices of stocks. The one time binomial model can be expanded to a multiple time Binomial model with some modifications. Based on certain assumptions and in view of the log normal distribution of returns, Black and Scholes developed a model in 1973 known as B-S model which is programmable into a computer and call and put prices can be found out easily by inputting some variables into the formula. Sensitivity calculations are possible to know the effect of rate of changes in some variable on value of an option. These are known as Greeks and the value can be calculated using derivatives/differential calculus. Similarly, implied volatility is the volatility that the option price implies.

There are strategies for the investors who use both call and put on the same asset. Straddles, strangles, strips and straps are such strategies. Straddle strategy involves simultaneous buying a call and put with the same exercise price and expiration date. A long straddle is created by buying an equal number of calls and puts with same stock at same exercise price same expiration date. On the other hand a short straddle involves simultaneous sale of a call and put the same stock, at the same strike price and exercise date. Strip strategy involves a long position with one call and two put options with same exercise price and expiration date. A strap is created by a long position with two calls and one put options with the same price and same expiration date. A strangle is just similar to a straddle except one difference i.e. in a straddle, the position is taken on the same strike price while in strangle the position is taken with different exercise prices. The investor buys a put and a call with the same expiration date but with different exercise price i.e. the exercise price of put and call is different strangle may be of long and short strangles. Different strategies have different profit patterns.

Key Words

Derivatives are the financial instruments whose pay-off is derived from some other underlying asset.

Forward contract is an agreement between two parties to exchange an asset for cash at a predetermined future date for a price specified today.

Future contracts are forward contracts traded on organised exchanges in standardised contract size.

Delivery price : The specified price in a forward contract will be referred to as the delivery price. This is decided or chosen at the time of entering into forward contract so that the value of the contract to both parties is zero. It means that it costs nothing to take a long or a short position.

Forward price : It refers to the agreed upon price at which both the counter parties will transact when the contract expires.

Future spot price : The spot price of the underlying asset when the contract expires is called the future spot price, since it is market price that will prevail at some futures date.

Long position : The party who agrees to buy in the future is said to hold long position. For example, in the earlier case, the bank has taken a long position agreeing to buy 3-month dollar in futures.

Short position : The party who agrees to sell in the future holds a short position in the contract.

Spot-price : This refers to the purchase of the underlying asset for immediate delivery. In other words, it is the quoted price for buying and selling of an asset at the spot or immediate delivery.

Option is the right (not obligation) to buy or sell an asset on or before a pre-specified date at a predetermined price.

Call option is the option to buy an asset.

Put option is the option to sell an asset.

Exercise price is the price at which an option can be exercised. It is also known as strike price.

European option can be exercised only on the expiration date of option.

American option can be exercised on or before the expiration date of option.

In-the-money : An option is called in-the-money if it benefits the investor when exercised immediately.

Out-of-the money : An option is said to be out-of-the money if it is not advantageous for the investor to exercise it.

At-the-money : When holder of an option neither gains nor looses when the exercises the option.

Option premium is the price that the holder of an option has to pay for obtaining a call or put option.

Intrinsic value of an option is the fundamental or underlying value which denotes a difference between the market price and the exercise price of the underlying asset.

Time value of an option is the difference between the value of an option at a particular time and its intrinsic value at the time.

Implied volatility is the volatility that the option price implies. It is different from actual volatility observed in market place.

Butterfly spread is a spread by taking a particular position in options with three different strike prices. In this strategy the investor buys a call with a relatively lower exercise price say K1 and higher strike price K3 and selling two call options with an exercise price say K2 which lies between K1 and K3.

Combination is a position involving both calls and puts on the same underlying asset.

Covered call option writing is a technique used by investors to help funding their underlying positions, which is used in equity market.

Diagonal spread is a combination of both types of vertical as well as horizontal spreads in which both expiration dates and the strike prices of calls are different.

Horizontal/time/calendar spread is the spread which is created by selling an option with a relatively shorter period to expiration and buying an option of the same type with a longer period to expiration.

Long straddle is the strategy which is created by an equal number of calls and puts with the same stock at the same exercise price and the same expiration date.

Short straddle is the simultaneous sale of a call and a put on the same stock at the same strike price and on the same exercise date.

Spread is a trading strategy which can be created by taking a position into or more options of the same type i.e. by combining two or more calls or two or more puts.

Straddle is a combination strategy in which the position is taken in the same number of puts and calls with the same strike prices.

Strangle is a position where an investor buys a put and a call option with the same exercise date but with different strike prices.

Strap is a long position with two call and one put options with the same exercise price and same date of expiration.

Strip is a long position with one call and two put options with the same exercise price and expiration date.

Vertical Bearish call option is an option in which an investor buys the option with a higher strike price and sells at a lower strike price both having same expiry periods.

Vertical Bullish spread is the spread in which an investor buys a call option on a stock with a certain strike price and selling a call option on the same stock with a higher strike price. Both options have the same expiry date.

Vertical spread is buying and selling puts or calls having same expiration date but with different strike prices.

Assignment

Short Answer-Type Questions

1. Differentiate between a forward contract and a futures contract. (See Subsection 4.3.2)
2. Differentiate between a futures contract and an options contract. (See Subsection 4.3.3)
3. Why does an options contract have an intrinsic value ? (See Subsection 4.3.3)
   or, If the premium on a call option has declined recently, does this decline indicate that the option is a better buy than it was previously ? [C.U. B.Sc. (H), Sem-V, 2020]
4. Why does commodity price risk need to be hedged by a firm ? (See Subsection 4.3.2)
5. How does interest rate risk affect a firm ? (See Section 4.12)
6. Discuss the impact of exchange rate risk on the value of a firm. (See Section 4.12)
7. What is meant by hedging? How does hedging improve the effectiveness of the operations of a business? (See Subsection 4.3.3)
8. What factors determine the need to hedge ? (See Subsection 4.3.3)
9. What is meant by risk-free rate of interest ? (See Section 4.12)
10. What is meant by LIBOR? Why can it be used as a proxy for risk-free rate while taking loans from a bank? (See Section 4.12)
11. Forward contracts are used to hedge future uncertainty. With respect to commodities, when would a party enter into a long forward contract to buy and when would a party enter into a short forward contract, i.e., a contract to sell ? (See Subsection 4.3.2)
12. FRAs are cash-settled. Explain the meaning of FRA and cash settlement. (See Subsection 4.3.7)
13. An Indian vegetable merchant exports fruits and vegetables to Singapore, pricing them in Singapore dollars. What price risks does he face and how can he reduce the risks ? (See Section 4.12)
14. What is meant by basis and basis risk ? (See Subsection 4.3.2)
15. Under what conditions would a hedger not be able to get a perfect hedge using futures ? (See Subsection 4.3.2)
16. Explain what happens to the position of a short hedger if the basis strengthens and if the basis worsens. (See Subsection 4.3.6)
17. If the minimum variance hedge ratio is 1, does it mean that you can completely eliminate price risk ? (See Subsection 4.3.6)
18. Explain the rationale for introducing currency futures. (See Subsection 4.3.2)
19. Under what conditions would you make arbitrage profits ? (See Section 4.12)
20. What is the motivation behind an interest rate swap ? (See Subsection 4.3.5)
21. What is a currency swap ? (See Subsection 4.3.5)
22. What is a swaption? What are the uses of swaptions ? (See Section 4.12)

23. What are the major differences between an interest rate swap and a currency swap ? (See Subsection 4.1[illegible])
24. Under what circumstances would you enter into a forward swap ? (See Subsection 4.1[illegible])
25. A swap contract can be considered as a series of forward contracts. Explain why. (See Subsection 4.1[illegible])
26. Explain the rationale behind using a commodity swap. (See Subsection 4.1[illegible])
27. Explain the rationale behind using an equity swap. (See Subsection 4.1[illegible])
28. Explain how banks can manage their gap using interest rate swaps. (See Subsection 4.1[illegible])
29. Explain the circumstances under which an option would be exercised. (See Subsection 4.3.3 & Section 4.12)
30. Explain when a call option would be exercised. (See Subsection 4.3.3 & Section 4.12)
31. Explain when a put option would be exercised. (See Subsection 4.3.3 & Section 4.12)
32. Under what circumstances would you buy a call option ? (See Subsection 4.3.3 & Section 4.12)
33. Under what circumstances would you buy a put option ? (See Subsection 4.3.3 & Section 4.12)
34. Under what circumstances would you write a call option ? (See Subsection 4.3.3 & Section 4.12)
35. Under what circumstances would you write a put option ? (See Subsection 4.3.3 & Section 4.12)
36. What are the minimum and maximum values of a call option and a put option ? (See Subsection 4.3.3 & Section 4.12)
37. What is the advantage of writing a covered call over writing a naked call ? (See Section 4.12)
38. How can one achieve portfolio insurance using put options ? (See Subsection 4.[illegible])
39. What is the concept behind calendar spread transactions ? (See Section 4.6)
40. When would you enter into a butterfly spread transaction ? (See Section 4.6)
41. When would you enter into a bought straddle and a written straddle transaction ? (See Section 4.6)
42. When would you enter into a bought strip and a written strip transaction ? (See Section 4.6)
43. When would you enter into a bought strap and a written strap transaction ? (See Section 4.6)
44. When would you enter into a strangle transaction ? (See Section 4.6)
45. What is the major advantage of binomial options pricing models as compared to the Black-Scholes Model ? (See Section 4.12)
46. What are interest rate caps, interest rate floors, and interest rate collars? When would these be used? (See Section 4.12)
47. What is meant by the followings: (i) Delta hedging, (ii) Gamma hedging, (iii) Vega hedging. (See Section 4.5)
48. What is meant by the delta, gamma, theta, vega, and rho of options ? (See Section 4.5)
49. Explain how index futures can be used for speculation. (See Subsection 4.3.2)
50. What is the difference between using stock futures and stock index futures for speculative purposes ? (See Subsection 4.3.2)
51. What is meant by index arbitrage ? Explain. (See Section 4.12)
52. State Forward price formula. (See Subsection 4.3.1)
(C.U., B.Sc. (H), Sem-V, 2020)

Long Answer-Type Questions

1. What are derivative securities ? Discuss the uses and types of derivatives. (See Section 4.1)
2. Explain different types of financial derivatives along with their features in brief. (See Section 4.1)
3. Distinguish between futures and forward contracts with suitable examples. (See Subsection 4.3.2)
4. How can financial derivatives be helpful in hedging, speculation and arbitrage ? (See Section 4.12)
5. Explain the terms futures, forward, option and swaps. (See Section 4.3)
6. Throw light on evolution of derivatives. (See Section 4.1)

Give a detailed account of functions of derivatives. (See Section 4.1)

What do you understand by future market? What are the functions of futures markets ? Explain. (See Subsection 4.3.2)

Describe some important features of futures market with suitable examples. (See Subsection 4.3.2)

Explain the role of various participants of futures market. (See Subsection 4.3.2)

How do you determine futures prices ? Explain by giving suitable examples. (See Subsection 4.3.2)

Discuss various factors affecting pricing of an option ? (See Subsection 4.5.3)

Discuss the factors that affect stock option prices. (See Subsection 4.5.1)
(C.U., B.Sc. (H), Sem-V, 2020)

How does volatility affect the pricing of an option ? Discuss various methods of volatility measurement. (See Subsection 4.5.2)

What are assumptions of Binomial option pricing models in one time and multiple time dimensions ? (See Section 4.9)

Discuss the derivation of one time period Binomial option pricing model with some hypothetical example. (See Section 4.9)

What are different parameters to understand option pricing models ? (See Subsection 4.5.2)

Describe various assumptions of the Black-Scholes model for calculating the prices of an option. (See Subsection 4.5.2)

Discuss in detail the Black-Scholes model of pricing an option with suitable illustrations. (See Subsection 4.5.2)

Explain various positions of option? Discuss with suitable illustrations and diagrams. (See Subsection 4.3.3)

What do you know by profit diagrams? How profit diagram can be useful in making strategies of option trading ? (See Section 4.6)

Discuss different strategies of option trading with suitable examples. (See Section 4.6)

How do you differentiate spread and straddle strategies ? Explain with suitable examples. (See Section 4.6)

What are different types of vertical and horizontal spreads ? Explain with suitable examples. (See Section 4.6)

Write notes on the following — (i) Bullish call option spread, (ii) Bearish call option spread, (iii) Long straddle, (iv) Short straddle, (v) Features of diagonal spread, (vi) Profit diagrams, (vii) Butterfly spread, (viii) Straddle vs Strangle. (See Sections 4.6 & 4.12)

How option strategies can be utilised in trading of currency call and currency put options ? (See Section 4.6)

Since exchange of payments takes place in different currencies in a currency swap, a currency swap involves currency risk. Then why would anyone enter into currency swaps ? (See Section 4.11)

Explain the following terms : (i) Call option, (ii) Put option, (iii) European options, (iv) American options, (v) Exercise price, (vi) Exercise date, (vii) Option premium. (See Section 4.12)

Explain the difference between the positions of an option buyer and an option writer. (See Subsection 4.3.3)

Explain when a call option would be exercised. (See Section 4.12)

Explain when a put option would be exercised. (See Section 4.12)

What type of hedging would be undertaken under the following circumstances ? (i) An Indian company has exported products to the USA and expects to receive USD 10 million from the importer in the USA in three months' time. Indian rupee futures are available through banks in India. (ii) A tyre manufacturer wants to reduce the price risk of rubber, which they use in the manufacture of tubes, and rubber futures

are available in MCX India. (ii) An oil producer would like to reduce the unknown price risk of crude. Crude oil futures are available in the NCDEX. (See Subsection 4...)

32. Maggi Ltd. Imports a sophisticated machine from US on 1st April, 2019 and has to pay $ 8,00,000 after 3 months, on 1st July, 2019. The current spot rate of exchange is $1 = ₹ 69.59. It is expected that the exchange rate prevails at ₹ 70.54. In order to protect from foreign exchange rate fluctuation, the company arranges a forward exchange contract with his banker undertaking to by $ 8,00,000 at a fix rate of ₹ 71. If the spot rate prevails at ₹ 70.99 on 1st July 2019, what would be the cash flow of the company? (See Subsection 4...)

33. The stock of Machino Ltd. (FV ₹ 10) quotes ₹ 920 today on NSE and the 3 month futures price quotes at ₹ 950. The borrowing rate is given as 24% p.a. and the expected annual dividend yield is 15% p.a. payable before expiry. Calculate the price of 3 month Machino Futures. (See Subsection 4...)

34. Company A can borrow at a fixed rate of 8% or at a floating rate of MIBOR + 150 basis points. Company B can borrow at a fixed rate of 9% or at a floating rate of MIBOR + 50 basis points. Show that these two companies can improve their position through an interest rate swap. What would be the gain to the two parties ? (See Section 4...)

35. ABC Corporation can borrow at 6% fixed rate or at a floating rate of LIBOR + 50 basis points. GH Corporation can borrow at 8% fixed rate or at a floating rate of LIBOR + 100 basis points. Show that these two corporations can be better off by entering into an interest rate swap. Assume that the comparative advantage is equally shared by the two parties. (See Section 4...)

36. BHP, Australia, can borrow at 8% fixed rate in Australia and at 9% fixed rate in India. Tata Steel can borrow at a fixed rate of 7% in India and at a fixed rate of 11% in Australia. The current exchange rate is AUD 1 = INR 36. Explain how the two companies can engage in a five-year currency swap with payments every ... months. (See Section 4...)

37. A PNB share is selling for INR 2,500 on January 1. It has a call option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 85. Draw a diagram showing the terminal value of this option as well as the gains from buying this option for possible stock prices of INR 2,300 to INR 3,000. (See Section 4...)

38. A State Bank share is selling for INR 2,500 on January 1. It has a put option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 160. Draw a diagram showing the terminal value of this option as well as the gains from buying this option for possible stock prices of INR 2,300 to INR 3,000. (See Section 4...)

39. A Vaisya Bank share is selling at INR 2,500 on January 1. It has a call option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 85.

    (i) On February 14, the Vaisya Bank share price is INR 2,540. What is its intrinsic value? Is the option in-the-money? Would you exercise this option on February 14 ? Explain.

    (ii) On February 14, the Vaisya Bank share price is INR 2,620. What is its intrinsic value? Is the option in-the-money? Would you exercise this option on February 14 ? Explain.

(See Subsection 4.3.5 & Sections 4.6 & 4.12)

40. A State Bank share is selling at INR 2,500 on January 1. It has a put option with maturity on March 31 with an exercise price of INR 2,700. This option is selling for INR 160.

    (i) On February 14, the State Bank share price is INR 2,540. What is its intrinsic value ? Is the option in-the-money ? Would you exercise this option on February 14 ? Explain.

    (ii) On February 14, the State Bank share price is INR 2,620. What is its intrinsic value ? Is the option in-the-money ? Would you exercise this option on February 14 ? Explain.

(See Subsection 4.3.5 & Sections 4.6 & 4.12)

41. The contract size of Allahabad Bank options is 2,450. Allahabad Bank shares are selling at INR 95 on March 1. Call options and put options are available with expiry on April 28 and an exercise price of INR 100. The volatility of the stock price is 18%, and the risk-free rate is 8%. Using the Black-Scholes options pricing model, calculate the call option price on March 1. (See Subsection 4.5.2)

42. Assume that Asian Paints stock is currently selling for INR 1,750. There is a put option on Asian Paints with a maturity of 90 days and an exercise price of INR 1,800. The volatility of the stock price is 15%, and the risk-

free rate is 9%. Form a risk-less hedge and calculate the price of a call option and a put option on the stock using Black-Scholes model. (See Subsection 4.5.2)

Assume that a security is selling at INR 400 and call and put options are available on the stock with a maturity of 90 days and an exercise price of INR 420. The call is selling at INR 6, and the risk-free rate is 8% per annum. According to put–call parity, what should the put sell for? Assume that the stock does not pay any dividends during the life of the option. (See Subsection 4.5.3)

Assume that a security is selling at INR 400 and call and put options are available on the stock with a maturity of 90 days and an exercise price of INR 420. The call is selling at INR 6, and the risk-free rate is 8% per annum. According to put–call parity, what should the put sell for? Assume that the stock will pay a dividend of INR 5 per share after 30 days. (See Subsection 4.5.3)

Assume that the BSE Sensex Index is at 16,500 and call and put options are available on the index with a maturity of 90 days and an exercise price 17,250. The index multiplier is 10. The call is selling at INR 25, and the risk-free rate is 8% per annum. According to put-call parity, what should the put sell for ? Assume that the index has a dividend yield of 2%. (See Subsection 4.5.3)

INDIGO requires 2,000,000 barrels of aviation fuel every month. Since the price of aviation fuel depends on the price of crude oil, INDIGO faces price risk. At the beginning of each month, INDIGO goes for a long hedge in crude oil futures contract for 2,000,000 barrels, with expiry by the end of that month.

(i) What is meant by a long hedge ?

(ii) What is the purpose of the long hedge undertaken by INDIGO ?

(iii) Would INDIGO be able to completely eliminate the price risk of aviation fuel ? Explain.

(See Sections 4.6 & 4.12)

5

Patterns of Corporate Financing

CONTENTS

5.1. Introduction

Before entering into the domain of a corporate financial system, we must have a clear idea regarding the term 'finance'. We often come across three closely connected variables, viz. 'money', 'credit' and 'finance'. There are some differences between these variables. While 'money' refers to the medium of exchange prevailing in the economy, 'credit' normally denotes a particular sum of money that has to be returned to the lender along with the interest amount. It is popularly known as 'debt'.

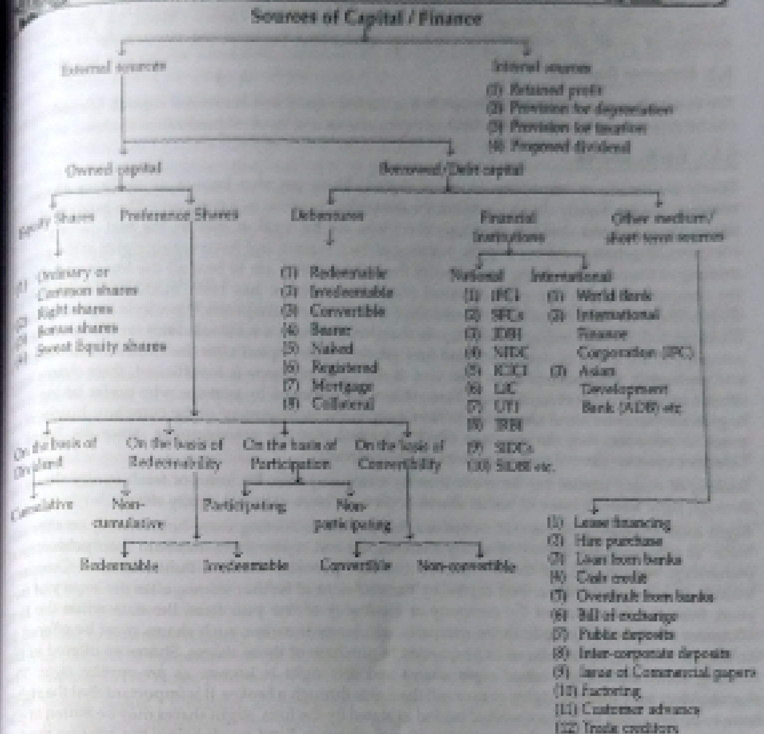
The term 'finance' on the other hand, implies monetary resources comprising both debt and ownership funds of any business enterprise. In corporate finance literature, both debt and equity (i.e. ownership fund) are considered as two broad categories of the pattern of corporate financing. The structure of Balance Sheet of a company shows that there are different sources from which it can mobilise funds. The relative share of each source in total funds reveals the importance attached to a particular source of funds and determines the financing pattern.

5.2. Patterns / Sources of Corporate Financing

Financing pattern of corporate sector has been examined from the liabilities side of the balance sheet. The liability side of the balance sheet represents a portion of capital which is invested by the owners, i.e., it is the owner's capital or it is popularly known as 'Shareholders Fund'. The other portion of capital may be termed as Debt Capital. Hence, capital of corporate bodies may be raised from different alternative sources. One of the methods of classification is with respect to basic pillars of sources which are :

- External Sources
- Internal Sources.

The following chart shows the different alternative patterns/sources wherefrom finance can be made available for a company.



The sources of capital can also be classified according to the repayment period, as Short-term sources, Medium-term sources and Long-term sources of capital.

(1) **Short-term sources :** Short-term sources are the sources which are required to be repaid within a period of one accounting year. These include short-term loans from banks or financial institutions, bill of exchange, trade credit, advances from customers etc.

(2) **Medium-term sources :** Medium term sources ordinarily include sources which are required to be liquidated within a period exceeding one accounting year but not exceeding five years. e.g., public deposits, bank loans, loan from financial institutions etc.

(3) **Long-term sources :** These include sources which are to be repaid within a period exceeding five years e.g., share capital, debentures, retention of profit, long-term loan from financial institutions etc. It is to be mentioned here that equity capital can be repaid only at the time of liquidation of a company.

Again, according to the ownership, sources can be classified as *Owned Capital*, eg. share capital, retention of profit, etc. and *Borrowed Capital* eg., debentures, loans, public deposit etc.

5.3. External Sources

The external sources of finance are classified as owned capital and borrowed capital. Owned capital can be raised from two sources — issue of equity shares and issue of preference shares.

5.3.1. Equity Shares

Equity shares are an important source of capital. These are also known as ordinary shares or common shares. Equity shares represent the ownership position in a company. The holders of equity shares are called shareholders or stockholders who are the legal as well as the real owners of the company. They have a control over the working of the company and have voting rights at the general meeting of the company. Being the owner of the company they are to bear all the losses and risks of the company but they are paid dividend only after payment has been made to the preference shareholders. So they are paid dividend only if there is a positive profit. If profit is high, the rate of dividend will be high and *vice versa*. Equity shareholders have a residual claim on the assets of the company in the event of liquidation and they get back their capital after the claims of the creditors and preference shareholders have been met. If the value of assets is insufficient, their claims may remain unpaid in case of liquidation. These shares are preferred by persons who prefer taking risk to gain a better return and also desire to have a say in the management of the company. In spite of much importance, they do not have effective control over the management since the Board of Directors exercise control over the affairs of the company.

Raising of equity capital can be made directly from the public by issue of fresh equity shares or through right issue or issue of bonus shares or through issue of sweat equity shares.

Right issue represents the issue of shares of a firm among its existing share-holders only on a pro rata basis. To safeguard the interest of equity shareholders and enable them maintain their proportional ownership, section 62(1) of the Companies Act, 2013 provides that if a Public Limited Company intends to increase its subscribed capital by the allotment of further shares, after the expiry of two years from the formation of the company or the expiry of one year from the date when the first allotment of shares was made in the company, whichever is earlier, such shares must be offered to holders of existing equity shares in proportion to purchase of those shares. Shares so offered to existing shareholders are called *right shares* and this right is known as *pre-emptive right*. The shareholders may exercise rights or may sell the rights through a broker. It is important that the rights have to be exercised within a specified period as stated by the firm. Right shares may be issued at par or at a premium. Generally the issue price of such shares is fixed much below the existing market price.

Issue of Bonus Shares is not in fact an issue of shares to raise further capital. Through bonus issue a firm capitalises a portion of accumulated profits and reserves. So, it involves transfer of money from reserves and surplus to equity share capital. Bonus shares are issued to the existing shareholders by the existing company without any consideration, i.e., free of cost. These shares are also issued in a fixed proportion only to the fully paid equity shareholders. The bonus issue is not made to the partly paid shares until they are fully paid up.

There are different sources from where issue of bonus shares can be made. They are *(i)* accumulated profits, *(ii)* general reserve, *(iii)* capital reserve (reserve created out of revaluation of fixed assets can not be utilised), *(iv)* capital redemption reserve u/s 55(2) of the Companies Act, 2013, *(v)* Security premium u/s 52(2) etc.

The total amount of bonus shares issued out of free reserves shall not exceed the total amount of paid up capital. The issue of bonus shares have to be made according to the guidelines issued by SEBI

(Securities and Exchange Board of India). These shares cannot be issued in lieu of cash dividend.

As a result of bonus issue, earnings per equity share comes down as the same amount of profit has to be distributed among the increased number of equity shares. If earnings do not increase after the issue of shares, future earnings per share is affected.

Other than the issue of equity shares to the public, a company can issue same class of equity shares among its employees or directors to ensure their loyalty and participation. According to section 54 of the Companies Act, 2013, a company can issue equity shares to its employees or directors at a discount or for consideration other than cash for providing know-how or making available rights in the nature of intellectual property rights or value additions, by whatever name called. These equity shares are called as *Sweat Equity*. The issue of these shares should be authorised by a special resolution passed by the company in general meeting. The company cannot issue these shares within a period of one year from the date on which the company is entitled to commence business.

Features of Equity Share :

Equity shares have a number of features which distinguishes them from other shares and securities. These shares have some special features in terms of rights and claims of their holders. Some of the basic characteristics of equity shares may be summarised as follows :

(a) Claim on income : The equity shareholders have residual claim on the income of the company. The earnings of a company after deduction of interest, tax and preference dividend are the earnings of the equity shareholders. The rate of dividend on these shares are not fixed, it depends upon the earnings of the company. For this reason, equity shares are also known as 'variable income security'. Again, equity shareholders may not get the full amount of residual earnings as the company may retain a portion of it for future developments. The payment of dividends to the equity shareholders depends on the discretion of the management.

(b) Claim on assets : The claim of the equity shareholders on the assets of the company is also residual and is relevant only when the firm is being liquidated. Other claimants, i.e., creditors, preference shareholders, etc. are paid first in the event of liquidation and after satisfying their claims if anything is left, it belongs to the equity shareholders. If the value of the assets is insufficient, their claims may remain unpaid.

(c) Right to control : As equity shareholders are the real owners of the company, they have the right to control the operations of the company. But they have indirect control over the affairs of the company. The major policies and decisions are taken by the board of directors.

(d) Voting rights : Ordinary shareholders have voting rights in the meeting of the company. An ordinary shareholder has votes equal to the number of shares held by him. They exercise their right to control through voting. They are required to vote on a number of important matters which include election of directors, changes in the memorandum of association etc. Though the members of the board are elected by the holders of the majority of ordinary shares, the proportionate voting system enables the minority shareholders to have some representation on the board.

(e) Pre-emptive right : The ordinary shareholders of a company enjoy pre-emptive right which gives them the first opportunity to subscribe to the issue of additional shares (right shares) offered by the company on pro rata basis. To safeguard the interest of equity shareholders and enable them maintain their proportional ownership, the law grants shareholders the right to purchase new shares in the same proportion as their current ownership. Shares so offered to existing shareholders are called right shares and their prior right to such shares is known as pre-emptive right. This right protects ordinary shareholders by ensuring that management cannot issue additional shares to strengthen its control by selling them to persons of their choice.

(f) **Limited liability :** This is another important feature of equity shares. Although equity shareholders are the true owners of the company, their liability is limited to the value of shares they have subscribed. In the event of liquidation they are liable to pay only the issue price of shares purchased. If a shareholder has already paid the full price of shares purchased, he cannot be held liable to contribute anything more in the event of a financial distress or liquidation. Thus, the equity shareholders enjoy the ownership without taking the risk of unlimited liability. This feature encourages unwilling investors to invest in the company and helps companies to raise funds.

(g) **Maturity :** Equity share do not have any maturity date. Equity share capital is a permanent source of capital and it is not repayable during the life time of the company. Equity shareholders are paid back only at the time of liquidation of a company after meeting all other claims including the claim of preference shareholders. But the company can buy back its own shares as per Sections 68, 69 and 70 of the Companies Act, 2013.

(h) **Rules for issue :** The issue of equity shares should be made strictly according to the Companies Act and SEBI guidelines. The issue of these shares should be made within the limit of authorised equity share capital as mentioned in the memorandum of association of the company. These can be issued at par, at a premium or at a discount.

(i) **Capital gain :** Ordinary shareholders may have an opportunity of gain in capital appreciation of shares. As the listed equity shares can be traded freely in the stock market, they have high marketability. Thus, an equity shareholder can easily sell his holding in favourable market condition and can earn capital gain besides earning the normal dividend.

(j) **Tax on dividend :** A differential rule exists regarding tax on dividend for the company which declares dividend and for the recipient, i.e., the equity shareholders who actually receive the dividend. A dividend tax is to be paid on dividend payable by the company u/s 115O, 115P and 115Q of the Income Tax Act, 1961 and, thereby, such dividend is tax-free in the hands of the recipient. Thus, it encourages investors to invest their funds in the company for earning a regular source of non-taxable income. But, the payment of dividend entails an additional burden on the company in the form of dividend tax.

(k) **Right to receive annual report :** Equity shareholders have the right to receive the annual report of the company at least 21 days prior to the annual general meeting as per section 136 of the Companies Act, 2013. This enables the equity shareholders to be aware of the affairs and the operations of the company.

Advantages of Equity Share :

Advantages of the equity shares can be stated from the company's point of view and from the shareholders' point of view.

From the company's point of view :

(1) **Permanent capital :** It represents a permanent source of capital. Equity share is available for use as long as the company is a going concern. It does not require any repayment except in the case of liquidation.

(2) **No financial burden :** Financing through equity shares does not impose any financial burden on the company as the company is not legally obliged to pay dividend at the time of financial difficulties. Dividend is payable only if there remains a surplus profit after the payment of tax and preference dividend.

(3) **Issue of other securities :** Equity shares do not carry any charge against the assets of the company. This helps the company to raise funds easily by issuing other securities like preference shares and debentures, i.e., the assets are free for raising additional finance.

(4) **Creditworthiness :** It enhances the creditworthiness of the company. As the equity shares have no charge on the assets of the company, additional funds can be raised through borrowings, with the assets as the security.]

(5) **Stability in control :** Controlling power remains in the hands of the equity shareholders. Hence, instability in control gets reduced.

(6) **New company :** Issue of equity shares is the most important source for raising funds for a new company having no market familiarity and a sound financial structure.

From the shareholders' point of view :

(1) **Ownership :** Equity shareholders are the true owner of the company. By investing in equity shares, investors become owner of the company.

(2) **Controllability :** Equity shareholders have the right to control the operations of the company. They have an indirect control over the affairs of the company.

(3) **Voting rights :** Ordinary shareholders have the voting rights in the meeting of the company. They exercise their 'right to control' through voting.

(4) **Pre-emptive right :** Ordinary shareholders enjoy pre-emptive right which enables them with the first opportunity to subscribe to the issue of additional/right shares offered by the company on pro rata basis before they are offered to the public.

(5) **Scope of getting bonus shares :** Equity shares get the benefit of acquiring high-priced shares without any consideration as Bonus shares.

(6) **Liability :** Liability of equity shareholders is limited to the face value of equity shares held by them. They are not liable to contribute anything at the time of liquidation if their shares are already paid up in full.

(7) **Return :** Equity shareholders share the residual income. If the company runs successfully and profitably, the equity dividend can be very high. Not only do they enjoy a high rate of dividend, they also have a regular source of income.

(8) **Non-taxable dividend :** Equity shareholders also enjoy the benefit of non-taxable dividend as the dividend received by them is a fully tax-free income.

(9) **Marketability :** Equity shares have high marketability. They are readily saleable.

(10) **Capital gain :** As the equity shares have high marketability, equity shareholders always have an opportunity of earning capital gain transferring their shares at high market price.

(11) **Liquidity :** Whenever equity shareholders need money, they can dispose the shares in the stock market which increases their liquidity position.

Disadvantages of Equity Share :

Equity capital has some disadvantages compared to other sources of capital. This can also be stated from the company's point of view and from the shareholders' point of view.

From the company's point of view :

(1) **High cost :** The equity share capital has the highest specific cost of capital among all the sources. The rate of return required to be paid to the equity shareholders is generally higher than the rate of return required to be paid to the other investors.

(2) **Cost of issue :** The cost of equity issue which comprises underwriting commission, brokerage and other issue expenses, is generally very high. Moreover, the dividend on equity shares is not deductible as an expense out of the profits of the company for taxation purpose.

(3) **Rigidity in capital structure :** As equity capital is a permanent source of capital, it provides no flexibility in the capital structure.

(4) **Over-capitalisation :** If excess amount of capital is invested in the business by way of equity financing, the same may-g result in the accumulation of idle capital which earn nothing and at the same time leads to over-capitalisation.

(5) **Trading on equity :** Excessive reliance on financing through equity shares reduces the capacity of the company to trade on equity. It cannot get the advantage of use of cheaper sources of debt capital for magnifying the return of equity shareholders if the amount of equity capital is more in the capital structure.

(6) **Creation of inefficiency :** Substantially owned funds create inefficiency in the organisation, as it does not have a permanent obligation either to repay the principal amount or the interest, as in the case of borrowings.

(7) **Passive investors :** A new company having no reputation in the market may find it difficult to raise funds through equity financing since equity shares do not attract passive investors who always prefer to have a steady income and the safety of their investments.

(8) **Capital market :** Issue of equity capital is dependent on the efficient system of the capital market.

▸ *From the shareholders' point of view :*

(1) **Uncertain return :** As the rate of return to equity shareholders depend on the amount of profit, there is no certainty of their income. Even if the company makes substantial profit, they do not have any guarantee of receiving dividend. Equity shareholders cannot contest the dividend decision of the board of directors.

(2) **Risky investment :** Equity investment is a risky investment. Equity stock prices tend to fluctuate widely which makes equity investments risky. Downward price movement may result in a considerable capital loss for the equity shareholders.

(3) **Controllability :** Though theoretically equity shareholders enjoy the controlling power over the firm, small equity investors cannot really exercise such power over the firm. As the equity shareholders are scattered and ill-organised, it is not possible for them to control the affairs of the company in the real sense.

(4) **Lowest priority :** As the equity shareholders are the last claimants of the assets of the company, they may not get anything in the event of liquidation.

(5) **Earnings dilution :** New issue of equity capital may reduce the earnings per share and thus may have an adverse effect on the market price of the equity share if the profits do not increase immediately in proportion to the increase in the number of equity shares.

(6) **Dilution of ownership and control :** If the equity shareholders are unable to exercise their pre-emptive right for any reason, i.e., if they are unable to invest in additional shares, the shares are issued to outsiders. This may result in dilution of ownership as well as dilution of control of the existing shareholders.

5.3.2. Preference Shares

The second important source of finance is the issue of preference shares. As the name itself implies, preference share is a type of security through which a company obtains funds in exchange for certain types of preferential treatment to its holders which are not usually given to the holders of the equity shares. They have preferential right over equity shareholders with respect to payment of dividend and repayment of capital in the event of liquidation of the company. These shareholders do not enjoy any voting rights in the meeting of the shareholders. Preference shareholders can exercise voting rights only on resolutions placed before the company which directly affect the rights attached to preference shares. Preference capital represents a hybrid form of financing possessing some characteristics of debt, such as, fixed dividend rate, no voting right, priority over equity capital etc. and some characteristics of equity, such as, payment of preference dividend is made out of distributable profits etc. As the preference shareholders do not enjoy any voting right, they do not have to bear any risks and hence, ownership is not affected. These shareholders are not treated as the true owners of the company. The investors who are not willing to take any risk and are satisfied with the fixed lower rate of dividend like to invest in preference shares.

▸ Features of Preference Share :

The following are some of the features of preference shares :

(1) **Ownership :** Preference shares are a part of ownership of the company. But they do not have any controlling power on the affairs of the company. Again, they enjoy only a fixed rate of dividend. Hence, it can be said that they are the owners of the company but not in its true sense.

(2) **Preference dividend :** Dividends at a fixed rate are payable on preference shares, the rate of which is declared at the time of issue of such shares. Preference dividend is an appropriation of profit and not a charge against profit.

(3) **Cumulative dividends :** Unless otherwise stated in the terms of issue, the preference shares are cumulative in the sense that all unpaid dividends are carried forward, gets accumulated and are paid in future whenever the company wants to pay equity dividend.

(4) **Claim on Income :** Preference capital has a prior claim on income over equity capital. Whenever a company has distributable profits, the dividend on preference share capital is to be paid first before the payment of equity dividend. Thus, preference share is referred to as a 'senior security'.

(5) **Claim on assets :** Preference shares also have a prior claim on assets of the company. They get a preference for the repayment of capital in the event of liquidation of the company. Their claim is to be settled first before making any payment to the equity shareholders.

(6) **Redeemability :** According to section 55(1) of the Companies Act, 2013, company cannot issue any preference shares which are irredeemable in nature. Section 55(2) states that a company, if so authorised by its articles, issue preference shares which are liable to be redeemed within a period not exceeding 20 years from the date of their issue. However, for a company engaged in infrastructural projects may issue preference shares for a period more than 20 years but not exceeding 30 years subject to the redemption of a minimum 10% of such preference shares per year from the 21st year onwards or earlier on proportional basis.

(7) **Voting right :** Preference shareholders ordinarily do not possess any voting right. However, preference shareholders can exercise the voting right on a resolution which directly affects the rights attached to preference shares. So, the voice of the preference shareholders in the management of the company is quite limited.

(8) **Hybrid form of security :** As it is already mentioned, preference capital represents a hybrid form of security, which satisfy some of the characteristics of equity capital and some of the characteristics of debt capital.

(9) **Cost :** Preference share capital is much costlier than debt capital. The cost of debt capital is the interest amount which is an admissible charge against profit for taxation purpose. So, tax benefit can be obtained for the payment of interest. But preference dividend is an appropriation of profit, i.e., dividend is declared from the post-tax profit. Hence, the full amount of preference dividend is treated as the cost of preference capital as no tax benefit is obtained for such payment.

(10) **Security :** Though preference capital has some of the features of debt capital, no collateral or mortgage is required for obtaining capital by issuing preference shares.

Classification of Preference Shares :

Preference shares may be of different classes. According to the rights and advantages enjoyed by the preference shareholders, it can be classified under four categories :

(A) According to the right to get dividend at a fixed rate :

Under this category preference shares are of two types :

(i) **Cumulative preference shares :** Preference share gets a fixed amount of dividend every year. In the absence of profit or insufficient profit for a particular year if the dividend is not paid, the arrear dividend will accumulate and the whole amount of arrear dividend is to be paid in the year when the company earns sufficient amount of profit. So, the dividend on these preference shares is guaranteed. The preference shares which carry the right to get arrear dividends are known as cumulative preference shares.

(ii) **Non-cumulative preference shares :** The preference shares which have no right to carry arrear dividends are known as non-cumulative preference shares. If preference dividend is not paid in any particular year due to loss or insufficient profit, the dividend for that year will not accumulate to be paid in future. Thus holders of such shares have only a preferential current dividend right.

(B) According to redeemability :

Preference shares under this classification fall under two heads :

(i) **Redeemable preference shares :** When preference share capital is redeemed after a stipulated time, such shares are called redeemable preference shares. As per section 55(1) of the Companies Act, 2013, the stipulated period within which preference shares are to be redeemed is 20 years. However, a company engaged in infrastructural projects may issue preference shares for a period exceeding 20 years.

(ii) **Irredeemable preference shares :** These shares are also called ordinary preference shares. Preference shares which cannot be redeemed during the lifetime of the company are known as irredeemable preference shares. It is already mentioned that irredeemable preference shares cannot be issued in India as per section 55(1) of the Companies Act, 2013.

(C) According to the right to participate in the surplus profit :

Preference shares are divided into two heads under this category :

(i) **Participating preference shares :** Participating preference shares are those shares which participate in the surplus profits of the company. A company having participating preference shares first pays the preference dividend at a fixed rate, and then pays a reasonable rate of dividend to the equity shareholders. If some profits still remain after paying both these dividends, then preference shareholders participate in the surplus profits in a stated manner. The mode of dividing surplus profits among preference shareholders and equity shareholders is given in the Articles of Association. Similarly, these shares are also entitled to participate in the residual assets after the payment of their normal claim at an agreed rate in the event of liquidation of company.

(ii) *Non-participating preference shares :* When the preference shareholders get only preference dividend at a fixed rate and do not enjoy the right to participate in the surplus profits, such preference shares are called as non-participating preference shares.

(D) According to convertibility :

Preference shares under this category fall under two heads :

(i) **Convertible preference shares :** The holders of preference shares which are given a right to convert their holdings into equity shares after a stipulated period of time are called convertible preference shares. The option of conversion and the specific period after which the conversion may take place should be mentioned by the company at the time of the issue of these shares. The right of conversion must also be authorised by the Articles of Association.

(ii) **Non-convertible preference shares :** When the right of conversion of preference shares to equity shares is not permissible to the preference shareholders, such preference shares are known as non-convertible preference shares.

Cumulative Convertible Preference Shares (CCP) :

Apart from the above classification a new financial instrument was introduced in the capital market in March, 1985. This instrument is issued as preference share carrying cumulative dividend right at 10 per cent. These shares are compulsorily to be converted within a period of 5 years but conversion may take place at any time after a period of 3 years. According to the guidelines for issue of these shares, such shares ordinarily be of the face value of ₹ 100 and it could be listed in one or more stock exchanges in the country. These shares should be issued only by the public limited companies.

Following are the various set of objectives for the issue of this instrument. These are :

(i) Setting up new projects.

(ii) Expansion or diversification of existing projects.

(iii) Normal capital expenditure for modernization, and

(iv) Working capital requirement.

When the projects are assisted by the financial institutions, it is required to take the approval of those institutions before the issue of CCP shares.

The CCP shares assure investors a minimum fixed return together with the prospect of capital appreciation and high equity dividend after conversion. But it also faces certain limitations, such as, it represents an expensive source of finance, it does not provide any benefit under the Income Tax Act etc.

Advantages of Preference Share :

Preference shares provide a number of advantages both to the company as well as investors or shareholders.

From the company's point of view :

The Company has the following advantages by issuing the preference shares :

(i) **Payment of dividend :** There is no legal obligation to pay preference dividend. The non-cumulative shares need not to pay any dividend if there is no profit.

(ii) **Fixed rate of dividend :** Preference shareholders are entitled to a fixed rate of dividend which enable the equity shareholders to get higher dividend.

(iii) **Cost of capital :** The company also prefers such shares for raising finance because the company's burden for cost of capital is comparatively small as the dividend payable is fixed at a certain moderate percentage.

(iv) **No charge on assets :** A company can raise long term capital by issue of preference shares without creating any charge over its assets.

(v) **Retention of control :** The management can retain control over the affairs of the company by issuing preference shares as these shareholders do not have any voting rights except only on resolution which directly affects their right.

(6) **Flexibility in capital structure :** Issue of preference shares brings flexibility in the capital structure of the company as they can be redeemed after a specific period of time.

(7) **Future expansion :** These shares can be issued to finance capital expenditure involved in expansion programmes and to strengthen the present equity base with a view to future expansion.

(8) **Trading on equity :** The company can take the advantage of trading on equity by the issue of preference shares. In case the company's rate of return is more than the cost of capital of preference shares, the financial leverage generated by the issue of preference shares produces magnified increase in Earnings Per Share (EPS) for the equity shareholders.

(9) **Creditworthiness :** Preference shares add to the net worth of the company and thereby strengthen the financial position of the company. Additional net worth enhances the ability of the company to borrow in future.

(10) **Hedge against inflation :** As a fixed financial commitment is given to the preference shareholders which is unaffected by inflation, hence financing through these shares provide a hedge against inflation.

(11) **No threat of liquidation :** If the company fails to pay the preference dividends, it does not face any threat of liquidation or other legal proceedings as there is no legal compulsion to pay preference dividend.

▸ ***From the shareholder's point of view :***

Investors in preference shares enjoy the following advantages :

(1) **Fixed return with less risk :** These shares have a special appeal to investors who are not inclined to take great risk and are satisfied with a fixed return on their investments.

(2) **Preferential right :** Preference shareholders always enjoy the preferential right in respect of payment of dividend and repayment of capital, in the event of winding up of the company, over other classes of shares.

(3) **Guarantee of refund :** Ordinary investors prefer to invest in preference shares as there is a guarantee of refund of capital after a definite period.

(4) **Protection of right :** Although preference shareholders carry no voting rights, they can vote on matters which affect directly their rights and on all resolutions if the dividend due on the shares is unpaid.

▸ Disadvantages of Preference Share :

Inspite of many advantages, preference shares suffer from many shortcomings which are discussed both from the company's point of view and investors'/shareholders' point of view.

▸ ***From the company's point of view :***

The following are the main disadvantages of preference shares from the company's point of view :

(1) **Cost of capital :** The issue of preference shares is costlier than the issue of debt capital. These shares are to be given a rate of dividend which is higher than the prevailing rate of interest on debt capital.

(2) **Financial burden :** Cumulative preference shares require payment of arrear dividend. This is no doubt a great burden on the company and the equity shareholders suffer on this account.

(3) **Claim on assets :** The issue of preference shares mean diminution of equity shareholder's claim over the assets of the company.

(4) **Repayment :** Unlike equity capital, preference capital cannot be used permanently by a company. Like borrowed capital, it is to be repaid.

Affects credit worthiness : Although there is no legal obligation to pay dividend on preference shares, but frequent delays or non-payment adversely affect the credit worthiness of the firm.

Affects value of the firm : For the same reason as mentioned above, it adversely affects the market position of the company and may reduce the market price of the equity shares and therefore, it affects the value of the firm.

Cash outflow : As the preference shares are to be redeemed compulsorily within a period of 20 years, it entails a substantial cash outflow from the company.

No tax benefit : Like interest on debt capital preference dividend is not a deductible expense for taxation purpose. This causes heavy strain on the company.

Decrease in EPS for the equity shareholders : If the company is not able to earn a return at least equal to the cost of preference share capital, it may result in decrease in EPS for the equity shareholders.

(10) **Liquidation :** The payment of dividend on preference shares in times of bad trade cause serious strain on the finance of the company and may ultimately bring about liquidation.

(11) **Affects flexibility in company management :** When the consent of preference shareholders is required to incur further indebtedness, it means a restrictive clause in regard to the flexibility of company management.

▸ ***From the shareholders' point of view :***

Shareholders suffer from the following demerits of preference shares :

(1) **Rate of dividend :** Preference shareholders are to remain satisfied with a fixed rate of dividend and that too at a moderate rate. So, investment in preference shares is less attractive than investment in equity shares.

(2) **No voting right :** Preference shareholders are deprived of voting rights except in cases which directly affects their right. They remain at the mercy of the management for the payment of dividend and redemption of their capital.

(3) **No participation in management :** As the preference shareholders do not enjoy any voting right, they do not have any voice in the management of the affairs of the company.

(4) **Status :** Preference shareholders do not enjoy any right of either owners or creditors. They cannot claim dividend at a higher rate in case the firm earns high profit. Again, if there is a loss, preference shareholders do not get any dividend. So, they neither enjoy the right of owners or the right of creditors.

(5) **Capital appreciation :** The prospect of capital appreciation in preference share is lower than the equity shares.

(6) **Marketability :** Preference shares are not easily marketable as equity shares.

(7) **Market price fluctuation :** The market prices of preference shares fluctuate much more than that of debentures.

(8) **Market situation :** In a good market situation, the return on equity shares is much higher than the return on preference shares as the preference shares do not get anything more than the preference dividend which has a fixed rate. So, in a good market situation, investment in these shares is not much attractive.

▸ Difference between Equity Shares and Preference Shares :

Both equity shares and preference shares are the part of ownership securities, yet there are numerous differences between these two shares. These are enumerated in the next pages :

Different Aspects	Equity Shares	Preference Shares
1. Object of issue	1. Equity shares are issued to provide long-term finance.	1. Preference shares are issued to provide long and medium-term finance.
2. Ownership	2. Holders of these shares are treated as the real owners.	2. Holders of these shares are the owners but not in true sense.
3. Risk	3. Being true owners of the company, equity shareholders bear all related risks.	3. Usually preference shareholders have no such risk except in case of non-cumulative preference shareholders who bear the risk of not getting dividend in a no-profit year.
4. Control	4. Equity shareholders exercises full control over the management of the company.	4. Preference shareholders exercise no control over the management of the company.
5. Voting right	5. Equity shareholders always enjoy full voting right in the meetings of the company.	5. Preference shareholders do not have any voting rights except in the cases which directly affects their right.
6. Redeemability	6. Equity shares are not redeemble before liquidation but a company can buy back its own equity shares u/s 68, 69 and 70 of the Companies Act, 2013.	6. Preference shares are to be redeemed within a period of 20 years.
7. Legal compulsion	7. A joint stock company must issue some equity shares as per Companies Act.	7. Issue of preference shares is not compulsory. It depends upon the discretion of the company.
8. Claim on income	8. Equity shareholders have residual claim on the income of the company.	8. Preference shareholders enjoy preferential claim over the equity shareholders on the income of the company.
9. Rate of dividend	9. Rate of dividend is not fixed for these shares. It depends upon the profit earned by the company and the dividend policy adopted by the company.	9. As per the directives of the Articles of Association, there is a fixed rate of dividend for these shares. In case of no profit, preference shareholders do not get any dividend except cumulative dividend.
10. Claim on assets	10. In case of liquidation, equity shares enjoy residual claim on the assets of the company.	10. They enjoy preferential claim on assets of the company over equity shareholders in the event of liquidation. Participating shareholders gets a part of surplus profit, if any, after meeting the dues to the equity shareholders.

Different Aspects	Equity Shares	Preference Shares
11. Types	11. Equity shares are of single type. There is no classification of equity shares.	11. Preference shares may be of different types, such as cumulative, non-cumulative, participating, non-participating, convertible, non-convertible, redeemable and irredeemable preference shares.
12. Burden	12. It causes least burden on the finance of the company.	12. It causes fixed burden on the finance of the company.
13. Participation in meeting	13. Equity shareholders are entitled to participate in the general meeting of the company.	13. Preference shareholders do not have such right.
14. Nomination of directors	14. Equity shareholders are empowered to nominate the board of directors of the company.	14. Preference shareholders do not have any power to nominate the members of the board of directors.
15. Participation in prosperity	15. The holders of equity shares get the scope for participation in the prosperity of the company.	15. There is no scope for participation in the prosperity of the company.
16. Capital reduction	16. Reduction of equity share capital is possible through reorganisation.	16. Reduction of preference share capital is possible only through payment.
17. Marketability	17. These shares are marketable even to investors of small means because of its low denomination.	17. These shares are marketable only to investors of moderate means because of its higher denomination.
18. Market price	18. The market price of equity shares fluctuates very widely.	18. Price fluctuation of preference shares is relatively less.
19. Capital appreciation	19. In case the market value of equity shares goes up, the holders get the benefit of capital appreciation of their investment.	19. Preference shareholders do not get the benefit of capital appreciation of their investment as they usually do not enjoy any right in the surplus profit earned by the company.
20. Capital structure	20. Equity shares make the capital structure rigid as the redemption of equity shares cannot be possible.	20. As the preference share capital can be redeemed easily in case of necessity, it provides much flexible capital structure.
21. Borrowing strength	21. It strengthens the borrowing capacity of the company.	21. It lessens the borrowing capacity of the company.
22. Trading on equity	22. Trading on equity is possible with preference shares and debentures.	22. Trading on equity is not possible without preference shares.
23. Choice of investors	23. Investors of adventurous spirit and risk-bearing capacity are mainly interested to invest in equity shares.	23. Conservative investors of less adventurous spirit and risk-bearing capacity and who prefer to have a fixed earning, like to invest in preference shares.

5.3.3. Debentures

So far we have dealt with 'owned capital'. But a company may wish to borrow money from persons who are willing to lend instead of buying shares. Money received as a loan is called 'Borrowed capital' and we shall now deal with it. The money lent to the company must be recorded and acknowledged by the issue of a document which is called a 'Debenture'. A company may raise long-term loans by the issue of debentures.

According to the Companies Act, debentures include debenture stocks, bonds and other securities of a company, whether constituting a charge on the assets of the company or not. From the view point of the company, a debenture may be defined as an instrument executed by a company under its common seal acknowledging indebtedness to some person or persons to secure the sum advanced. Debentures are termed as "Creditorship Securities". According to **Thomas Evelyn**, *"A debenture is a document under the company's seal which provides for the payment of a principal sum and interest thereon at regular intervals, which is usually secured by a fixed or floating charge on the company's property or undertaking and which acknowledges a loan to the company's property or undertaking and which acknowledges a loan to the company". A debentureholder is a creditor of the company. They get a fixed rate of interest even if the company makes no profit. The debentures are generally given a floating charge over the assets of the company. In order to meet its initial needs and also for extension and development, a company supplements its capital by the issue of debentures. They are normally repayable at the end of the period for which the loan is taken.*

Features of Debentures :

As already mentioned, a debenture is a creditorship security with a fixed rate of interest, a fixed maturity period and a certainty in income. The salient features of debentures are as follows :

(1) ***Trustee :*** *A trustee is appointed through an indenture/trust deed when a debenture issue is sold to public. The trustee is usually a financial institution or bank or insurance company or a firm of attorneys, who protects the interest of debentureholders. The trustee is responsible to see that the borrower fulfills all its contractual obligation.*

(2) ***Security :*** *Generally, debentures are secured through a charge on the present and future immovable assets of the company. This is called equitable mortgage.*

(3) ***Interest :*** *A fixed rate of interest is paid annually or half yearly or quarterly on the value of debentures. The rate is fixed at the time of issue of debentures. It is called the contractual or coupon rate of interest. A company has a legal obligation to pay the interest on due date, irrespective of its level of earnings. Debenture interest is tax deductible for computing the company's corporate tax.*

(4) ***Maturity :*** *Although debentures provide long-term finance to a company, they mature after a specific period at a definite time as stipulated in the issue. In India, a debenture is typically redeemed after 7 to 10 years in installments. The debentureholders may force winding up of the company as creditors, if the company does not pay back the principal amount on the specific date.*

(5) ***Debenture redemption reserve :*** *A Debenture Redemption Reserve (DRR) is created with at least 50% of the amount of issue/redemption before commencement of redemption for the purpose of redeeming all debentures which have maturity period of more than 18 months.*

Sinking fund : A sinking fund may be created under the control of the trustee for which cash is set aside periodically for retiring debentures. Sometimes, the company can itself handle the retirement of debentures using these funds.

Call and put provision : Issue of debentures sometimes provide a call feature which entitles the company to redeem its debentures at a certain price before the maturity date. The call price may be more than the issue price which is generally 5% of the par value in India. This difference in prices is called the call *premium*. This provision is also called a 'buy-back' provision.

The put option gives a right to the debentureholder to seek redemption of debentures at a predetermined price on a specific date.

Credit rating : Debentures are rated by professional bodies, such as, CRISIL, ICRA, CARE etc. to indicate the degree of their safety. Credit rating ensures timely payment of interest and redemption of principal by a borrower. Credit rating should be done compulsorily.

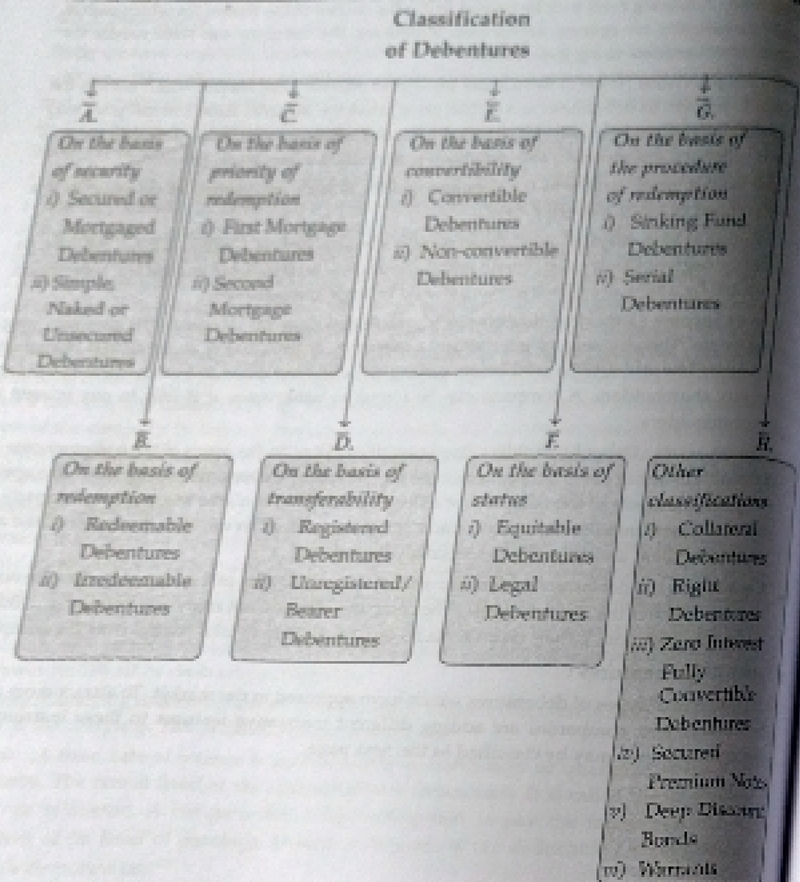
(9) **Claim on income :** Debentureholders enjoy preferential claim on income of the company over shareholders. The payment of interest and repayment of principal is a contractual obligation enforced by law and is to be paid before paying any dividend either to preference shareholders or equity shareholders. A company can be forced to bankruptcy if it fails to pay interest to debentureholders.

(10) **Claim on assets :** Debentureholders also enjoy prior claim on the assets of the company over the shareholders in the event of liquidation of the company. Debentureholders may have specific charge on the assets of the company or a floating charge over all the assets of the company. The claim of debentureholders on assets ranks parispassu with other unsecured creditors if the assets pledged to them are not sufficient to satisfy their claims.

(11) **Control :** As the debentureholders are treated as the creditors of the company, they do not have any control over the management of the company. They do not enjoy any voting right. But at the time of liquidation, if their claim remains unpaid, they may take control over the company.

Types of Debentures :

There are several types of debentures which have appeared in the market. To attract more and more debentureholders, companies are adding different innovative features to these instruments. The types of debentures may be classified in the next page :



(A) On the basis of security :

On the basis of security, debentures may be classified under two heads :

*(i) **Secured or Mortgaged Debentures :** If a debenture is issued against mortgage of an asset, it is called the Mortgaged Debenture. Mortgaged debentures again can be classified into two categories :*

(a) Debentures with a fixed charge, the holders of which have prior right over the particular asset, and

(b) Debentures with floating charges, the holders of which carry a charge on all the assets of the company.

The security helps reducing the risk of debentureholders.

(ii) **Simple, Naked or Unsecured Debentures :** When no asset is pledged under mortgage to raise loan by issuing a debenture paper, it is called the simple or naked or unsecured debenture. They have no priority as compared to other creditors. They are treated as the ordinary creditors at the time of winding up of the company.

(B) On the basis of redemption :

On the basis of redemption, debentures may be classified under two heads :

(i) **Redeemable Debentures :** Redeemable debentures are those on which the repayment of the principal is to be made on a specific date or in installments either at the company's option or at fixed intervals as long as the company is a going concern. The time and mode for redeeming the debentures are fixed at the time of their issue.

(ii) **Irredeemable Debentures :** Irredeemable debentures or perpetual debentures are those in respect of which no time is fixed within which the company is bound to pay, although it may pay back at any time it chooses. So, generally, it is not paid back during the life time of the company. The debentureholders cannot demand payment as long as the company is a going concern and does not default in the payment of interest. But all debentures, whether redeemable or irredeemable, becomes payable if the company goes into liquidation.

(C) On the basis of priority of redemption

Under this category also debentures are classified under two heads :

(i) **First Mortgage Debentures :** The debenture that should be redeemed first on priority basis at the time of liquidation is called the first mortgage debenture. Such a situation arises when a single asset is mortgaged to raise loans more than once.

(ii) **Second Mortgage Debentures :** These debentures are redeemed only after the redemption of the first mortgage debenture. These debentures are also called as ordinary debentures.

(D) On the basis of transferability :

On the basis of transferability, the debentures are classified under two heads :

(i) **Registered Debentures :** If the names and addresses of debentureholders and the serial numbers of debenture papers are recorded in the register book of the company, such debentures are called registered debentures. These debentures are not transferable by delivery. They can be transferred by a regular transfer deed and the transfer must be registered with the company. Only the registered holders are entitled to receive interest and repayment of principal sum on maturity.

(ii) **Unregistered/ Bearer Debentures :** If no records of the names and addresses of debentureholders are maintained in the register of the company such debentures are known as bearer debentures or unregistered debentures. Bearer debentures are just like bearer cheques and currency notes. Interest coupons are attached to these debentures and interest is paid to a person who holds the debenture at the time of payment. Refund of money at the time of redemption is also made to the bearer of debenture paper, whether the bearer is the actual owner or not. No stipulated formalities are to be obtained for transfer of such debentures.

(E) On the basis of convertibility :

On the basis of convertibility, the classification of debentures are :

(i) **Convertible Debentures :** Convertible debentures are those in which an option is given to the debentureholders to exchange their debentures for shares in the company under certain conditions and limitations imposed regarding the period during which the option may be exercised. The right to convert and the terms are mentioned in the Articles of Association. The conversion may take place at a given rate or at a given ratio. Convertible debentures

give an investor the privilege of being a secured creditor of the company and then to change his status to that of a shareholder.

Convertible debentures again may be of two types :

(a) **Fully Convertible Debentures (FCD) :** When the full amount of debentures is converted into equity shares after the lapse of a certain period as specified at the time of issue, such debentures are called fully convertible debentures.

(b) **Partly Convertible Debentures (PCD) :** When a part amount of debentures is converted into equity shares and the balance part is not converted, the debentures are called partly convertible debentures. The inconvertible part of the PCD is redeemed after the lapse of the specified period. The offer for debenture contains the details of the conversion option. The investors of these debentures get the advantage of both convertible and non-convertible debentures blended in one debenture.

(ii) **Non-convertible Debentures :** If there is no option to convert debentures into equity shares, the debentures are called non-convertible debentures. So, a holder of this debenture cannot change his status to a shareholder. All debentures, generally, are non-convertible unless the holders of them specifically get the option for conversion.

(F) On the basis of status :

The classification under this category are :

(i) **Equitable Debentures :** Equitable Debentures are those debentures which are secured by deposit of title deeds of the property with a memorandum in writing, creating a charge against that property.

(ii) **Legal Debentures :** Legal Debentures are those in which the legal ownership of the property of the company is transferred by a deed to the debenture holders as security for loans.

(G) On the basis of the procedure of redemption :

Debentures are classified under two heads in this category :

(i) **Sinking Fund Debentures :** Sinking Fund debentures are those debentures which are redeemed out of sinking fund created for the purpose of redemption. Such redemption takes place in installments at a regular interval.

(ii) **Serial Debentures :** Serial debentures are those debentures which are redeemed serially. A part of the total debentures are redeemable according to their serial numbers in installments.

(H) Other classifications :

Apart from the above classifications, debentures may also be divided into some other categories. These are :

(i) **Collateral Debentures :** Collateral debentures are those debentures which are issued as collateral or secondary security by a company to raise a loan or overdraft etc. generally from the banks and financial institutions. These debentures are issued to provide security against raising a loan. Company does not get cash or anything else against the issue of such debentures. The lender simply holds the debentures and no interest is payable on these instruments. These become effective only when the company makes a default in the repayment of loan against which these have been issued. At that time, the lender becomes a debentureholder of the company like other debentures.

(ii) **Right Debentures :** Right debentures are issued to raise the long-term working capital requirement of a company. These debentures are issued to the existing resident Indian shareholders on a right basis in proportion to their shareholding. These debentures cannot be converted into equity shares but can be redeemed after a specific period of time as indicated in offer document.

(iii) **Zero Interest Fully Convertible Debentures (ZFCD) :** Zero interest fully convertible debentures are those debentures which are to be converted compulsorily into equity shares at the expiry of a given period (not exceeding 3 years) from the date of issue. No interest is payable by the company on these debentures from the date of issue to the date of conversion. If the company wants to convert these debentures into equity shares just after 18 months from the date of issue, the debentures are to be credit-rated by an approved credit rating agency as per the SEBI guidelines. The return to debentureholders is available in the form of difference between the issue price of the ZFCD and the market price of the converted shares.

(iv) **Secured Premium Notes (SPNs) :** Secured premium notes is a tradable instrument in which debentureholders do not get any scope of conversion of their holding to equity shares, rather a detachable warrant is attached against which the holder gets the right to purchase equity shares at a pre-fixed price after a specific 'lock in' period. No interest is payable on such debentures. In India, Tisco Ltd. had first issued SPNs during the month of August in the year 1992.

(v) **Deep Discount Bonds (DDBs) :** Another innovative debt instrument is the deep discount bond which is issued at a discount to the face value. It is also a type of zero interest bond and not convertible. The DDB is redeemed at the expiry of a specified period at the face value. The return available to the holders of DDB is the difference amount between the face value and issue price. Industrial Development Bank of India was the first institution to issue DDB in the year 1992 for a 'deep discount' price of ₹ 2,700, the face value of which is ₹ 1 Lakh over the maturity period of 25 years. The investor of DDB were given an option to redeem at the end of 5th, 10th, 15th and 20th year at different values.

(vi) **Warrants :** A warrant is a security or a right that permits the holder to buy a specified number of shares of common stock during or at the expiry of a specified period and at a given price. So, a warrant gives the holder the right to purchase a fixed number of shares in the future at a pre-determined price from the company. The holder of a warrant may or may not exercise his right for purchasing the company's share. If he exercises his right by paying the specified amount, he becomes an equity shareholder of the company and if he does not, then the warrant lapses. The holder of a warrant is also allowed to transfer or sell his right in the secondary market. Warrants are brought and sold in the same way as stocks. Warrants are long-term rights. They may have expiry dates that lie between 5 to 10 years.

Warrants are generally issued with other securities, i.e., a bond or a preference share, in a package. Warrants are often employed as "sweeteners" to a public issue of bonds or debts, to add to the marketability of the issue. During a period of financial crunch, a company may decide to use warrants to make their debt issue attractive to the investors and to obtain the required amount of capital easily. The investor not only obtains the fixed return associated with the debt but also gets an option to purchase common stock at a stated price.

Warrants are attached with the bond or with the preference shares in a definite proportion which is known as the exercise ratio. It reflects the number of shares that can be acquired per warrant. A warrant can either be detachable or non-detachable. Detachable warrants can be sold separately from the bond as a result of which the holder can continue to retain the instrument to which the warrant was tied up. But a non-detachable warrant cannot be sold separately from the bond with which it was tied up. The holders of the warrants are not the shareholders of the company until they exercise their options.

For many years, warrants were considered speculative instruments rather than investment securities, since the warrant has no value other than as a right to purchase other securities. Deepak Fertiliser and Petrochemical Corporation Ltd. issued debentures with warrants attached in 1987. In 1992, the Tata Iron and Steel Company issued secured premium notes with warrants attached to attract investors ; Ranbaxy and Reliance Ltd. issued securities with warrants attached in 1995.

A warrant is different from convertible securities (i.e., convertible debentures and convertible preference shares). A holder has the right to receive equity shares through exchange of the convertible securities. But in case of a warrant, the security holder has right to purchase equity shares at a specified price. In the case of convertible securities, the securities and the right are not separable but a warrant may be detachable. In case of exercising right, convertible securities have to be surrendered. But in case of detachable warrants the right can be exercised separately and there is no need to surrender the securities with which the warrants were actually tied up. Warrants are exercisable against cash only. But in case of convertible securities there is no need to pay any cash. Conversion of securities against equity shares is made at a conversion rate.

So, warrants play a crucial role in funding expansion and growth programmes of an organisation. Warrants enable companies to raise funds cheaply and provide an opportunity for them to issue equity shares in future at a premium over the current price.

There are also some other innovative debt instruments which are issued to raise funds from the market, such as Callable Bond, Adjustable Rate Bonds, Inflation Adjusted Bonds (IAB), etc.

Advantages of Debentures :

Debentures as the source of capital have many advantages that can be discussed from the point of view of the company and from the point of view of debentureholders.

From the point of view of the company :

The advantages from the company's view point are :

(1) **Financial plan :** Company can raise medium term and long-term finance through the issue of debentures. Thus, adjustment in financial plan can be possible by the company to suit its requirements.

(2) **Dilution in control :** Debentures enable the company to raise finance without giving any control to the debentureholders. They do not weaken the control of existing shareholders as they do not have any voting rights. So, controls on ownership and on affairs of the company remain in the hands of the shareholders.

(3) **Rate of interest :** The rate of interest payable on debentures is fixed as well as lower than the rates of dividend paid on shares.

(4) **Tax benefit :** Interest paid on debentures is a tax deductible expenditure and thus reduces the tax burden of the company.

(5) **Cost of financing :** As the rate of interest on debentures is low as well as it is a tax deductible expenditure, the effective cost of financing by debentures becomes very low for the company.

(6) **Flexibility in capital structure :** Inclusion of debentures in the capital structure makes the capital structure of the company flexible. It enables a company to correct a situation of over capitalisation through the redemption of debentures.

(7) **Possibility of Trading on equity :** The company is able to trade on equity because of low effective cost of debentures. As a result, the company is able to pay better rates of dividend to equity shareholders.

(8) **Hedging against inflation :** Debentures provide a hedge against inflation as the interest amount as well as the principal repayment amount are fixed in monetary terms. If there is inflation at the time of repayment then the creditors are not benefited but the debtor company get the benefit of hedging.

(9) **Fund raising during depression :** A company is able to raise funds through issue of debentures even during depression since the risk of investing in debentures is low and debentures are considered to be a source of stable income.

From the point of view of the debentureholders :

The advantages from the debentureholders point of view are :

(1) **Fixed regular income :** Debentureholders get a regular fixed rate of interest which is payable by the company even out of capital if profits are not available. So, debentures provide a stable source of income to its investors.

(2) **Less risky investment :** Debentures are a good investment from the point of view of cautious investors who do not want to risk their investments too much and yet wish to earn high and regular income.

(3) **Security :** Debentures offer a definite security to the investors and so it appeals to the conservative minds. The better the security, the greater will be the chance of a successful debenture issue.

(4) **Protection of interest :** The interest of debentureholders is protected by various provisions of the Debenture Trust Deed. There are trustees to protect their interest. Their interest is also protected by the guidelines issued by the Securities and Exchange Board of India in this regard.

(5) **Maturity period :** Generally debentures have a fixed maturity period and many investors prefer this instrument because of a fixed maturity period.

(6) **Claim on assets :** In the event of liquidation, debentureholders enjoy a preferential claim on the assets of the company over the shareholders.

(7) **Scope for becoming the owner :** Holders of convertible debentures get the scope for becoming the owners of the company after conversion of debentures into equity shares either in part or full. Not only that, they also get an additional benefit of capital appreciation and higher income in the form of dividend after conversion.

(8) **Liquid investment :** A debenture is more a form of liquid investment and an investor can obtain loan from financial institutions by mortgaging or selling debentures.

Disadvantages of Debentures :

Debenture issue is an important source of long-term financing no doubt. However, its advantages are set off, at least to some extent by its limitations. These are also observed from the company's point of view and from the debentureholders' point of view.

From the point of view of the company :

A company suffers from the following disadvantages of financing through debentures :

(1) **Permanent liability :** Payment of interest on debenture as well as repayment of the principal amount on maturity are the permanent liabilities of the company. These have to be paid even when there are no profits. Hence, it is a permanent burden of the company.

(2) **Winding up :** Failure in payment of interest and in repayment of the principal amount adversely affect the credit worthiness of the company and may force a company to go into liquidation.

(3) **Threatens liquidity position :** As the debentures are to be redeemed compulsorily at the time of maturity, it threatens the liquidity position of the company except in case of convertible debentures.

(4) **Restriction to raise other loans :** Charge on assets of the company for issuing debentures restrict a company from getting further loans against the security of assets already mortgaged to debentureholders. If the capital structure of a company is heavily loaded with debentures, banks and other financial institutions do not show favourable attitude towards the company. The goodwill of the company falls in the eyes of public too.

(5) **Cost of capital :** The use of too much debt financing may push the market price of equity shares down. So, it usually increases the risk perception of the investors in the firm and as a result, the overall cost of equity capital of the company will increase.

(6) **Trading on equity :** If the company earns a rate of return on capital employed which cannot cover the debenture interest, then the company cannot take the advantage of debt financing. This will have a magnified adverse effect on Earnings Per Share (EPS).

(7) **Cost of issue :** Cost of issue of debentures for raising finance is also high because of high stamp duty.

(8) **Financial burden for sinking fund :** Creation of sinking fund for the purpose of redemption of debentures results in a regular financial burden on business. An instalment amount has to be set aside every year from the profit of the company and that has to be invested in some outside securities. This creates a regular financial burden.

(9) **Not suitable for all companies :** A company whose expected future earnings are not stable cannot issue debenture for raising long-term capital. If market demand for products of a business enterprise fluctuates severely causing heavy fluctuations in profit, then the issue of debenture is not suggested. Again a company which does not have sufficient fixed assets to offer as security to raise finance through the issue of debentures, cannot use this source of raising funds to its benefit.

(10) **Conditions in the capital market :** Conditions prevailing in the capital market is also very important in case of debenture issue, especially when the required rate of return on new debentures is too high. This may cause delay in debt financing and/or the company may face to equity financing.

▶ ***From the point of view of the debentureholders :***

Many investors do not find debentures or bonds as an attractive investment opportunity because of the following limitations :

(1) ***No control :*** *Having no right of voting, debentureholders do not have any controlling power over the management of the company.*

(2) **Tax burden :** The interest received by the debentureholders is fully taxable in their hands which they can avoid by way of equity dividend.

(3) **Participation in surplus profit/asset :** Debentureholders get interest only at a fixed rate. They cannot participate in the surplus profit of the company nor can they participate in the surplus assets of the company since they are treated merely as creditors and not as the owners of the company.

(4) **Type of Investors :** *Normally the face value of each unit of debenture paper is high. Therefore, it is not possible for a middle income group of people to invest in debentures.*

(5) **Fluctuation in debenture price :** The prices of debentures in the market fluctuate with the changes in the interest rate.

(6) **Favourable market condition :** In a favourable market condition, investors would like to invest in equity shares because of high expected return from the stock investment and not in debentures which earns only a fixed return.

(7) ***New company :*** *In case of a new company it is very difficult for a prospective investor to decide on the financial future. They cannot have faith in investment of debentures inspite of the guaranteed fixed regular interest income, rather they like to invest in shares of such a company.*

Differences between Shares and Debentures :

The following are the differences between Shares and Debentures.

Different Aspects	Shares	Debentures
1. Object of issue	1. Shares are mainly issued to provide long-term finance.	1. Debentures are issued for both long and medium-term finance.
2. Ownership	2. Shareholders are treated as the owners of the company.	2. Debentureholders are the lenders of the company.
3. Nature	3. The amount of share capital is treated as the ownership capital of the company.	3. The amount received from the issue of debentures is treated as debt or borrowed capital.
4. Return	4. The return on shares is known as dividend.	4. The return on debentures is known as interest.
5. Rate of return	5. The rate of dividend depends upon the availability of profit though the rate of preference dividend is fixed.	5. The rate of interest on debenture is fixed. It does not depend on the availability of profit.
6. Accounting treatment	6. Dividend paid is not an expense of the company. It is an appropriation of profit.	6. Debenture interest is a compulsory payment and it is treated as a charge against profit.
7. Tax benefit	7. As the dividend paid to shareholders is an appropriation of profit, it is not tax deductible.	7. As the debenture interest is a charge against profit, the company gets the benefit of tax deductibility.
8. Burden	8. It causes least burden on the finance of the company.	8. It causes fixed burden on the finance of the company.
9. Voting right	9. Equity shareholders enjoy full voting rights in the meetings of the company, whereas the preference shareholders enjoy restricted voting rights.	9. Debentureholders have no voting right in any circumstances.
10. Control	10. Equity shareholders exercise full control over the management of the company, whereas preference shareholders do not have any power to control.	10. Debentureholders exercise no control over the management of the company.
11. Type of investors	11. Investors who are of adventurous spirit and risk-bearing capacity invest in shares.	11. Cautious investors who are reluctant to take risk generally invest in debentures.
12. Charge on assets	12. It does not create any charge on assets as security at the time of issue.	12. It creates charge on assets in many cases as security at the time of issue.
13. Claim on assets	13. At the time of liquidation, shareholders have residual claim on assets of the company after meeting the outsiders' claim.	13. Debentureholders enjoy preferential claim on assets of the company over the shareholders in the event of liquidation.

Different Aspects	Shares	Debentures
14. Other rights	14. Shareholders enjoy different rights and privileges as are mentioned in the Articles of Association.	14. Debentureholders have only the right to get the contractual interest and repayment of the principal amount at the stipulated period.
15. Marketability	15. Marketability is possible to small and moderate investors.	15. Marketability is generally possible to the big investors because of the high denomination.
16. Compulsion to issue	16. It is compulsory for a joint stock company to have share capital. So, issue of shares is compulsory as per statutory requirement.	16. There is no such legal compulsion to issue debentures for a joint stock company.
17. Risk	17. Shareholders are to bear all financial risks of the company as they are the owners of the company.	17. Debentureholders are treated as the creditors of the company. So, they are not supposed to bear any risk related to the affairs of the company.
18. Trading on equity	18. Trading on equity is possible with preference shares and debentures.	18. No trading on equity is possible without debentures.
19. Redeemability	19. There is no question of redemption of equity shares before winding-up of the company. However, preference shares are to be redeemed within a period of 20 years.	19. Debentures are always required to be redeemed at a stipulated period of time or at any time as per the choice of the company.

5.3.4. Term Loans

Term loans, also referred to as Term Financing, are generally raised by the business concerns from financial institutions to meet their medium-term and long-term financial needs. Medium-term loans are granted for periods ranging from 1 to 5 years and long-term loans are granted for periods beyond 5 years. Term loans are also known as term or project finance. The amount raised from term-loans is used mainly for the purpose of financing fixed assets and to meet permanent working capital. It can also be used for the purpose of expansion, diversification, modernisation as well as to redeem preference capital, debentures or bonds. The term loan carries a moderate rate of interest and is paid in installments. A term loan is granted on the basis of a formal agreement between the borrower and the lending institution. The main features of term finacing are security, interest, principal repayment and conditions of the lender. Term loans may take the form of an ordinary loan or a revolving credit. In India term loans are being provided mainly by commercial banks and specialised financial institutions or development banks. Some major financial institutions which provide term loans are :

(1) **Industrial Finance Corporation of India (IFCI) :** IFCI is the first term financing institution in India. It was set up as a statutory corporation in the year 1948 under the IFC Act, 1948. The objective of the corporation as laid down in the preamble of the IFC Act, 1948, *is to make medium and long-term credits more available to individual concern in India, particularly in circumstances when normal banking accommodation is inappropriate or recourse to capital issue method is impracticable.* The corporation pays due attention to the need for dispersal of new industries, industrial development

of relatively developed districts/areas in the country, growth of industries in the cooperative sector and reasonably well-developed investment portfolio for itself. The IFCI was converted into a company in 1993 as IFCI Ltd. under the provisions of the Industrial Finance Corporation (Transfer of undertaking and Repeal) Act, 1993.

(2) **State Financial Corporation (SFCs) :** The Government of India passed the State Financial Corporation Act in 1951 to facilitate the formation of State Financial Corporations. The first SFC was setup in Punjab in 1953. At present, there are altogether 18 SFCs functioning in the country. These corporations are expected to be complementary to the IFCI. The IFCI was setup to offer financial assistance to only large and medium sized undertakings while SFCs were set-up to offer financial assistance to small and medium sized industrial concerns. SFCs render assistance to all kinds of industries which may be in the form of private limited companies, partnership firms or sole-trading concerns.

(3) **Industrial Development Bank of India (IDBI) :** The Industrial Development Bank of India was established under the IDBI Act, 1964 as a wholly owned subsidiary of the Reserve Bank of India on July, 1964. However, in February, 1976, it was delinked from the Reserve Bank; its ownership was transferred to the Government of India. For greater operational flexibility, a portion of its share was offered to the public in 1995 and as a result of which government holdings had come down to 72 per cent. It serves as an apex financial institution. The Bank has been assigned a special role in respect of co-ordinating the activities of other financial institutions and to act as a reservoir from which the other financial institutions can draw. It also provides direct financial assistance to industrial units for planning, promoting and developing industries in order to fill the gaps in the industrial structure by providing medium and long-term finance.

(4) **National Industrial Development Corporation (NIDC) :** NIDC was set up in 1954 as a statutory corporation owned by the Government of India. The main objectives for setting up the NIDC were to formulate and execute projects for setting up new industries, to provide consultancy services and to finance the rehabilitation and modernisation of certain industries, such as cotton and jute textiles, machine tools etc.

(5) **Industrial Credit and Investment Corporation of India Ltd. (ICICI) :** The ICICI was set up on January 5, 1955 as a public limited company by the Government of India, the World Bank and representatives of the Indian private industry. It is the second all India development bank after IDBI. It was established mainly for developing medium and small industries of the private sector. It provides foreign currency loans to industrial projects and provides finance in the form of long or medium term loans.

ICICI also started leasing operations in 1983 and took up merchant banking activities too. Other than the companies, institutions and individuals, its equity capital is also owned by public sector institutions, such as LIC, GIC, Banks etc. Since 2002, ICICI Ltd. has been merged with the ICICI bank, and it functions like a universal bank, i.e., it provides both short-term loans to small investors and long-term loans to large industrial houses.

(6) **Life Insurance Corporation of India (LICI) :** The LICI was set up in 1956 with the nationalisation of insurance business in India. It took over the assets and liabilities of 245 private insurers engaged in the business of life insurance in India. LICI also provides industrial finance to different types of organisations. LIC is very much suited for participation in industrial financing as it has long-term funds available at its disposal. Generally, LIC takes part in industrial financing through the subscription of shares and bonds of developing financial institutions, by direct lending to industry and by purchasing securities of joint stock companies from the industrial securities market.

(7) **Unit Trust of India (UTI) :** UTI came into existence on February 1, 1964 under the Unit Trust of India Act, 1963. It was a closed-end mutual fund to mobilize the resources/savings of small investors. Its establishment has been a land mark in the history of investment trusts in India. The main objective of the UTI was to pool the savings of the middle and low-income groups and to develop the savings habit of the people. As the UTI has long-term available funds it can assist financially the industrial organisations by subscribing directly to their equity shares, preference shares, debentures, bonds and also by providing short-term loans.

(8) **Industrial Reconstruction Bank of India (IRBI) :** Industrial Reconstruction Bank of India was established on March 20, 1985 under the IRBI Act, 1984. It emerged as the principal credit and reconstruction agency for assisting the rehabilitation of sick and closed industrial units. Other than reviving sick and closed industrial units, it also acted as the prime loan and reconstruction agency. The Government of India reconstituted the IRBI into a new company which is known as Industrial Investment Bank of India (IIBI) on March 27, 1997. Now it acts as an autonomous development finance institution like IFCI, IDBI, ICICI etc.

(9) **State Industrial Development Corporation (SIDCs) :** In order to accelerate the pace of industrial development in their states, many State Governments have set up State Industrial Development Corporations to supplement the efforts of SFCs. Andhra Pradesh and Bihar were the first states to set up such corporations in 1960. At present there are 28 such SIDCs working in the country. SIDCs were established as wholly owned undertakings of the State Governments but two SIDCs have been set up under the statutes of the legislative bodies. Other than granting financial assistance they also help in promotion and management of an Industrial concern.

(10) **Small Industries Development Bank of India (SIDBI) :** SIDBI was set up in April 1990 under the Small Industries Development Bank of India Act, 1990, passed by the parliament. It came into existence on April, 1990. The main objective of SIDBI has been to work as a principal financial institution for the promotion, financing and development of industries in the small scale sector. It is also expected to co-ordinate the functions of the financial institutions, viz., SFCs, State Small Industries Development Corporations, Scheduled Banks and State Co-operative Banks etc., engaged in the promotion, financing and developing the small-scale industries.

There are also some other financial institutions, such as, Infrastructure Development Finance Co. Ltd. (IDFC), National Bank for Agriculture and Rural Development (NABARD), Export Import Bank of India (EXIM Bank), National Small Industries Corporation Ltd (NSIC) etc. which also provide term loans.

Besides the above national financial institutions there are also some international financing institutions which provide industrial finance through their member countries while some of them directly to the companies. The assistance rendered by all such institutions have geared up the pace of industrialisation. This includes the institutions like the World Bank and its affiliates such as International Bank for Reconstruction and Development (IBRD), International Finance Corporation (IFC), International Development Association (IDA), Multilateral Investment Guarantee Agency (MIGA) etc.

Features of Term Loans :

There are certain cardinal features of term loans which are sharply distinct from short-term as well as long-term loans. These features have therefore made it imperative to study it separately. These are :

(1) **Maturity :** In India Financial Institutions (FIs) provide term loans generally for a period of 6 to 10 years. A grace period (moratorium) of 1 to 2 years may be allowed in some cases. The repayment starts 2 or 3 years after sanctioning of loan but the payment should be made only in accordance with the specified schedule. Commercial banks advance term loans generally for a period of 3 to 5 years.

Cost : No flotation cost is associated with the raising of term loans as the lender institutions grant loans after a thorough and detailed appraisal of applications made by the borrower for the purpose of taking loans.

Negotiation : The term loans are negotiated loans between the borrowers and the lenders. The sanction of loan depends on the negotiation made between them.

Security : Term loan is a secured borrowing. If the term loan is secured by the assets acquired using term loan funds, it is called *primary security* and when it is secured by company's existing and future assets, it is called *secondary or collateral security*. Again, the lender may create a fixed charge on specific assets or a floating charge, i.e. a general mortgage covering all assets. Assets with floating charge may be dealt with freely in the normal course of business without obtaining the approval of the lender.

Interest : A fixed rate of interest is associated with any kind of term loans. The borrower has to pay the interest compulsorily. But deferment of interest payment is also possible during the project stage. If the interest amount is deferred, it will accumulate along with the amount of the principal and the total accumulated amount is required to be repaid at the time of maturity.

Repayment of loan : Repayment of loan can be made annually, half-yearly or quarterly as per the requirements of the lending institutions after allowing an initial grace period of 1-2 years. Repayment of loan includes payment for interest as well as the principal amount.

Penal interest : If the borrower is in default in respect of both the interest and principal amount, a penal interest at a specified rate for the period of default is to be paid on the amount of total default.

Commitment charge : A borrower may have to pay a commitment charge if he does not utilise or draw the total amount of loan sanctioned by the lender in his favour. Though he has to pay a commitment charge, such deferment saves interest payment.

Restrictive covenants : To protect the interest of the lenders, loan agreement may contain a number of restrictive terms and conditions which are known as covenants. These covenants create some restrictions in the conduct of the operations of the borrowing enterprises and which are mainly related to assets, liabilities, cash flows and control. The borrowing firm generally has to keep the lender informed by furnishing financial statements and other information periodically.

Advantages of Term Loan :

Term loans have merits both for the borrowers and for the lenders.

For the borrowers :

Advantages of term loans from the borrower's point of view are :

(1) **Low cost :** Cost of term financing is much lower than the cost of equity capital or preference capital financing. Again, no flotation cost is associated with term financing.

(2) **Control :** Term loan financing does not result in dilution of control since the lenders of term financing are not entitled to vote. So, it does not affect the control of existing shareholders.

(3) **Tax-deductibility :** Interest paid on term-loans is a compulsory payment and thus it is a charge against profit. So, the borrowers can get the benefit of tax-deductibility regarding interest payment which also lowers the cost of term financing.

(4) **Flexibility :** The agreement made between the borrowers and the lenders are quite flexible in a sense that the term of loan, drawal of loan, the repayment schedule etc. are adjustable according to their requirement.

For the lender :

Advantages from the lender's point of view are :

(1) **Rate of interest :** Term loans earn a fixed rate of interest which signifies a permanent regular source of income.

(2) **Maturity period :** Term loans have a definite maturity period which helps the lender to plan for a proper roll over of the amount.

(3) **Risk :** The lender does not face any risk to provide term loans as this, together with interest and other charges, remain fully secured.

(4) **Restrictive covenants :** The interest of the lender is adequately protected by incorporating restrictive covenants in the loan agreement.

Disadvantages of Term Loans :

Term loans are also not free from drawbacks. These are :

To the borrower :

Disadvantages of term loans from the borrower's point of view are :

(1) **Legal compulsion :** The interest payment and capital repayment are a compulsory statutory obligation. Failure to meet these payments can cause a lot of embarrassment. It can even threaten the existence of the borrower's business.

(2) **Penal interest :** Not only in the case of failure, even for a slight default in payment for interest or repayment of loan, the borrower is required to pay penal interest.

(3) **Commitment charge :** A borrower may have to pay a commitment charge if he does not utilise or draw the total amount of loan sanctioned by the lender in his favour.

(4) **Interference :** Sometimes financial institutions may force the borrower company to induct their nominee in the board which may cause interference in the decision making process of the company.

(5) **Financial risk :** Term loan financing enhances the financial risk associated with the firm for which the market price of equity shares may fall. This may increase the overall cost of capital.

(6) **Restrictive covenants :** Term loan agreement may contain a number of restrictive terms and conditions, known as covenants. These may be derogatory to the interest of the borrower and may reduce managerial freedom.

To the lender :

Disadvantages from the view point of the lender are :

(1) **No voting right :** Term loans do not carry the right to vote. So, the lender of term loans cannot participate in the management of the company.

(2) **Negotiable securities :** Term loans are not represented by negotiable securities. Term loans cannot be securitized like debentures or bonds against the loan which can be negotiated in favour of others.

To conclude, it can be said that apart from the advantages and disadvantages stated above, term loans provide all the advantages and disadvantages of debenture financing to the lender as well as to the borrowers. It carries low cost but involve high risk. It does not affect control but reduces managerial freedom to some extent.

5.3.5. Lease Financing

In addition to debt and equity financing, leasing has emerged as another important source of intermediate and long-term financing of corporate enterprises. A lease is a financing device which

developed rapidly during 1960s and 1970s in the U.S. and in India just before the middle of the ...

... specialised means of financing which focuses on equipment leasing. If a firm wishes to get the ... of a specific asset, they can choose from two alternatives — the particular asset can be purchased or the asset can be taken on lease. If the firm wishes to purchase the asset it has to incur a ... capital expenditure and on the other hand, if the asset is taken on lease the firm gets the asset's services without necessarily incurring any capital liability. Leasing is therefore, a source of finance as it enables the firm to obtain the use of assets in exchange of agreeing to pay a periodic rent without purchasing the asset in exchange of a huge capital outlay.

Leasing is an arrangement under which a company acquires the right to make use of the asset without purchasing the asset or holding title to it. *A lease, thus, is the written agreement under which the owner gives the right to economic use of the assets to the user for a stated period of time against consideration.* A lease is essentially the renting of an asset for some specific period. The owner of the asset is called the '*lessor*' and the other party that uses the asset is known as the '*lessee*'. The periodic payment which the lessee has to pay to the lessor is known as the '*lease rental*'. A lessor may be a dealer, supplier, manufacturer or a finance company while a lessee can be an individual, a firm or a company interested in the use of assets without owning it. Thus, lease can be defined as a specialised means of acquiring the use of assets without ownership. So, in the leasing procedure, the ownership of the leased property is retained with the owner/lessor. Thus, as a legal owner, the lessor is also entitled to claim depreciation on the assets. At the end of the leasing period, the asset generally reverts back to the lessor unless the lease contract contains a provision for the renewal of the contract. Hence, leasing is a source of finance as it enables the firm to obtain the use of assets against payment of lease rentals without necessarily incurring capital expenditure for the purchase of the same asset.

According to **Woolf, Tanna & Singh** "*Leasing is effectively a source of finance but it always relates to a specific assets. Under a lease contract, the ownership of the assets remains with the lessor whilst the use of assets is available to the lessee in return for the payment of a fixed rental. Lease finance is very similar to debt in that the lease payments are fixed financial contractual obligations. Leasing will therefore, increase the level of gearing and financial risk of the company. There is need to look at the financial implications of leasing and especially at the tax implications for both the lessor and the lessee*". As per **Accounting Standard – 19** issued by the Council of the Institute of Chartered Accountants of India, *a lease is an agreement whereby the lessor conveys to the lessee in return for a payment or series of payments the right to use an asset for an agreed period of time.* It comes into effect in respect of all assets leased during the accounting period that commences on or after 1.4.2001. Lease is often called as *off balance sheet* financing because neither the leased assets nor the liabilities under lease contracts appear on a firm's balance sheet. The terms and conditions regulating the lease arrangement are given in the lease contract made between the lessor and the lessee.

Features of Lease Financing :

From the above discussion we can see that lease financing has a number of features which distinguishes it from debt and equity financing. The following are the most significant features of lease financing :

(1) **Equipment leasing :** It is a specialised means of financing in which the asset, property or equipment is leased instead of providing direct cash to the company.

(2) **Related party :** There are two parties involved in lease financing. The owner who gives the right of economic use of the assets to the user is known as the lessor and the user who uses the asset is known as the lessee.

(3) **Ownership :** Ownership is retained in the hands of lessor. The title of holding the asset does not pass to the lessee. They are only eligible to use the asset and at the end of the lease period, the particular asset reverts back to the lessor.

(4) **Depreciation :** As the lessor is the legal owner of the asset, he is entitled to claim depreciation on the assets.

(5) **Lease rentals :** A payment which has to be made periodically by the lessee to the lessor for using the asset is known as the lease rental.

(6) **Period :** The arrangement of leasing has to be made for a specific period of time which may cover the entire economic life of the asset or may cover a shorter period than the useful life of the asset.

(7) **Legal aspect :** There is no definite statute which governs lease financing. The provisions relating to bailment in the Indian Contract Act govern equipment leasing.

(8) **Taxation :** Lease rentals paid by the lessee is a fully tax deductible expense whereas the lease rentals received by the lessor are taxable as Business Income in the hands of lessor.

(9) **Off-balance sheet item :** As neither the leased assets nor the liabilities under lease contract appear in the balance sheet, it is often called as an off-balance sheet item.

(10) **Lease contract :** A contract is made between the lessor and the lessee which contains the terms and conditions regulating the lease arrangements.

Classification of Lease Financing :

The classification of leases adopted in AS-19 is based on the extent to which risk and rewards incident to ownership of a leased asset lie with the lessor or the lessee. Broadly speaking, the lease may be classified as a 'Finance Lease' and an 'Operating Lease'.

(A) Finance Lease :

A finance lease is also known as a 'Capital Lease', 'Full pay-out lease', 'Long-term lease', 'Close ended lease' etc. It is generally a non-cancellable contractual obligation which usually covers the full useful economic life of the assets or a period that is close to the economic life. It transfers substantially all the risks and rewards of ownership of an asset. As the lessee gets an uncontested right to use the asset over its entire working life, the lessee's position is quite similar to that of owner though the lessor retains title of the asset.

The lessor receives a lease rental during the non-cancellable period or primary lease period which covers fully the cost of the asset as well as a reasonable return on the funds used to purchase that particular asset. As the lease rental covers fully the cost of asset over the term of lease, financial leases are, therefore, also called as capital or full payment leases. The responsibility of repairs, maintenance and insurance of the assets generally lies with the lessee. The lessee has also to bear the risk of obsolescence. Lessee has the right of uninterrupted use of the asset till lease payments are made but he has no right to sell the asset without the permission of the lessor. At the end of the lease period, the assets may be returned to the lessor or handled as per the lease contract.

Financial lease is essentially a form of borrowing. The lessor buys the asset which is identified by the lessee as per his requirement. The lessor may not be involved in dealing with the vendor in respect of the negotiation of price, delivery schedule etc. It may be fixed by the lessee with the manufacturer or supplier of asset. The lessor only pays the purchase price to the manufacturer or supplier and signs contract to lease it out to the lessee.

According to Financial Accounting Standards Board Statement No. 13 (FASB No. 13), if at its inception, a lease meets one or more of the following criteria, the lease shall be classified as a financial or capital lease by the lessee :

(i) The lessor transfers title to the lessee at the end of the lease period.

(ii) That lease contains an option to purchase the asset at a bargain price.

(iii) The lease period is equal to or greater than 75% of the estimated economic life of the asset.

(iv) At the beginning of the lease, the present value of the minimum lease payments/rentals equals or exceeds 90% of the fair value of the leased property to the lessor (less any investment or tax credit realised by the lessor).

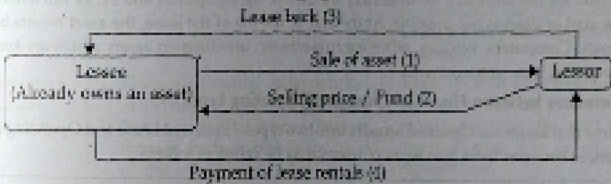
On the basis of how the lessee acquires the asset, financial lease may be of different types :

(a) **Direct Lease :** Most financial leases are direct leases. The lessor purchases the asset identified by the lessee from the manufacturer or supplier and hands it over to the lessee. A manufacturer instead of entering in a sale agreement to act as a seller can act as a lessor under the lease agreement and can deliver the asset to the lessee. This direct lease again be classified into two groups :

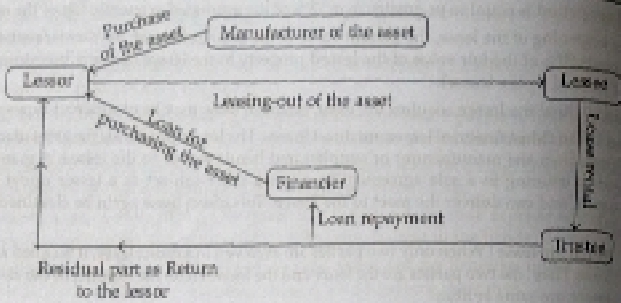
(a) **Bi-partite lease :** When only two parties are involved in a direct lease, it is called a bi-partite lease. Here, the two parties are the *lessor* and the *lessee*. Here, the supplier of the asset and the lessor are same entities.

(b) **Tri-partite lease :** When three parties are involved in a direct lease, it is considered as a tripartite lease. Here, the three parties are the *lessor*, the *lessee* and the *supplier* of the leased-out asset.

(b) **Sale and Lease Back :** The sale and lease back arrangement is essentially helpful for the firms having fixed assets but there is a shortage of funds. This is a special financing lease arrangement in which the lessee who is already the owner of the asset, sells the asset to the lessor, who, in turn, leases the asset back to the owner in return of periodic lease payments. Under this arrangement, lessee not only retains the economic use of the assets but also gets the fund from the sale of the assets to the lessor which augments the liquidity position of the company. The mechanism of sale and lease back can be shown in the following figure :



(c) **Leveraged Lease :** Leveraged lease involves three parties to the transaction (i) lessor (equity investor) (ii) lessee and (iii) financier or lender. When the cost of the asset is too high, lessor may borrow a substantial portion (almost 75%) of the purchase price of the asset from some lender which may be a financial institution or a bank. The loan taken by the lessor is secured by the assets and lease rental. Then the asset is given to the lessee who in return pays lease rentals to the lessor or to the financier of the asset directly as a part of the loan repayment as per the lease agreement. The surplus, if there is any, which arises out of the difference between the lease rental and the repayment portion goes to the lessor. So, lease payment must be large enough to meet the loan repayment, payment of interest to the lender as well as provide a return to the lessor. The lease rentals are distributed first to the lender to repay the loan amount taken by the lessor to purchase the leased asset and the balance, if any, goes to the lessor. The transaction is routed through a trustee who looks after the interest of the lender and lessor. This whole arrangement is called leveraged lease. Under this arrangement, the lessor acts as an equity participant who supplies only a part of the cost of the assets and the lender supplies the balance. This process of leveraged lease can be shown by a flow-chart.



III) Operating Lease :

As per AS-19, the operating lease is a lease which is not a finance lease. Any lease which does not satisfy any one of the four conditions stated in FASB No. 13, is known as 'Operating Lease' or a 'True Lease' or 'Non-financial Lease' or 'Open-ended Lease' or 'Service lease'. An operating lease or service lease is an arrangement under which the lessee acquires the economic use of the asset on a time to time basis. It is a short-term, cancellable lease agreement. Under this agreement the lessor is responsible for repairs, maintenance and insurance of the assets. The lessor is also responsible to pay tax on lease rentals. The asset may be given on lease to different lessees one after another on time to time basis. The lease rentals payable by different lessees during the lease period are not sufficient to cover fully the cost of the equipment and hence full amortisation of the cost of asset is not possible. At the end of the life of the lease, the asset reverts back to the lessor. Computers, vehicles, office equipments etc. are common assets which are leased under operating lease arrangements.

➧ *Differences between Financial Lease and Operating Lease :*

We know that leases are classified broadly into two types – Financial Lease and Operating Lease. The differences between these two types of lease may be stated as follows :

Points of Difference	Financial Lease	Operating Lease
1. Nature	1. Financial lease is basically a form of borrowing. The lessee has to decide whether to lease or borrow and buy the asset.	1. An operating lease is basically a rental agreement. The lessee has to decide whether to lease or buy the asset.
2. Role of lessor	2. The lessor takes the role of a financier. They cannot render specialised service in connection with the asset.	2. The lessor is specialised in handling and operating the particular asset and generally provides specialised services.
3. Purchase of asset	3. The lessor buys the asset which is identified by the lessee as per his requirement i.e., the leased asset is use-specific.	3. The lessor buys the asset of common-use activity and leases it out to different lessees successively.
4. Coverage period	4. Financial lease covers the full useful economic life of the assets or a period that is close to the economic life.	4. Operating lease covers significantly less than the useful economic life of the asset. It is a short-term lease.
5. Cancellability	5. It is generally a non-cancellable contractual obligation.	5. The lease is usually cancellable at short notice.
6. Risk and rewards	6. Financial lease transfers substantially all the risks and rewards of ownership to the lessee. The lessor only remains the legal owner of the asset.	6. Under this lease agreement lessee is given only the right to use the asset for a certain period of time. Hence, the risks and rewards associated with the ownership remain with the lessor.
7. Risk of obsolescence and maintenance	7. As the equipment is chosen by the lessee, the risk of obsolescence and the liability for repair, maintenance and insurance of the equipment rests with the lessee.	7. Operating leases normally include the maintenance clause requiring the lessor to maintain the leased asset and also to bear the risk of obsolescence.
8. Number of lessees	8. As the lease period covers most the economic life of the asset, the lessor relies on a single lessee to recover his investment.	8. Since the lease period is shorter than the expected life of the asset, the lessor does not rely on the single lessee for the recovery of his investments. So, the number of lessee should be at least two or more than two.
9. Amortisation of the cost of the asset	9. The lessors' capital outlay is fully amortised during the primary lease period. The lessor recovers through the lease rentals the cost of the leased asset along with interest and profit. Hence, it is called *full-pay-out lease*.	9. Since the lease periods are shorter, the lease rentals are not sufficient to totally amortise the cost of the assets. Hence, it is called a *non-payout lease*.
10. Types of assets	10. This type of lease is generally suitable for equipment which is tailor made and does not have ready resale or re-lease market. e.g., Heavy machines.	10. This type of lease is suitable for equipment having longer economic life and ready resale or re-lease market. e.g., Automobiles, Computers, Office equipments etc.
11. Classification	11. On the basis of how the lessee acquires the asset, financial lease may be of different types.	11. There is no such classification in respect of an operating lease.
12. Purchase of the asset by the lessee	12. A finance lease may provide a right or option, to the lessee, to purchase the equipment at a future date.	12. Under the operating lease, no such right is given to the lessee.

Advantages of Lease Financing :

Lease financing brings actual benefit for both the lessor and the lessee. These benefits can be summarised as follows :

To the lessor :

The lessor can be benefitted out of lease financing in the following ways :

(1) **Ownership :** The lessor retains ownership of the assets and the asset reverts back to the lessor at the end of the lease period. Moreover, the lessor can take repossession of the asset in the event of any default on the part of lessee.

(2) **Security :** As the lessor retains full ownership of the asset, his interest is fully secured. The lessor can take back the asset leased at the end of the lease period or in case of default payment conveniently realise an asset secured against a loan payment.

(3) **Depreciation :** As the lessor is the actual owner of the asset, he can charge depreciation on the asset leased. Under the existing provisions of the Income Tax Act, the owner can charge depreciation on the basis of diminishing method and in respect of ships, straight line depreciation is permissible.

(4) **Tax relief :** Depreciation is a charge against profit and hence the lessor can take the benefit of tax relief by way of charging depreciation on leased assets to a great extent.

(5) **Lease rentals :** Lessor gets a fixed amount of lease rentals regularly which is a steady source of income over the lease period.

(6) **Cash inflow :** When the asset is reverted back to the lessor, he can sell the asset in the market which enables inflow of cash.

(7) **High profitability :** Generally, lease rentals received by the lessor is higher than the interest paid by them on the amount of borrowings. The difference between these two amounts shows a positive figure which signifies profitability.

(8) **Growth potentiality :** Lease financing is regarded as a more convenient form of financing than debt financing. Every year many companies are coming up. Leasing maintains the economic growth even during a period of depression as the lessee can get the benefit of economic use of the asset without actually purchasing the asset during a recession. Thus, there is a potentiality of a very large growing market for leasing companies in India.

To the lessee :

The benefits which arises to the lessee are :

(1) **Capital outlay :** The lessee is not required to incur any capital expenditure for purchase of fixed assets, such as, land, building, plant, machinery, heavy equipments etc. So, the lessee can start his business without making any initial investment to acquire fixed assets.

(2) **Convenience :** Lease financing is regarded as a more convenient form of financing than debt financing. Borrowing from banks and financial institutions involve a long and complicated procedure. In comparison, leases are less restrictive and can be negotiated faster.

(3) **Short period lease :** Buying of an asset for a short period is inconvenient, costly and time consuming because arrangements have to be made to resell the asset. In such a case, it is better to take the asset under the lease arrangement.

(4) **Tax shield :** Leasing can provide the tax advantages to the lessee if the lessee is a tax paying entity. When a company acquires an asset on lease, the full amount of lease rentals is deductible for tax purposes.

Cost of repairs, maintenance, insurance etc. : Under lease agreement, a lessee may be able to pass on a number of 'nuisance costs' to the lessor, such as, legal fees, taxes, insurance, repairs and maintenance etc.

Off-balance sheet financing : The lease is considered as a hidden source of financing or off-balance sheet financing as the lessee is not required to show the assets acquired on lease in the balance sheet as the other debts and loan appear. Thus, window dressing of balance sheet is possible which mislead the investor regarding the financial leverage of the firm.

Borrowing capacity : As the lease does not appear in the balance sheet as debt, the firm's borrowing capacity remains intact. In otherwords it may be argued that the lease enhances the real borrowing capacity of the firm.

Risk of obsolescence : Risk of obsolescence can be averted under the lease arrangement. The chances of technological obsolescence is highly associated with assets, such as computers, electronic goods etc. Under the lease arrangement, the lessee can transfer the risk of obsolescence to the manufacturer-lessor.

Specialised services : With a full service lease, the lessee can get the advantage of specialised services from the lessor at a lower cost for assets like computers.

(10) **Dilution of control :** Dilution of control is not possible from the lessee's view point under the lease agreement. Sometimes, financial institutions, while lending funds to the borrower may insist for conversion of loan into equity or may nominate directors on the board. Such dilution of control is not possible under lease financing.

(11) **Costly assets :** Lease financing is very much effective for the assets which are very costly. Under this arrangement, a lessee can get the benefit of the same without going to purchase it.

(12) **Testing of assets :** Sometimes, it may become essential to test the asset before actually going to purchase the same. Lease arrangement provides this facility to the prospective buyer of the asset.

(13) **Flexibility in lease rentals :** The lease rentals are structured in such a way that it is convenient for the lessee to pay the rentals from the funds generated from operations. Not only that, the lease period is chosen in such a way so as to enable the lessee to pay rentals conveniently.

Disadvantages of Lease Financing :

Like other sources of financing, it is also not free from demerits. The disadvantages of lease financing are noted below :

To the lessor :

The disadvantages from the point of view of the lessor are :

(1) **Cost of repairs, maintenance etc. :** In lease financing the lessor has to bear a number of nuisance costs that usually accompany ownership including legal fees, insurance, repair, maintenance etc.

(2) **Taxes :** The lease rentals received by the lessor is fully taxable in the hands of lessor.

(3) **Control :** Lessor cannot impose any power of control on the lessee's business which may be possible in the case of direct loan given to a borrower firm by nominating a director on the board or by imposing a condition of convertibility of loan into equity capital.

(4) **Risk of obsolescence :** The lessor has also to bear the risk of technological obsolescence in the near future in case of assets such as computers, electronic goods etc.

(5) **Sales tax :** Sales tax may be charged twice in case of a lease financing transaction. The lessor has to pay sales tax at the time of buying the asset and again it has to be paid at the time of leasing the asset to the lessee. However, in case of VAT (Value Added Tax), the lessor gets an input tax credit.

To the lessee :

The disadvantages from the point of view of lessee are :

(1) **Restriction on use :** The lessor may impose, as the owner of the asset, certain restrictions on use of the asset. Again, lessee cannot make any additions or alterations with the leased asset to fit his requirements.

(2) **Default :** If the lessee defaults in making payment of lease rentals or in complying with any terms and conditions of the lease agreement, the lessor can take back the leased asset.

(3) **Technological changes :** In case of technological changes, an upgradation of the asset is required to maintain efficiency. But, the lessor may not allow such upgradation or alteration or modification being the owner of the asset or the lessee may not even want to invest in making the changes.

(4) **Understatement of asset :** As the lessee does not show the leased asset in the balance sheet, it may lead to underestimation of the asset. But it has to be mentioned as a foot note to the balance sheet.

(5) **Higher payout obligation :** In case of finance lease if the lessee opts for premature termination of the lease agreement because of any reason, he may have to pay higher rentals.

(6) **Residual value :** The lessee cannot enjoy the residual value of the assets which may increase due to inflation because of his non-ownership of the asset.

The growth of equipment leasing is of recent origin and its volume in India is quite modest. Many private sector non-bank financial companies are engaged in lease financing. Some of them are Infrastructure Leasing and Financial Services Ltd. (IL & FS), ICICI, IFC, LIC, NDFC, IRBI etc. whereas the lessee companies include many leading corporations in both public and private sectors and many manufacturing companies.

5.3.6. Hire Purchase

Hire purchase involves a system under which term loans for purchase of goods and services are advanced to be liquidated in stages through a contractual obligation. Hire purchase credit may be provided by the seller himself or by any financial institution.

Under the hire purchase system the customer called the *hire purchaser*, gets the possession of the goods immediately, can use it and pay the price in instalments. However, the ownership of the goods remain with the seller who is called the *hire vendor* and passes to the hire purchaser only after the payment of last instalment. Usually, a certain amount is paid at the time of delivery and the balance amount is paid together with interest on the unpaid amount in different instalments. Each instalment paid by the hire purchaser is treated as the hire charges for using the asset. In case of default in payment of any instalment, the seller can repossess the assets without compensating the hire purchaser.

In India the Hire Purchase Act, 1972 was promulgated to govern hire purchase agreements. Though the Act was passed by the parliament and had got the assent of the President it did not become operational. Hire purchase agreements are rather being governed by the laws of contracts.

Section 2(C) of the Hire Purchase Act has defined a hire purchase agreement as *an agreement under which goods are let out on hire and under which the hirer has an option to purchase them in accordance with the terms of the agreement and includes the agreement under which :*

(i) Possession of goods is delivered by the owner thereof to a person on condition that such person pays the agreed amount in periodical instalment ;

(ii) The property in the goods is to pass to such a person on payment of the last instalment ; and

(iii) Such a person has right to terminate the agreement at any time before the property passes.

AS-19 which deals with the leases is also applicable to Hire Purchase agreement. Para-4 of AS-19 states that *the definition of a lease includes agreements for the hire of an asset which contains a provision giving the hirer an option to acquire title to the asset upon the fulfillment of agreed condition. These agreements are commonly known as hire purchase agreements. Hire purchase agreements include agreements under which the property in the asset is to pass to the hirer on the payment of the last instalment and the hirer has a right to terminate the agreement at any time before the property so passes.*

The hire purchaser shows the assets in his balance sheet and can charge depreciation on such assets. The amount of interest which is included in the payment of different instalments are eligible for deduction from the taxable income.

Features of Hire Purchase System :

The essential features of hire purchase system are :

(1) Possession : The hire vendor transfers only the possession of goods to the hire purchaser immediately after signing the hire purchase contract.

(2) Use of the asset : The hire purchaser is entitled to start using the goods immediately after receiving the possession.

(3) Payments made : On signing the agreement the hire purchaser may pay a certain initial amount which is called "Down Payment". The balance amount along with interest is paid in different instalments at a regular interval for a specific period of time.

(4) Hire purchase price : Hire purchase price is the total amount which is to be paid as per the hire purchase agreement. It includes the down payment and all the instalments together with the amount of interest.

(5) Title of the goods : Title or ownership of the goods passes to the buyer only after the payment of the last instalment.

(6) Termination : The hire purchaser has the right to terminate the agreement at any time before the title of the goods passes to the hire purchaser.

(7) Default in payment : The hire vendor has the right to repossess the goods without making any compensation in case of default in payment of any instalment. Even if only the last instalment is not paid, the seller can take back the goods from the buyer.

- Difference between Hire Purchase and Lease Financing : Now, we can indicate some of the differences between the procedures of hire purchase and lease financing. These are as follows :

Points of Difference	Hire purchase	Lease financing
1. Ownership	1. According to the hire purchase agreement, the hire purchaser gets the ownership of assets after the payment of last instalment.	1. According to the lease contract, the lessee does not get the ownership of the assets even after the completion of the lease period.
2. Salvage value of the asset	2. The hire purchaser also owns the salvage value of the asset after the completion of hire purchase agreement period.	2. The lessee, however does not own the salvage value of the asset after the completion of lease period.

(Contd.)

Points of Difference	Hire purchase	Lease financing
3. Depreciation cost	3. The hire purchaser can charge depreciation on the asset.	3. The lessee, however, does not have any right to charge depreciation on the asset.
4. Tax concessions	4. The hire purchaser enjoys tax concessions only on 'interest' amount paid in the instalments.	4. The lessee enjoys tax concessions on the whole amount of lease rental paid to the lessor.
5. Reflections of transaction in the balance sheet	5. This transaction is reflected in the balance sheet of the 'hire purchaser'	5. This transaction, however, is not reflected in the balance sheet of the "lessee".

5.3.7. Commercial Banks (i.e., Bank Financing)

The commercial banks have rendered great service to Indian business in meeting its need of current financial and short-term capital. The major portion of working capital loans are provided by commercial banks. They provide a wide variety of loans to meet the specific requirements of a concern. Commercial banks provide loans and advances in the following forms :

(i) Loans
(ii) Cash credits
(iii) Overdrafts
(iv) Discounting of bills

(i) **Loans :** When a bank makes an advance in lump sum with or without security, the whole of which is withdrawn in cash immediately by the borrower who undertakes to repay it in instalments, is called a loan. The entire amount of loan is paid to the borrower either in cash or by credit to his deposit account. Loans are sanctioned for definite purpose and periods. The borrower is required to pay the interest at an agreed rate on the whole amount from the date of sanction whether he draws the full amount from the loan account or not. Repayments of loan may be made in instalments or at the expiry of a certain period. The rate of interest on loan is generally lower than the rate of interest on cash credits or bank overdrafts. Commercial banks usually provide short-term loans up to a period of one year for meeting working capital requirements of business firms. But term loans may also be provided for medium-term (which is repayable within 1 to 5 years) and long-term (which is repayable after 5 years). There is yet another type of loan sanctioned by the commercial banks, viz. *demand loan*. Demand loan is a loan which is to be payable on demand.

(ii) **Cash credits :** Cash credit is the most popular method of financing by commercial banks. It is an arrangement under which a borrower is allowed an advance up to a certain limit against the security of tangible assets or guarantees. The borrower need not borrow the entire amount of advance at one time, rather he can draw as often as required provided the limit of cash credit is not exceeded. It is also known as *secured credit*. But if the cash credit is not backed by any security, it is known as clean cash credit. The borrower gives a promissory note which is signed by two or more sureties in case of *clean cash credit*. Interest on cash credit is not charged on the full amount of the advance but on the amount actually availed of by the borrower. The Reserve Bank of India issued a directive to all scheduled commercial banks on 28th March, 1970, prescribing a

commitment charge which banks should levy on the unutilised portion of the credit limits. The borrower also enjoys the facility of repaying the amount, partially or fully, as and when he desires. These accounts are repayable on demand, but banks usually do not recall such advances, unless they are compelled to do so by adverse factors. Cash credit operate against the security of inventory and accounts receivables in the form of hypothecation/pledge.

(iii) **Overdrafts :** Under this arrangement, the commercial bank allows its customer to overdraw his current account upto a certain limit so that it shows a debit balance. So, opening of an overdraft account requires that a current account will have to be formally opened. Any business concern can enter into this arrangement to tide over a temporary shortage of funds. The customer is charged interest on daily overdrawn balances and not on the limit sanctioned. Overdraft accounts can either be clean overdraft, partly secured or fully secured. The security in an overdraft account may be shares, debentures and government securities. In special cases, life insurance policies and fixed deposit receipts are also accepted. So, there is no difference as such between overdrafts and cash credit. The main difference between these two is that overdraft is allowed for a shorter period and it is a temporary arrangement whereas the cash credit is allowed for a longer period of time.

(iv) **Discounting of bills /purchase of bills :** Bill arises out of trade transactions. The seller of goods draws the bill on the purchaser. The bill may be either clean or documentary, i.e., supported by a document of title to goods like a railway receipt, and may be payable on demand or after a period not exceeding 90 days.

Commercial banks finance the business concern by discounting their bills at a price lower than their face value. The bankers however, collect the full amount on maturity. The difference between these two amounts represent the earnings of the bankers for the period. This item of income is called a 'discount'. In case the bill discounted is dishonoured by non-payments, the bank recovers the full amount of the bill from the customer along with expenses in that connection.

Bills are sometimes purchased from approved customers. Although the term 'bills purchased' gives the impression that the bank becomes the owner of such bills, in actual practice the bank holds the bill only as a security for the advance. A bank has to be very cautious and grant advances against the purchase or discount of a bill only to those customers who are creditworthy and have established a steady relationship with the bank.

There may be some other form of advances which are granted by the commercial banks, such as :

(v) **Advances against goods :** The term goods include all forms of movables, such as agricultural commodities, industrial raw materials or partly finished goods etc. A banker accepts them as security and allows advances against them.

(vi) **Advance against documents of title to good :** These documents include a bill of lading, dock warehouse keeper's certificate, railway receipt etc. An advance against the pledge of such documents is equivalent to an advance against the pledge of goods themselves.

(vii) **Advances against supply-bills :** Advances may be granted by banks against bills for supply of goods to government or semi government departments which is obtained against an order after the acceptance of a tender. Again, advances against bills from contractors for work executed either wholly or partially, entered into with the government agencies also come under this category.

5.3.8. Public Deposits

Public deposits are the fixed deposits accepted by the public companies directly from the public, popularly it is a source of short-term and medium-term finance. After the commencement of Companies Act, 2013, no company shall invite, accept or renew deposits from the public except in manner provided in Section 73, 74, 75 and 76 of this Act. However, these sections do not apply to a banking company and non-banking financial company as defined in the Reserve Bank of India Act, 1934 and to such other company as the Central Government may, after consultation with Reserve Bank of India, specify.

Features of Public Deposits :

The important features of public deposit are :

(1) **Advertisements :** While inviting deposits the company must issue an advertisement (which indicates its financial position) in a leading english newspaper and also in a vernacular newspaper circulating in the state in which the registered office of the company is situated.

(2) **Form of application :** No company can accept or renew any deposit unless an application is made by the intending depositor for the acceptance of such deposits.

(3) **Ceiling on deposits :** Deposits from the general public must not exceed 25% of the aggregate of the paid-up share capital and free reserves. But it is 10% as against unsecured debentures or any deposits from its shareholders or any deposits guaranteed by its directors.

(4) **Maturity period :** The minimum period for acceptance of deposits is normally 6 months and the maximum period is 36 months. However, for meeting short-term requirement of funds, a company may accept deposits for a period of 3 months, the total amount of which should not exceed 10% of share capital and free reserves.

(5) **Repayability :** No company shall accept any deposit which is repayable on demand. Once a deposit is accepted for a certain period, the company cannot repay the same before the expiry of six months.

(6) **Receipts to depositors :** It is necessary for every company to give receipts for the amount received by them to the depositors or their agents for accepting or renewing any deposit.

(7) **Register :** A register should be maintained by every company mentioning the names and addresses of the depositors, amount of deposit, date of deposit, maturity date, rate of interest and the date of interest payment etc.

(8) **Interest :** The rate of interest may vary from 8% to 12% depending upon the tenure of deposit.

(9) **Return :** Companies accepting public deposit must regularly file return giving detailed information regarding such deposits to the Registrar of Companies. A copy of return is also required to be furnished to the Reserve Bank of India.

(10) **Brokerage :** For mobilising deposits, companies may require to pay brokerage to the brokers, managers or consultants which is usually 1% of such deposit.

(11) **Liquid assets :** Companies must maintain a part of assets in a liquid state. Companies should deposit or invest by the 30th day of April every year, a sum which shall not be less than 10% of deposits maturing during the year ending on 31st March next year in a scheduled bank or in government securities or in trust securities. The amount so set aside should be utilised for the purpose of repayment of deposits only.

(12) **Penalty on default :** If a company fails to repay any deposit or a part thereof, the Company Law Board may direct the company by order to make the repayment forthwith or within a stipulated time. If the company fails to comply with the order of the Company Law Board, it shall be punishable with imprisonment upto 3 years and shall be liable to pay a fine of not less than ₹ 50 for every day during which such non-compliance continues.

Advantages of Public Deposits :

The merits of borrowing by public deposits are as follows :

From the company's point of view :

Public deposits offer the following advantages to the company :

(1) **Simple Procedure :** Financing through public deposit is simple without much complicated formalities.

(2) **Less costly :** It is beneficial for the company since it receives funds on lower rates of interest than charged by banks and financial institutions.

(3) **Cost of collection :** The cost of collecting deposits from the public is less too.

(4) **Security :** The public deposits are usually not backed by any security or assets of the company. So, the company can use its assets as security for raising capital from other sources.

(5) **Flexibility :** Public deposits introduce flexibility in the financial planning. These can be repaid when they are not required.

(6) **Tax benefit :** As the interest paid on public deposits is a charge against profit, the company can have tax benefit on such interest.

(7) **Dilution of control :** The depositors do not have any right to interfere with the internal management of the company. Thus, there is no dilution of control of shareholders.

(8) **Trading on equity :** It helps in trading on equity if the company is earning more than the rate of interest paid on public deposits. Thus, the shareholders can get higher rate of dividend.

From the investor's point of view :

The investors also find certain advantages in public deposits, which are :

(1) **Rate of interest :** The rate of interest is usually higher than several alternative sources, such as, banks, post offices etc.

(2) **Maturity period :** As the maturity period is short ranging from 6 months to 36 months, investors are in a position to utilise their money in different alternative sources, if necessary, just after a maximum of a 3 year time period.

Disadvantages of Public Deposits :

Inspite of many advantages, the following are the demerits of public deposits :

From the company's point of view :

The disadvantages of public deposit from the company's point of view are :

(1) **Dependability :** Raising funds through public deposits is not a reliable and dependable source of finance. It is difficult to predict whether public deposits would be forthcoming to the desired extent. The depositors may not respond when the conditions in the company are uncertain. Therefore, such deposits are termed as 'fair weather friends'.

(2) **New companies :** New companies and companies with uncertain earnings cannot raise finance through public deposits.

(3) **Limitation of amount :** The amount that can be raised through public deposit is limited as per the Sections 73 to 76 of the Companies Act, 2013. Maximum of 25% of the paid up capital and reserves can be raised from the general public, whereas the percentage is only 10 if it is raised from shareholders, directors etc.

(4) **Short maturity period :** The maturity period of public deposit is relatively short though they can be renewed, but it is not a wise thing to depend on them for long-term financing.

(5) **Unhealthy trend in capital market :** There are numerous rates of interest offered by different companies which create unhealthy trends in the capital market and this is detrimental to the development of the capital market too.

From the investor's point of view :

The disadvantages from the investor's point of view are :

(1) **Security :** Investors do not get any security for their deposits. Money deposited by them may be used by the management in any way it likes. So, the risk of investment in public deposit is much higher.

(2) **Taxability :** Interest income on public deposit is not exempted from tax. Hence, many investors do not like to invest in public deposits.

(3) **No guarantee :** Public deposits are neither covered by any insurance nor guaranteed by the government as opposed to bank deposits. Inspite of many safeguards, there is a danger of losing money to the mismanaged companies.

(4) **Liquidity :** Public deposits are not very liquid assets. It is possible for an investor to withdraw his deposit from a bank easily but not from a company.

(5) **Narrowed the investment market :** The method has narrowed the investment market by restricting the supply of good securities such as shares and debentures to an ordinary investor.

5.3.9. Inter-corporate Deposits

A company can borrow funds for a short period upto 6 months from other companies which have surplus liquidity. Such borrowings are known as inter-corporate deposits. Inter-corporate deposits can be of three types : (*i*) call deposit, (*ii*) three-months deposits and (*iii*) six-months deposit.

A deposit is said to be a *call deposit* if it is withdrawable by the lender any time by giving a day's notice. But the time requirement to mobilise the process is at least 3 days.

To overcome short-term requirement generated out of dividend payment, excessive import, unplanned capital expenditure etc., a company can take deposits for 3 months from the other companies. This deposit is known as *three-months deposit*. In practice this inter-corporate deposit is more popular.

If a company receives deposits for a period upto 6 months from the another company, it is known as *six months deposit*. Lending companies cannot extend deposits beyond this time limit.

The rate of interest on inter-corporate deposits varies depending upon the amount involved and the time period. Since 1973, the market for these deposits have been expanding in India as the restrictions on working capital finances were imposed by the Reserve Bank of India in that year. There are no limits on borrowings for inter-corporate deposits made for short term.

Inter-corporate deposit transactions are very easy as there are neither any legal restrictions nor any rules and regulations binding such transactions. This transaction can be done secretly, so that there will be no chance of unhealthy competition and/or any possibility of undercutting rates of interest. Sometimes inter-corporate deposit transactions are made based on personal contacts.

5.3.10. Commercial Paper

An emerging source of financing working capital requirements of the corporate enterprises is the Commercial Paper (CP). It is an unsecured promissory note issued by the large listed joint stock companies to raise short-term funds under the approval of the Reserve Bank of India with a fixed maturity.

Commercial Paper was first introduced in the Indian money market in the year 1990 on the recommendations of the Working Group on Money Market (Vaghul Committee, 1988) and the Reserve Bank of India's announcement in its credit policy statement dated March 27, 1989.

The companies which issue a commercial paper must have a minimum tangible net worth of ₹ 5 crore as per the latest audited accounts, sound financial health and should enjoy a high credit rating. A credit rating agency called CRISIL (Credit Rating Institution Services of India Limited) has been set up in India by ICICI and UTI which has been approved by the RBI to rate commercial papers.

Commercial paper has a fixed maturity period mostly ranging from 91 to 180 days. But it can also be issued for a maximum period of one year and a minimum period of 15 days. It has to be issued in multiples of ₹ 5 lakhs and the minimum size of an issue to a single investor is of the face value of ₹ 25 lakhs, though it can be issued at a discounted value too. The face value of a commercial paper issued by any company should not exceed 30 per cent of its working capital limit and the company should have a minimum current ratio of 1·33 as per the latest audited balance sheet.

Government companies, companies governed by FERA with prior approval of the government and Non-resident Indians (NRIs) are also eligible to issue commercial papers. The participants or the investors of commercial paper can be corporate bodies, banks, UTI, LIC, GIC etc.

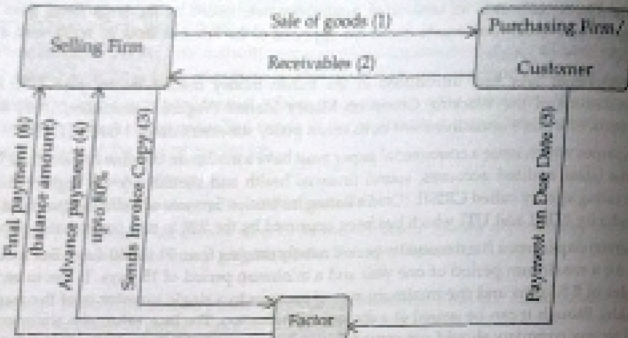
A commercial paper is a cheaper source of raising short-term finance as compared to the bank credit and proves to be effective even during a period of tight bank credit. It provides a diversified source of capital to the lender. It cannot be redeemed before the period of maturity even if the issuing firm has surplus funds to pay back.

5.3.11. Factoring

Another method of raising short-term capital is through account receivable credit offered by factors. *A 'factor' is a financial institution that manages the collection of bills receivables of business enterprises.* Factoring is a unique financial innovation. Factoring service is an arrangement between the seller of goods and a financial institution, called the 'factor' under which the later takes over the credit collection, purchase and recovery function of the former. Being a financial service it is designed to help business firms on the management of receivables. A businessman or a firm can obtain cash for invoices he sends to his customers in respect of supply of goods and services to them through factoring. Thus, factoring is also termed as 'Invoice Discounting'. In a nutshell, factoring is the collection and finance service designed to improve the cash flow position of the sellers by converting sales invoices into ready cash. Hence, funds can be obtained under this arrangement at the moment of sales in respect of credit sales. The factor works between the seller and the buyer for realisation of credit sales, once a sale transaction is completed. Factors provide all these services against a charge which is worth paying, as compared to the other sources of finance.

The procedure of factoring starts when the seller makes an agreement with the factor. After signing an agreement regarding the terms and conditions, the client passes all credit sales to the factor and informs their customers that the payment has to be made to the factor. The factor purchases the invoices and makes the advances, generally upto 80% of the invoice amount. Once the customer makes payment to the factor, the balance amount due to the client is paid by the factor. The factor also

conducts a follow up if the customer does not pay by the due date. The mechanism of factoring can be shown under the following figure :



In addition to purchasing the receivables, factors may also render other services which include :

(i) *Bill discounting facility*

(ii) Maintenance of sales ledger

(iii) Credit control

(iv) Protection from bad debts

(v) *Advisory services to their clients etc.*

Factoring may be on a recourse basis or on a non-recourse basis. In a recourse basis factoring, the factor does not provide any protection to the client against a customer's failure to meet debts. But in a non-recourse factoring, the credit risk or the bad debt risk is borne by the factor.

In case of a recourse factoring, the finance provided by the factor appears in the balance sheet only as a contingent liability and is treated as an off-balance sheet item. But in case of non-recourse factoring the borrower is not required to show it anywhere in the financial statements.

At present, factoring is rendered by only a few financial institutions in India on a recourse basis. The first factoring company in the country was set up by Can Bank Financial Services Ltd. and the Rashtriya Chemical and Fertilisers Ltd. The State Bank of India and a few other commercial banks formed SBI Factors and Commercial Services Ltd. Similarly Punjab National Bank, Allahabad Bank also started factoring services. The Small Industries Development Bank of India (SIDBI) introduced its own direct factoring services to help the small-scale sector in timely recovery of their sale proceeds. The report of the working group on Money Market (Vaghul Committee) constituted by the Reserve Bank of India has recommended that banks should be encouraged to set up factoring divisions to provide speedy finance to the business firms.

5.3.12. Customer Advances

This source of capital will be discussed in details in chapter 7 of this book.

5.3.13. Trade Creditors

This source will also be discussed in chapter 7 of this book.

Internal Sources

...than the external sources of capital there are also some internal sources of capital. Expansion or diversification of production capacity is carried out by the established companies primarily through internal resources such as retained profit or ploughing back of profits, provision for depreciation, proposed dividend etc. But a new company can raise funds only through external sources such as shares, debentures, loans, public deposits etc.

Retained Earnings / Ploughing Back of Profit

...already mentioned, retained earnings or ploughing back or reinvestment of a part of the profits is an ideal method of financing expansions and improvements. Every growing concern requires capital for expansion and improvement. The concept of ploughing back of profits is a management tool under which the entire profits are not distributed among the owners of capital but a part of the earned profits is 'ploughed back' or retained to be utilised in future. It is also called 'self financing', 'inter-financing' or 'financing from internal resources'. This is the only internal source of long-term financing. A company may before the declaration of any dividend in any financial year transfer such percentage of profits for that financial year as it may consider appropriate to the reserves of the company as per section 123(1) of the Companies Act, 2013.

Other than expansion and improvements, retained earnings can be utilised for the long-term purposes, such as purchase or construction of assets, modernisation, repayment of loan, redemption of preference share capital or debentures, meeting the long-term working capital etc. This is the cheaper source of financing and also strengthens the financial position of the company.

The practice of ploughing back of profits is influenced by many factors, such as :

(i) **Net profit :** Ploughing back of profits depends largely upon the amount of net profit that the company earned in a particular year. If a company has earned huge profits in any year, its capacity to retain profits will be higher.

(ii) **Dividend policy :** Dividend policy of the company determines the extent to which profits can be retained for ploughing back.

(iii) **Taxation policy :** Taxation policy of the Government also affects ploughing back of profit as retention is available only from the profit after tax.

(iv) **Statutory requirement :** According to section 123(1) of the Companies Act, 2013, a company may transfer such percentage of profits as it may consider appropriate to the reserves of the company before the declaration of dividend in any financial year.

(v) **Future Financial Requirement :** Future financial requirement of the company also affects retention. If there are highly profitable investment opportunities in the future, the company can go for more retention.

(vi) **Desires of shareholders :** Retention also depends on the shareholders' desires. If they want to have a regular income they may desire maximum distribution of profit and in such a situation retention may be low.

(vii) **Age of the company :** Age of the company also affects retention. A new company wants to grow big in future, so their retention will be high. But if the company is an old profit making company, their retention may be lower in amount.

(viii) **Attitude of the management :** The quantum of retention also depends upon the attitude of the management towards financing by retained earnings.

Advantages of Retained Earnings :

This method of raising finance is very useful as it does not cost anything to the company and also strengthens the financial position of the company. The main advantages of this method of raising long-term internal capital are :

(1) **Permanent source of capital :** As the equity shareholders are the owners of profit retained, the liability is to be paid off only at the time of liquidation. Hence, ploughing back of profit is identified as a permanent source of capital.

(2) **No fixed obligations :** It is a very economical method of financing because no return is to be paid on retained earnings and no fixed obligations are created.

(3) **Financial position :** It strengthens the company's financial position, increases credit-worthiness and enables a company to float new securities in the market and to raise debt capital without any difficulty.

(4) **Legal formalities :** Retained earnings are readily available internally. They are not required to comply with any legal formalities as in the case of other sources of financing.

(5) **Dilution of control :** There is no dilution of control when a firm relies on retained earnings.

(6) **Stable dividend policy :** Retained earnings can be a good source by which the rate of dividend on equity shares can be stabilised.

(7) **Development, extension etc. :** Re-investment of profit is the most advantageous method for raising additional capital required for development or extension of a project, for replacement of an asset, etc.

(8) **Depression :** Retained earnings enable the company to survive at a time of economic depression and uncertainty in the capital market.

(9) **Indicator of efficiency :** If a large amount of money is accumulated as reserves and retained profits, it indicates the efficiency of the management. It increases the confidence of the existing shareholders as well as the prospective shareholders.

(10) **Market value of shares :** Retained earnings help in increasing the market value of shares as it creates a good impression in the minds of the investor.

(11) **Cheaper source :** As compared to other sources of financing, this method is least costly as there is no flotation cost involved here.

(12) **Capital formation :** Reinvestment of earnings increases capital formation which is necessary for the economic development of the country.

(13) **Charge on assets :** Financing through retained earnings does not create any charge on the assets of the company. The assets remain free.

(14) **Self dependence :** Retained earnings makes the company self dependent as it does not have to depend upon outsiders, such as banks, financial institutions etc.

(15) **To make good of the deficiencies :** If there are any deficiencies in the provisions for depreciation, bad and doubtful debts etc., retained earnings can be utilised to make good such deficiencies.

(16) **Redemption of debt and replacement of asset :** It enables the company to redeem debentures as well as preference shares. It also helps in the replacement of assets after the expiry of the life of the assets.

(17) **Safety of investments :** It assures the investors a minimum rate of dividend and enhances the earnings capacity of the concern. This ultimately benefits the real owners of the company.

(18) **Increase in equity shareholders' fund :** As the equity shareholders are the real owners of the retention, it helps in increasing their fund in the company beyond the equity share capital.

(19) **Increased productivity :** It indirectly helps in increasing productivity since ploughing back of profit acts as a very economical method of financing. As it increases productivity, it helps in greater, better and cheaper production of goods and services.

Disadvantages of Retained Earnings :

If the policy of ploughing back is ill-planned, irrational and excessive, it may lead to the following disadvantages :

(1) **Low rate of equity dividend :** If there is a huge amount of undistributed profit, the rate of equity dividend will be low which may create dissatisfaction among the shareholders.

(2) **Misutilisation :** The management of a company may misutilise the retained earnings by investing them in unprofitable areas or by spending them unnecessarily.

(3) **Over-capitalisation :** It also creates threats of over-capitalisation of the company if the management utilises the retained earning for issuing bonus shares at a regular interval.

(4) **Creation of monopolies :** Excessive use of retained earnings continuously for a long period of time may encourage the formation of monopolies with all its inherent evils.

(5) **Manipulation in the value of shares :** The practice of retained earnings may be used to manipulate the prices of shares with a view to purchase the shares. As a result the genuine investors are deprived and economic wealth is concentrated in a few hands.

(6) **Evasion of tax :** Retention can also be made for the evasion of super-profit tax, thus reducing the revenue of the Government.

(7) **Interfering with the freedom of the investors :** Some investors may desire to withdraw the whole profit for other alternative uses. But by keeping a part of profit as retention in the business, shareholders cannot enjoy their full freedom.

(8) **Social waste :** Excessive ploughing back may be regarded as a social waste because the funds retained might have been utilised in a more productive way in alternative investment opportunities.

The advantages and disadvantages as stated above can also be categorised under the three broad heads :

(1) Advantages/disadvantages to the company.

(2) Advantages/disadvantages to the shareholders, and

(3) Advantages/disadvantages to the society or nation.

5.4.2. Provision for Depreciation

Depreciation policy is a matter of considerable importance to the financial executives. Depreciation means permanent decrease in the value of assets due to wear and tear, lapse of time, obsolescence, exhaustion, accident etc. Depreciation is a non cash expense as it does not involve any cash outflow. So, it can be treated as a very good source of internal financing for long-term purpose. Again, if the amount of depreciation is not used for buying the asset, it increases the size of cash inflows.

In a real life situation, depreciation is used as a tool for savings in payment of tax and dividends which results in withholding a part of the funds generated through normal trading operations. This will be clear with the following example :

	Case I ₹	Case II ₹	Case III ₹
Profit before depreciation and tax / PBDT	50,000	50,000	(50,000)
Less : Depreciation (say)	—	10,000	10,000
Profit after depreciation but before tax	50,000	40,000	(60,000)
Less : Income tax (say 50%)	25,000	20,000	—
Profit after Tax / PAT	25,000	20,000	(60,000)
Net Flow of Funds after tax (Depreciation + PAT)	25,000	30,000	(50,000)

The above example shows that in Case II the net flow of funds is the maximum which [illegible] In Case I and Case II, PBDT shows the same figure but charging of depreciation in case II reduces the tax liability and ultimately enhances its flow of fund. From case III, it is clear that if a company is running into losses, then any amount of depreciation charged will neither affect tax liability nor the payment of dividends. In such a situation depreciation does not mean withholding of fund and [illegible] treated as a source of fund. So, depreciation is a source of fund as long as the company makes profit. To conclude, it may be said that true fund flow from depreciation is the opportunity of saving cash outflow through taxation.

5.4.3. Provision for Taxation

It is also an important internal source of capital which will be discussed in chapter 7 of this book.

5.4.4. Proposed Dividend

Proposed dividend is a very short-term source of internal financing. The portion of profit which can legally be distributed to the shareholders of the company is called the dividend. When, after determining the amount of divisible profits at the end of the financial year as per the provisions of the Articles of Association of the company, dividend is declared, it is called proposed dividend. According to section 123(5) of the Companies Act, 2013, a company has to pay to the concerned shareholders by cheque or dividend warrant or in any electronic mode within 30 days from the declaration of dividend. Hence, there is a time gap between declaration of dividend and the payment of dividend. So, as long as the amount is not paid, it may be treated as the internal source of fund but for a very short span of time.

5.5. Venture Capital

There is another form of equity or debt financing designed specially for funding high-risk and high-reward projects known as venture capital.

Venture capital as a source of long-term business financing has emerged as a necessity for the potentially growth undertakings of new entrepreneurs. The term 'venture capital' refers to capital invested in a business or industrial enterprise which carries the elements of risk and insecurity and the chances of business hazards. According to Pratt, it may be referred to as *"the early stage financing of new and young enterprises seeking to grow rapidly."* Thus, the risk involved in venture capital investment is high and shareholders may not get good result in terms of dividends etc., but chances of capital gains may be there in the long-run.

[illegible]stor or financier is known as the 'Venture Capital Firm' or the 'Venture Capitalist' and the [illegible] where investment is made is known as the 'Venture Capital Undertaking'. A statutory [illegible] or a finance company or a mutual fund may act as the Venture Capitalist. They may invest [illegible] funds in a venture capital undertaking either in the form of equity share capital or as debt [illegible]. The venture capital firms 'by financing the risky proposals' can play a crucial and innovative [illegible] the development of small scale enterprises. The equity participation of venture capital firms [illegible] does not exceed 49% of the total equity capital of the undertaking and hence, the ownership [illegible] effective control remains with the entrepreneur. As the venture capital financing is risky, the [illegible] capital firm should be very careful, analytical and selective in financing the venture capital [illegible]takings. Before financing, the venture capital firm should take into consideration the feasibility, [illegible] competence, commercial viability, managerial skill of the undertaking, etc.

[illegible] financing may be provided for —

- Supporting a new concept or idea, i.e., development of new process/product.
- Assisting initial marketing activities.
- Technological upgradation.
- Adopting foreign technical know-how.
- Carrying out research and development activities for product development etc.

[illegible] India, the idea of venture capital was first initiated by the Industrial Finance Corporation of India [illegible] when it set-up the Risk Capital Foundation in 1975 with a view to assist entrepreneurs, [illegible] technologists and professionals who have skills but lack of finance. The concept of [illegible] capital financing was recognised for the first time in the fiscal budget for the year 1986-87. The finance minister in his budget speech for 1988-89 declared that a scheme will be formulated [illegible] which venture capital firms will be enabled to invest in new companies and be eligible for the [illegible] treatment of capital gains available to non-corporate entities. On December, 4, 1996, [illegible] approved the Venture Capital Funds (Regulations) 1996. Again, to facilitate the growth of a [illegible] capital industry in India, the Government of India has constituted Chandrasekhar Committee [illegible] recommendations which submitted its report in April, 2000.

[illegible] from Risk Capital Foundation which is renamed as IFCI Venture Capital Funds Ltd. (IVCF), [illegible] venture capitalists have been set up in India to undertake venture capital activities such [illegible] Technology Development and Information Company of India (TDICI), Gujarat Venture Financial [illegible], Can Bank Venture Capital Fund, State Bank Venture Capital Fund etc.

[illegible].6. Euro Issue

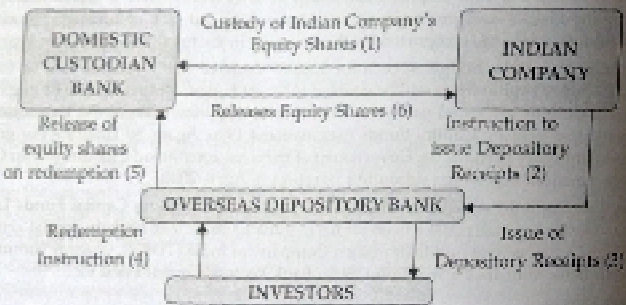
[illegible] from raising funds through an issue of securities or debt capital from the domestic market, the Government of India allows the Corporate sector to access funds from abroad through Euro Issue. [illegible] it may be treated *as an international source of finance*. The term 'Euro Issue' means the issue which is listed on European Stock Exchange but subscriptions for which can come from any part of the world except India. The Government of India permitted in the year 1992 to raise funds by issue of equity/debt capital in the international market. Over the last few years, more and more Indian corporates have begun to look at the Euro markets for meeting their financial needs. The Indian companies having a consistent track record of good performance, financial or otherwise, for a minimum period of three years are eligible to raise funds through Euro Issue. The Ministry of Finance [illegible] guidelines for Euro Issue w.e.f. April 1, 1992 on the basis of the proposal made in the Budget 1992-93. The most popular sources of Euro Issues include :

(1) Foreign Currency Convertible Bonds (FCCBs) and

(2) Depository Receipts (DRs)

Foreign Currency Convertible Bonds (FCCBs) : These are bonds issued to and subscribed by non-resident investors in any freely convertible foreign currency. FCCBs have fixed maturity date and carry an option for conversion into a fixed number of ordinary shares of the issuing company. The conversion may take place fully or partly on the basis of any equity related warrants attached to the instruments. These are unsecured instruments and carry a fixed rate of interest which is payable in terms of dollars. If the investors do not exercise the conversion option, then the redemption price of the instrument is also payable in dollars.

Depository Receipts (DRs) : An indirect method of raising equity capital from the capital markets of developed countries is to issue depository receipts. A Depository Receipt is a negotiable instrument representing a certain number of equity shares which are tradeable on stock exchange in Europe or in the US or both. It is quoted and traded in the currency of the country in which they trade, and are governed by the trading and settlement procedures of the market. These receipts are issued to the non-resident investors outside India against the issue of ordinary shares denominated in Indian rupees in the domestic market by the issuing company through an overseas intermediary (Overseas Depository Bank) called 'Depository'. The equity shares are registered in the Depository's name while such shares are physically custodized in the home market with another intermediary (Local Bank) called the 'custodian', who is the agent of Depository. The process of issuing Depository Receipts can be shown in the following chart —



When the funds are raised from the retail market in United States through Depository Receipts, it is called **American Depository Receipts or ADRs** and when they are tradeable on major International Stock Exchanges outside the United State — mainly the London Stock Exchange (LSE) — and in the US Over-the-counter Market is called **Global Depository Receipts or GDRs**. GDRs have evolved as European based instruments while ADRs evolved as US based instruments. Apart from ADRs and GDRs, there are also some other form of DRs, such as, International Depository Receipts (IDRs), European Depository Receipts (EDRs) etc., but these are rarer forms of DRs. From a legal and settlement point of view, there is no difference between various types of DRs.

An ADR is a dollar denominated negotiable certificate and falls within the regulatory framework of the USA. A company issuing ADRs should have to comply with US Generally Accepted Accounting Principles (GAAP) and 'full disclosure requirements' of the US Securities and Exchange Commission (SEC). There are five principal types of ADR programmes — Unsponsored ADR programme,

Sponsored ADR programme — Level I, Level II, Level III and Rule 144(A) ADRs. The investors of ADRs enjoy rights which are comparable to those of holders of the underlying securities and also they enjoy the benefits, convenience and efficiency of trading in the US Securities market. The main drawbacks in raising money through ADRs are the requirement to meet the US GAAP, parting with the voting rights to individual investors as well as the personal liability of the directors of the company to the shareholders.

Infosys was the first Indian company to tap the ADR market for raising capital in March 1999 followed by ICICI, Satyam Infoway etc.

A GDR is also a negotiable instrument denominated in dollars or some other freely convertible currency. It is used as a funding vehicle for raising capital through equity route simultaneously in two or more markets. The holders of GDR are entitled to receive dividend in dollars but they have no voting rights. Once a GDR is issued, it can be traded freely among international investors. The GDR holder has an option to convert the GDR into a fixed number of equity shares and become an equity shareholder of the company. If they exercise their option to convert GDR into equity shares, such shareholders are entitled to receive dividend in Indian rupees, carry voting rights as well as the shares are tradeable like other equity shares on Indian Stock Exchanges. Thus, these are as liquid as the underlying securities because these two are interchangeable. Unlike ADRs, GDRs are not required to comply with US GAAP or the requirements of the US Securities and Exchange Commission (SEC). Thus, the companies issuing GDR can enjoy the benefits of an internationally traded security without changing their reporting practices.

The issue of GDR is permitted only for some specific purposes as mentioned in the guidelines issued by the Government of India on November 12, 1993. These include, financing the import of capital goods, financing domestic purchase/installation of plant, equipment and buildings, prepayment or repayment of earlier external borrowings etc. With a view to liberalise further the operational guidelines for the issue of GDRs/ADRs, the Government of India made several modifications from time to time.

The Reliance Industries Limited was the first company which raised money through GDR in May 1992 followed by Grasim Industries in November, 1992. Thereafter, many companies such as, Hindalco, Bombay Dyeing, Mahindra and Mahindra, State Bank of India have raised funds through GDRs from time to time.

Summary

Sources of raising capital may be classified under two broad categories, external sources and internal sources. Classification can also be made according to the repayment periods as short-term, medium-term and long-term sources of capital.

The principal sources of external capital are shares, debentures and term loans.

A share is the interest of a shareholder in a definite portion of the capital. Under the Companies Act, 1956, a company can issue two types of shares — equity shares and preference shares.

Equity shares or ordinary shares provide ownership rights to equity shareholders. They are the legal owners of the company. Equity shareholders enjoy the rewards, as well as bear the risks of ownership. They have residual claim on income and assets of the company and they enjoy voting right. They also enjoy pre-emptive right. It is a permanent source of capital and does not carry any fixed burden. But the cost of equity share is very high.

Preference shares are those shares which enjoy preferential rights with respect to fixed rate of dividend, claim on assets and with respect to repayment of capital either during the lifetime or on winding up of the company. It represents a hybrid form of financing — satisfy some characteristics of equity capital and some attributes of debt capital. They are not the owners, in true sense and do not enjoy any voting rights except the cases which directly affects the right of the preference shareholder. There may be different kinds of preference shares among which convertible preference share represents convertible securities.

Convertible preference shares are those preference shares which can be converted into equity shares after a stipulated period of time.

The next important source of capital is debenture which is an acknowledgment of a debt, given under the seal of the company and containing a contract for the repayment of the principal sum at a specified date and for the payment of interest at a fixed percentage. Debentures can also be of several types including convertible Debentures which represents convertible securities.

There are also some innovative sources of debt securities, such as, Secured Premium Notes (SPNs), [illegible] Discount Bonds (DDBs), Zero Interest Fully Convertible Debentures (ZFCD), Warrants etc.

Warrant is an option that permits the holder of it to buy a specified number of shares during or at the expiry of a specified period at a given price. Warrants are generally issued with a bond or a preference share.

A warrant is different from convertible securities (convertible debentures as well as convertible preference shares). A convertible security requires surrender of the security in exchange for the equity shares. On the other hand, a warrant requires a surrender of the warrant plus the payment of additional cash, known as *option price* or *exercise price* to obtain the equity shares.

Term loans are loans with maturity period of more than a year which are generally received from the financial institutions. Like debenture, it also carries a fixed rate of interest and contains a contract for the repayment of the principal amount at a specified date. As the providers of term loan are not the owners of the company, they do not enjoy any right or advantages of ownership.

Other than these main sources, leasing has emerged as an important source of intermediate and long-term financing. A lease is a contractual arrangement under which a company (lessee) acquires the right to make the use of the asset without purchasing it from the owner (lessor) at an agreed periodic payment (lease rental). Lease can be broadly classified as finance lease and operating lease.

Hire purchase system is a system under which the customer (hire purchaser) gets the possession of the goods immediately, can use it, but pay the price in instalments to the hire vendor, which may be a seller himself or may be a financial institution. The hire vendor repossess, the assets in case of default in any instalment payment without compensating the hire purchaser. Hire purchaser obtains the ownership of the asset only after the payment of the last instalment.

Commercial banks is one of the most important sources of capital in the Indian business scenario. They provide loans and advances in the form of term loans, cash credit, overdrafts, discounting of bills etc.

Public deposits are deposits taken from the public directly by the manufacturing and non-banking financial companies. Generally it provides short and medium-term capital, carries an interest rate from [illegible] p.a., depending upon the period of deposit.

Inter-corporate deposits are taken by one company from other companies, generally for a short-period of time upto 6 months. There may be three types of inter-corporate deposits : call deposit, three-months deposit and six-months deposit. There is no legal restriction to be followed for inter-corporate deposit transaction.

Commercial Paper is an emerging source of capital for working capital financing. When a company issues an unsecured promissory note to raise short-term funds under the approval of RBI, it is known as Commercial Paper. Generally its maturity period ranges from 91 days to 180 days.

Factoring is another method of raising short-term capital by which a financial institution takes over the credit collection, 'purchase and recovery function' of the seller of goods. The factor i.e., the financial institution acts between the seller and the buyer for realisation of credit sales.

The most important source of long-term internal finance is retained earnings. It is nothing but the reinvestment or ploughing back of a part of profit in the business to strengthen the financial condition, for expansion and improvements, for purchase of assets, for repayment of loans, for redemption of preference shares and debentures and also for meeting the long-term need of working capital. It is a permanent source of capital and there is no fixed obligation associated with it. It is also a cheap source of capital as no flotation cost is involved.

There may be other sources for internal financing. These are provisions for depreciation, provisions for taxation and proposed dividend. While provision for depreciation provides fund for medium and long period, provision for taxation and proposed dividend meet short-term finance purposes.

Another form of equity or debt financing for funding high risk and high reward projects is known as venture capital. It refers to financing new projects and funding risky and unproven technologies. Besides funding the technologies, venture capital is also involved in fostering the growth and development of enterprise. It plays an important role in the development of small scale enterprises in particular and economic development in general.

The Government of India permitted Indian companies in the year 1992 to raise finance from the international market through the issue of equity or debt capital which is known as Euro Issue. The most popular sources of euro issue are FCCBs, GDRs etc. FCCBs are bonds issued to the non-resident investors where as GDRs are negotiable instruments issued to the non-resident investors evidencing a fixed number of equity shares which are tradeable on major international stock exchange.

Assignment

Objective Type Questions

1. State whether the statements are true or false :
   (i) A fixed rate of dividend is paid to the equity shareholders.
   (ii) Preference shares are to be redeemed within a period of 10 years.
   (iii) Convertible preference share is also known as convertible security.
   (iv) Cost of debt is more than cost of preference capital.
   (v) Debentureholders have voting right in the meeting of the company.
   (vi) Warrant is not a convertible security.
   (vii) Debenture is the short-term source of capital.
   (viii) SIDBI provides term loan to the small-scale industries.
   (ix) Under lease arrangement, lessee gets the ownership of the asset.
   (x) The hire purchaser obtains ownership of the asset only after the payment of last instalment.
   (xi) Inter-corporate deposit can be taken for a period of one year.
   (xii) Commercial paper is a source of long-term capital.
   (xiii) Factoring is a method of raising short-term capital.
   (xiv) Retained earning is the cheapest source of financing.

(xv) Provision for taxation is an external source of financing.

[Ans. (i) False ; (ii) False ; (iii) True ; (iv) False ; (v) False ; (vi) True ; (vii) False ; (viii) True ; (ix) False ; (x) True ; (xi) False ; (xii) False ; (xiii) True ; (xiv) True ; (xv) False.]

Short Answer Type Questions

1. Define redeemable preference shares. (See Subsection 5.3.[illegible]
2. Do the preference shareholders have voting right in the meeting of the company ? If they have, then under what circumstances ? (See Subsection 5.3.[illegible]
3. Write a short-note on cumulative convertible preference shares. (See Subsection 5.3.[illegible]
4. Write a short note on Debentures. (See Subsection 5.3.[illegible]
5. Write short notes on three financial institutions. (See Subsection 5.3.[illegible]
6. Write short note : Bills discounting as a means of finance. (See Subsection 5.[illegible]
7. Write short note : Internal source of finance. (See Section 5.4. Subsections 5.4.1., 5.4.2., 5.4.3. & 5.4.[illegible]
8. Write short note : Public deposits as source of working capital funds for companies. (See Subsection 5.3.8[illegible]
9. What do you mean by pre-emptive right ? What is its relevance to the shareholders ? (See Subsection 5.3.[illegible]
10. Write a short note on warrant. Is there any difference between warrants and convertible securities ? (See Subsection 5.3.[illegible]
11. Write short notes on :
    (i) Commercial paper ; (See Subsection 5.2.[illegible]
    (ii) Factoring ; (See Subsection 5.2.[illegible]
    (iii) Hire purchase and (See Subsection 5.3.6[illegible]
    (iv) Leasing. (See Subsection 5.3.5[illegible]
    (v) Public deposits. (See Subsection 5.3.[illegible]
    (vi) Convertible debentures. (See Subsection 5.3.[illegible]
    (vii) Term Financing. (See Subsection 5.3.[illegible]
    (viii) Ploughing Back of profit / Retained Earnings (See Subsection 5.4.[illegible]
12. Write short notes : operating and financial leases. (See Subsection 5.3.5[illegible]
13. What do you mean by 'sale lease back' and 'hire purchase' system of financing ? (See Subsections 5.3.5. and 5.3.6[illegible]
14. Evaluate the following as a form of financing current assets :
    (a) Trade credit. (See Subsection 5.2.1)
    (b) Bills of exchange. (See Subsection 5.2.[illegible]
    (c) Bank loan. (See Subsection 5.2.[illegible]
    (d) Cash credit. (See Subsection 5.2.[illegible]
15. Write short notes on :
    (a) Venture Capital. (See Section 5.[illegible]
    (b) ADRs. (See Section 5.6.
    (c) GDRs. (See Section 5.6)

Essay Type Questions

1. Discuss the respective advantages and limitations of raising finance through each of the following sources :
   (a) Issue of equity capital. (See Subsection 5.3.1)
   (b) Issue of preference capital. (See Subsection 5.3.2)
   (c) Acquisition of machinery under deferred payment system. (See Subsection 5.3.6)
2. Recently companies are resorting to issue of convertible debentures for their long term funds. Explain with arguments whether it is a healthy development. (See Subsection 5.3.3)
3. What is capital employed ? How is it financed ? Discuss the merits and demerits of two major types of long-term sources of capital. (See Section 5.2)
4. Discuss the role played by convertible debentures in augmenting equity. (See Subsection 5.3.3)
5. Discuss briefly the available external sources of finance that can be tapped for a new project by an existing profitable company. (See Section 5.2)
6. "The equity share is different from a preference share". Illustrate in the light of preferences available to preference shareholders. (See Subsection 5.3.2)
7. Why is the equity shareholders considered the true owners of the company ? What are their risks which the other investors do not have ? (See Subsection 5.3.1)
8. What are the advantages and disadvantages of debenture as an instrument of financing from the point of view of the company as well as the investors ? (See Subsection 5.3.3)
9. What do you mean by public deposit ? What are the advantages and disadvantages of raising funds by means of public deposit ? (See Subsection 5.3.8)
10. Explain the systems of equipment leasing and hire purchase. (See Subsections 5.3.5. & 5.3.6)
11. What are the basic differences between Hire purchase and lease ? (See Subsections 5.3.5. & 5.3.6)
12. What are the sources of internal financing ? State the advantages and disadvantages of any one of them. (See Section 5.4, Subsections 5.4.1., 5.4.2., 5.4.3. & 5.4.4)
13. What are the short-term sources of capital ? Write notes on any two of them. (See Section 5.2)
14. What is capital ? Discuss the merits and demerits of three types of long-term source of capital, whether internal or external. (See Section 5.2)
15. Evaluate the following as a source of finance :
    (a) Equity shares ; (See Subsection 5.3.1)
    (b) Preference shares ; (See Subsection 5.3.2)
    (c) Debentures ; (See Subsection 5.3.3)
    (d) Institutional finance (Indian) ; (See Subsection 5.3.4)
    (e) Convertible cumulative preference shares. (See Subsection 5.3.2)
16. Why do you consider 'leasing' a type of financing ? What are the various types of lease financing ? (See Subsection 5.3.5)

6

Cost of Capital

CONTENTS

6.1. Introduction

The term cost of capital has received considerable attention both from theorists and practitioners in modern financial management. In this chapter our goal is to analyse the different sources of finance or capital and costs involved in raising this required fund for investment purpose. The basic object of estimating the cost of capital of a firm is to formulate an optimum capital structure and considering various investment decisions by comparing it with the prospective profits of new investments or projects. The process of capital investment decisions or the capital budgeting would be taken up in the sixth chapter and this chapter is a prelude to that.

In this chapter, we first present an overview of the concept of cost of capital, its importance and significance and then examine various methods used for estimating the cost of equity capital, preference capital, debt capital etc. and the overall cost of capital.

6.2 Definition, Meaning and Concept of Cost of Capital

A company may contemplate to obtain desired amount of funds at some point of time in its normal course of business for the purpose of its operations, expansion, modernisation, acquisition, replacement etc. The company obtains funds by means of different sources i.e., equity share capital, preference share capital, debentures or other long-term debts, deposits etc. These sources constitute components of funds. Each of these components of funds involves cost to the company. The cost of each component of funds is designated as specific cost of capital. Therefore, the term 'cost of capital' represents the rate of return which the company must pay to the suppliers of capital for the use of their funds. Therefore, from the company's standpoint, the term cost of capital may be defined as the cost of obtaining funds. According to **Hunt, William and Donaldson,** *"The cost of capital may be defined as the rate that must be earned on the net proceeds to provide the cost elements of the burden at the time they are due"*.

From the point of view of an investor, the term 'cost of capital' may be defined as the minimum required rate of return, an investment project must earn in order to cover the cost of raising funds being used by the company in financing of the proposal. Therefore, investors of funds will have an expectation of receiving a minimum return from the company. The minimum return as expected by the investors depends upon the risk perception of the investor as well as on the risk-return characteristics of the company. Therefore, while procuring funds, the company must pay this return to the investors in order to satisfy the expectations of its investor so as to make no reduction in the earning per share to the shareholders and its market price. According to **James C. Van Horne,** *"the cost of capital represents a cut-off rate for the allocation of capital to investments of projects. It is the rate of return on a project that will leave unchanged the market price of the stock."* According to **Ezra Solomon** *"the cost of capital is the minimum required rate of earnings or the cut-off rate of capital expenditures."* In the words of **Milton H. Spencer,** *"cost of capital is the minimum rate of return which a firm requires as a condition for undertaking an investment"*. In a nutshell, therefore, the term cost of capital from the investors standpoint is the minimum return which the company must earn on the proposals in order to achieve break-even and may be termed as 'target rate', 'cut-off rate' or 'hurdle rate'.

6.3 Importance, Relevance and Significance of Cost of Capital

The concept of cost of capital is very important in the financial management. It is important in the following managerial decisions :

(1) **Capital budgeting decisions :** The cost of capital plays a crucial role in the capital budgeting decision. The cost of capital is used as the discount rate in Net Present Value (NPV) calculations and as a target rate of return for comparing with a project's Internal Rate of Return (IRR). In present value method of capital budgeting, if the present value of all future streams of cash earnings from investments is greater than or equal to the cost of investment, the project may be accepted. Otherwise, the project may be rejected. Under IRR method, IRR is compared with the cost of capital. Thus the concept of cost of capital provides the criterion of accepting or rejecting the proposals in capital budgeting in the most judicious and rational manner.

(2) **Capital structure or capital mix decisions :** The cost of capital is a significant factor in designing the capital structure of a company. The company's objective is to maximise the shareholder's wealth and, therefore, a finance manager should raise capital from different sources ensuring maximum return to the shareholders by minimising risk and cost factors. For example, obtaining loan as a source of capital involves lower cost due to income tax benefits, but it involves heavy risk inviting a cash crunch situation due to regular drainage of profit through payment of

interest. It is, therefore, necessary that cost of capital from various sources should be measured and considered carefully while planning the capital structure of the company.

(3) **Evaluation of financial performance of top management** : The concept cost of capital can be used effectively to evaluate the financial performance of top management. The process involves a comparison of actual profitability of the project undertaken with the projected overall cost of capital and an appraisal of the actual cost of capital incurred in raising the required funds to finance the project. If actual profitability of the project undertaken is more than the projected overall cost of capital and actual cost of capital, the financial performance of the top management may be considered satisfactory.

(4) **Inventory management policy** : In respect of taking a decision regarding an inventory management policy, the cost of capital can be used as a guide rate to evaluate the financing cost of carrying inventory.

(5) **Receivables management policy** : In a similar manner, cost of capital may be used to calculate the cost of carrying the firm's investment in receivables and to evaluate the alternative policies and practices in respect of receivables.

(6) **Dividend policy** : The concept of cost of capital helps a firm to frame its dividend policy more logically. Apart from this, it is also useful in respect of capitalisation of profits through issue of bonus shares, right shares etc.

5.4. Classification/Types of Cost (of capital)

Cost (of capital) can be classified as follows :

(1) **Future cost and Historical cost** : Future costs are estimated costs for the future. It relates to the cost of funds intended to finance the expected project. In contrast, historical cost represents cost incurred in the past in acquiring funds. In financial decision making, future cost of capital is relatively more relevant and significant than the historical cost. For evaluating the viability of a project, the finance manager compares estimated earnings from the project with estimated cost of funds to finance that particular project. Future costs are, therefore, widely used in capital budgeting and capital structure designing decisions. This does not mean that historical cost is not relevant at all. Historical costs are useful for analysing the existing capital structure. It may act as a guide in predicting future costs and in evaluating the past performance of the company.

(2) **Specific cost and Composite cost** : Every company in its normal course of business at some point of time, requires funds for the purpose of its operations, expansion, modernisation, acquisition and replacement of long-term assets. The company obtains funds from various sources i.e., issue of equity shares, preference shares, debentures etc., other long-term debts and deposits. These sources constitute components of funds. Each of these components of funds involves costs to the company. Cost of each component of funds is designated as *component cost or specific cost of capital*. This concept of cost of capital is useful in those cases where only one type of capital is employed to judge the profitability of the project.

Generally, the capital funds come from a pool of different sources, none of the elements of which can or should be specifically identified with the particular proposals under review. When we combine all specific costs from all sources, the result will provide the *overall or composite or combined or weighted cost of capital*. The composite cost of capital, thus, represents the average of the costs of each sources of funds employed by the company and, therefore, known as *weighted average cost of capital*. In capital budgeting decision, it is an important criterion for accepting or rejecting the proposals.

Explicit cost and Implicit cost : The cost of capital of a firm can also be analysed as explicit cost and implicit cost of capital. The explicit cost of any source of capital is the discount rate which equates the present value of cash inflows (net of underwriting costs) with the present value of cash outflows. These outflows may be in the form of interest payment, repayment of principal and dividend payment that the company has to pay to the suppliers of funds. Thus, the explicit cost of capital is the internal rate of return of the cash flows. The explicit cost of an interest bearing bond will be the discount rate that equates the present value of the contractual future payments of interest and principal with the net amount of cash received today. This may be computed with the help of the following formula :

$$N_0 = \frac{I_1}{(1+K)^1} + \frac{I_2}{(1+K)^2} + \frac{I_3}{(1+K)^3} + \cdots\cdots + \frac{I_n}{(1+K)^n}$$

$$= \sum_{t=1}^{n} \frac{I_t}{(1+K)^t}$$

Where,

N_0 is the Net amount of funds received by the company i.e., net cash inflow at time '0' ;

I is the cash outflow in periods 1, 2, 3, ..., n. i.e., cash outflow in period t.

K is the discounting factor i.e. explicit cost of capital.

However, there is one source of funds which does not involve any payment or flow i.e., the retained earnings of the company. Had these retained earnings (i.e., undistributed profits) been distributed among the shareholders, they could have invested these funds elsewhere and would have earned some return. This return is foregone by the investors when the profits are not distributed and are ploughed back. Therefore, the company has an implicit cost of these retained earnings and this implicit cost is the opportunity cost of investors. Thus, the implicit cost of retained earnings is an opportunity cost since a shareholder is deprived of the opportunity to invest retained earnings elsewhere.

Except the retained earnings, all sources of funds have explicit cost of capital.

(4) **Average cost and Marginal cost** : Average cost represents the weighted average cost of the various specific costs of the different components of capital structure such as, debentures or other debts, preference shares and equity shares. The weights are in proportion of the share of each component of capital in the capital structure. Marginal cost of capital, by contrast, refers the weighted average cost of new funds obtained by the company. Marginal cost of capital is an acceptable criterion for various investment/capital budgeting decisions and, therefore, most significant factor to be considered. [Detail discussion will be made later on.]

5.5. Computation of Cost of Capital

Computation of cost of capital involves two steps :

1. Computation of specific cost of various sources/components of capital.
2. Computation of weighted average cost of capital by considering all the specific costs i.e., overall cost of capital.

I. Computation of Specific Cost of Various Sources/Components of Capital

Cost of each specific source of finance viz., debt, preference share capital, equity share capital and retained earnings can be computed as explained in subsections 4.5.1 – 4.5.4.

6.5.1. Cost of Debt (K_d)

Normally, the capital structure of a company includes a debt component in the form of Debentures, Bonds, Term Loans from financial institutions and Banks etc. The debt carries a fixed rate of interest payable to the suppliers, irrespective of the profitability of the company. The calculation of cost of debt depends on the terms and conditions relating to the rate of interest (i.e., coupon rate and normally fixed), timings of interest payment and repayment of principal amount etc. Nevertheless, following information are required to calculate the cost of debt :

(a) Net cash proceeds from the issue :

This may be calculated as follows :

	₹
Face value of debt (FV)	xx
Add : Premium charged on the issue of debt, if any	xx
Less : Discount allowed at the time of issue of debt, if any	(xx)
Less : Flotation cost i.e., underwriting, brokerage, printing of prospectus, advertisement etc. (It is calculated at the face value or the issue price whichever is higher)	(xx)
Net cash proceeds (N)	xxx

(b) Periodic interest payment and tax shield :

The debt is carried a fixed rate i.e., coupon rate of interest payable to the suppliers, irrespective of the profitability of the company. To simplify the calculation of cost of debt, the interest amount is assumed to be payable annually. It may be noted that interest on debt is always payable on the face value of debt instead of its issue price. The most important point to be noted in respect of payment of interest is that it is a *charge against profit*. Therefore, any payment towards interest will reduce the profit and ultimately the company's tax liability would decrease. This is called 'Tax shield'. Therefore, the effective payment of interest is less than the actual payment of interest made by the company to the suppliers of debt. It can be explained with the help of following example :

		₹
EBIT (Earnings Before Interest & Tax) of the company	=	1,00,000
Interest on debt per year	=	10,000
Tax rate	=	35%

Find out Net Income/Earnings After Tax (*EAT*) in both the following situations :

(*a*) When interest is paid, and

(*b*) When interest is not paid.

Calculation of Net Income/EAT

	Situation (a) ₹	Situation (b) ₹
EBIT	1,00,000	1,00,000
Less : Interest	(10,000)	Nil
EBT	90,000	1,00,000
Less : Tax @ 35%	31,500	35,000
EAT/Net Incomes	58,500	65,000

...ference between the net incomes is ₹ 6,500 (65,000 – 58,500). It is more when interest is not ... Thus, the effective cost of interest payment from the point of view of the company is not ...000 but only ₹ 6,500. After-tax interest payment, i.e. effective interest payment is ₹ 6,500 which ...ess than the actual interest payment of ₹ 10,000. This after-tax interest or effective interest payment may be computed with the help of following formula :

$I(1-t)$, where I = Interest payment

t = tax rate

or, $10{,}000\,(1 - 0{\cdot}35)$ = ₹ 6,500

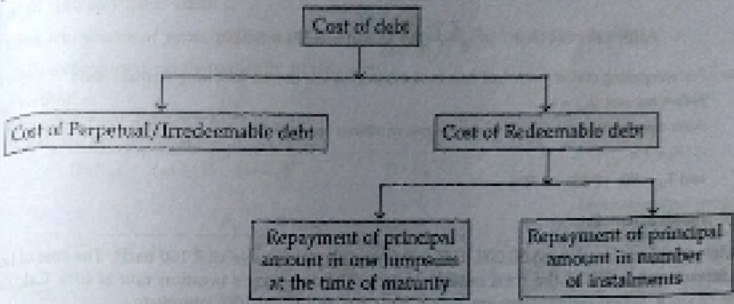
...payments of principal :

...repayments of principal do not qualify for tax deduction and, therefore, do not require any adjustment for taxes. But repayment may be made either in the year of maturity or in a number of instalments instead of one lumpsum payment made at the end of the year.

...Nature of debt :

The exact method of calculation would depend upon the type and nature of debt. The debt can be perpetual/irredeemable or redeemable.

On the basis of the above information, the cost of debt can now be ascertained as follows :



...Cost of Perpetual/Irredeemable Debt :

This may be ascertained as follows :

Cost of debt before tax, $K_i = \dfrac{I}{ND}$

Cost of debt after tax, $K_d = \dfrac{I}{ND}(1-t)$

Where, I = Fixed annual interest payment,

ND = Net cash proceeds from the issue of debt,

t = Applicable tax rate.

Illustration 1.

...company issues ₹ 10,00,000, 9% debentures of ₹ 100 each. The company is in 35% tax bracket. You are required to calculate the cost of debt assuming that the debt is issued at (*i*) par, (*ii*) 10% discount, and (*iii*) 10% premium.

Solution :

Cost of debentures :

(i) *When issued at par :*

$$\text{Before tax cost } (K_i) = \frac{₹\ 90{,}000}{₹\ 10{,}00{,}000} = 9\%$$

$$\text{After tax cost } (K_d) = \frac{₹\ 90{,}000}{₹\ 10{,}00{,}000} \times (1 - 0.35) = 5.85\%$$

(ii) *When issued at discount of 10% :*

$$\text{Before tax cost } (K_i) = \frac{₹\ 90{,}000}{₹\ 9{,}00{,}000} = 10\%$$

$$\text{After tax cost } (K_d) = \frac{₹\ 90{,}000}{₹\ 9{,}00{,}000} \times (1 - 0.35) = 6.5\%$$

(iii) *When issued at premium of 10% :*

$$\text{Before tax cost } (K_i) = \frac{₹\ 90{,}000}{₹\ 11{,}00{,}000} = 8.18\%$$

$$\text{After tax cost } (K_d) = \frac{₹\ 90{,}000}{₹\ 11{,}00{,}000} \times (1 - 0.35) = 5.32\%$$

Note : For computing cost of perpetual debt issued at par, we can use the following formula also :

Before tax cost $(K_i) = r$

After-tax cost $(K_d) = r(1 - t)$, where r = rate of interest/coupon rate.

$\therefore K_i = 9\%$

and $K_d = 9(1 - 0.35) = 5.85\%$

Illustration 2.

Jindal steel Ltd. has issued 30,000, 10% irredeemable debentures of ₹ 100 each. The cost of floatation of debentures is 2% of the total issued amount. The company's taxation rate is 40%. Calculate the cost of debt, when debentures are issued (i) at par and (ii) at 10% premium.

Solution :

(i) *When debentures are issued at par :*

Calculation of net cash proceeds from debenture issue (ND)

	₹
Total issued amount (30,000 × ₹ 100)	30,00,000
Less : Flotation cost (2% of ₹ 30,00,000)	(60,000)
Net cash proceeds (ND)	29,40,000
Annual interest charge (I) = 10% of ₹ 30,00,000 =	₹ 3,00,000

$$\therefore \text{Cost of Debt } (K_d) = \frac{I}{ND}(1 - t)$$

$$= \frac{₹\ 3{,}00{,}000}{₹\ 29{,}40{,}000}(1 - 0.40)$$

$$= 6.12\%$$

(ii) *When debentures are issued at 10% premium :*

Net cash proceeds from debenture issue (ND)

	₹
Total issued amount (30,000 × ₹ 100)	30,00,000
Add : Premium (10% of ₹ 30,00,000)	3,00,000
	33,00,000
Less : Flotation cost (2% of ₹ 33,00,000)	(66,000)
	32,34,000
Annual interest charge (I) = 10% of ₹ 30,00,000 =	₹ 3,00,000

$$\therefore \text{Cost of Debt } (K_d) = \frac{I}{ND}(1 - t)$$

$$= \frac{₹\ 3{,}00{,}000}{₹\ 32{,}34{,}000} \times (1 - 0.40)$$

$$= 5.56\%$$

Cost of Redeemable Debt :

(i) *When repayment of principal amount in one lumpsum at the time of maturity :*

This may be computed mathematically in two ways :

First method :

$$ND_0 = \frac{I_1}{(1+K_d)^1} + \frac{I_2}{(1+K_d)^2} + \frac{I_3}{(1+K_d)^3} + \cdots + \frac{I_n + P_n}{(1+K_d)^n}$$

$$= \sum_{t=1}^{n} \frac{I_t}{(1+K_d)^t} + \frac{P_n}{(1+K_d)^n}$$

Where,

ND_0 = Net cash proceeds from issue of debt.

I = Cash outflow on account of interest payment in periods 1, 2, 3, ..., n i.e., the time period 1, 2 and so on upto the year of maturity after tax.

P_n = Cash outflow on account of repayment of principal in the year of maturity.

K_d = Cost of debt after tax.

n = Number of years to maturity.

This method involves the trial and error approach and, therefore, a cumbersome procedure to find out the cost of debt. In order to avoid this long approach, the following formula may be applied instead of first method :

Second method :

$$K_d = \frac{I(1-t) + \frac{1}{n}(PD - ND)}{\frac{1}{2}(PD + ND)}$$

Where,

K_d = Cost of debt after tax.

I = Annual interest payment/Rate of interest.

t = Tax rate.

n = Number of years in which debt is to be redeemed.

PD = Principal value of debentures at the time of redemption.

ND = Net cash proceeds at the time of issue of debenture.

This method is popularly known as *short-cut method* for determining cost of debt. However, this method cannot be applied when repayment of principal amount is made in instalments instead of one lumpsum repayment.

Illustration 3.

Larsen Ltd. issued 15% debentures @ ₹ 100 each in order to raise ₹ 10,00,000 to finance a project. The flotation cost is being 10% and redeemable at par at the end of 5 years. The corporate tax rate is 35%.

Compute cost of debt by using (i) Trial and error method and (ii) Short-cut method.

Solution :

(i) **Trial and error method :**

The cost of debt capital under this method, may be computed as follows :

$$ND_0 = \frac{I_1}{(1+K_d)^1} + \frac{I_2}{(1+K_d)^2} + \frac{I_3}{(1+K_d)^3} + \dots + \frac{I_n + P_n}{(1+K_d)^n}$$

$$= \sum_{t=1}^{n} \frac{I_t}{(1+K_d)^t} + \frac{P_n}{(1+K_d)^n}$$

Where,

ND_0 = Net cash proceeds from issue of debt i.e. [₹ 10,00,000 – (10% of ₹ 10,00,000)] = ₹ 9,00,000.

I = Cash outflow on account of interest payment (after tax) in periods 1, 2, 3 ..., n, i.e. ₹ 97,500 [(15% of ₹ 10,00,000) (1 – 0·35)] in periods 1, 2, 3, 4 and 5.

P_n = Cash outflow on account of repayment of principal in the year of maturity i.e. ₹ 10,00,000 in the 5th year.

K_d = Cost of debt after-tax.

n = Number of years to maturity i.e. 5 years.

$$₹\,9{,}00{,}000 = \frac{₹\,97{,}500}{(1+K_d)^1} + \frac{₹\,97{,}500}{(1+K_d)^2} + \frac{₹\,97{,}500}{(1+K_d)^3} + \frac{₹\,97{,}500}{(1+K_d)^4} + \frac{₹\,97{,}500 + ₹\,10{,}00{,}000}{(1+K_d)^5}$$

$$\text{or, } ₹\,9{,}00{,}000 = \sum_{t=1}^{5} \frac{₹\,97{,}500}{(1+K_d)^t} + \frac{₹\,10{,}00{,}000}{(1+K_d)^5}$$

The value of K_d for this equation would be the cost of debt after-tax.

Now by applying *trial and error method* using present value tables, we can find out the value of K_d as follows :

Calculation of total present value at 12% and 13% rates of interest

Years	Cash outflow on account of interest and principal	At 12% rate		At 13% rate	
		PV factor	Total PV	PV factor	Total PV
1-4	₹ 97,500	3·6048	₹ 3,51,468	3·5173	₹ 3,42,937
5	₹ 10,00,000	0·5674	₹ 5,67,400	0·5428	₹ 5,42,800
			₹ 9,18,868		₹ 8,85,737

By applying simple interpolation technique, we get,

At 12%, total PV is ₹ 9,18,868

K_d → ← ₹ 9,00,000

At 13%, total PV is ₹ 8,85,737

$$\frac{K_d - 12}{13 - 12} = \frac{₹\,9{,}00{,}000 - ₹\,9{,}18{,}868}{₹\,8{,}85{,}737 - ₹\,9{,}18{,}868} \left[\frac{\text{Partial difference}}{\text{Total difference}}\right]$$

$$\text{or } \frac{K_d - 12}{1} = \frac{-18{,}868}{-33{,}131}$$

$$\text{or. } K_d - 12 = 0{\cdot}5695$$

$$\therefore \quad K_d = 12{\cdot}57\%$$

(ii) **Short-cut method :**

The cost of debt capital (K_d) under this method can be computed with the help of following formula :

$$K_d = \frac{I(1-t) + \frac{1}{n}(PD - ND)}{\frac{1}{2}(PD + ND)}$$

Where,

I = Annual interest payment i.e., ₹ 1,50,000.

t = Corporate tax rate i.e., 35%.

n = No. of years in which debt is to be redeemed i.e., 5.

PD = Principal value at the time of redemption i.e., ₹ 10,00,000 (since at par).

ND = Net cash proceeds at the time of issue of debentures i.e., ₹ 9,00,000 [₹ 10,00,000 – 10% (flotation) of ₹ 10,00,000].

$$\therefore \quad K_d = \frac{₹\,1{,}50{,}000(1-0{\cdot}35) + \frac{1}{5}(₹\,10{,}00{,}000 - ₹\,9{,}00{,}000)}{\frac{1}{2}(₹\,10{,}00{,}000 - ₹\,9{,}00{,}000)}$$

$$= \frac{₹\,97{,}500 + ₹\,20{,}000}{₹\,9{,}50{,}000}$$

$$= 12{\cdot}37\%$$

(b) *When repayment of principal amount is made in number of instalments instead of one lump sum repayment (Here short-cut method cannot be applied) :*

This may be computed mathematically as follows :

$$ND_0 = \frac{I_1+PD_1}{(1+K_d)^1}+\frac{I_2+PD_2}{(1+K_d)^2}+\frac{I_3+PD_3}{(1+K_d)^3}+\cdots\cdots+\frac{I_n+PD_n}{(1+K_d)^n}$$

$$ND_0 = \sum_{t=1}^{n}\frac{I_t+PD_t}{(1+K_d)^t}$$

Where,

ND_0 = Net cash proceeds from issue of debt.

I = Cash outflow on account of interest payment in periods 1, 2, 3 and so on upto the year of maturity after-tax.

PD = Cash outflow on account of principal repayment in periods 1, 2, 3, and so on upto the year of maturity.

K_d = Cost of debt after-tax.

n = Number of years to maturity.

Illustration 4.

L & T Ltd. issued 5-year 10% debentures @ ₹ 100 each in order to raise ₹ 10,00,000 to finance a project. The flotation cost is being 10% and has agreed to amortised equally over its life. The corporate tax rate is 35%. Find out the cost of debt.

Solution :

The cost of capital of debentures after tax (K_d) may be computed mathematically as follows:

$$ND_0 = \frac{I_1+PD_1}{(1+K_d)^1}+\frac{I_2+PD_2}{(1+K_d)^2}+\frac{I_3+PD_3}{(1+K_d)^3}+\cdots\cdots\frac{I_n+PD_n}{(1+K_d)^n}$$

$$\text{or,}\quad ND_0 = \sum_{t=1}^{n}\frac{I_t+PD_t}{(1+K_d)^t}$$

Where,

ND_0 = Net cash proceeds from issue of debentures i.e., ₹ 9,00,000 [₹ 10,00,000 – (10% of ₹ 10,00,000)]

PD = Cash outflow on account of principal repayment in periods 1, 2, 3 and so on upto the year of maturity i.e., ₹ 2,00,000 [₹ 10,00,000 ÷ 5].

I = Cash outflow after-tax on account of interest payment in periods 1, 2, 3 and so on upto the year of maturity i.e.,

At the end of year 1 → 10% of ₹ 10,00,000 (1 – 0·35) or ₹ 65,000

At the end of year 2 → 10% of ₹ 8,00,000 (1 – 0·35) or ₹ 52,000

At the end of year 3 → 10% of ₹ 6,00,000 (1 – 0·35) or ₹ 39,000

At the end of year 4 → 10% of ₹ 4,00,000 (1 – 0·35) or ₹ 26,000

At the end of year 5 → 10% of ₹ 2,00,000 (1 – 0·35) or ₹ 13,000

K_d = Cost of debt after-tax.

n = No. of years to maturity i.e., 5 years.

$$₹ 9,00,000 = \frac{(₹\ 65,000+₹\ 2,00,000)}{(1+K_d)^1}+\frac{(₹\ 52,000+₹\ 2,00,000)}{(1+K_d)^2}$$

$$+\frac{(₹\ 39,000+₹\ 2,00,000)}{(1+K_d)^3}+\frac{(₹\ 26,000+₹\ 2,00,000)}{(1+K_d)^4}$$

$$+\frac{(₹\ 13,000+₹\ 2,00,000)}{(1+K_d)^5}$$

Now by applying *trial and error method* using present value tables we can find out the value of K_d as follows :

Calculation of total present value at 10% and 11% rates of interest

Year	Cash outflow on account of interest and principal	At 10% rate		At 11% rate	
		PV factor	Total PV	PV factor	Total PV
1	₹ 2,65,000	0·9091	₹ 2,40,912	0·9009	₹ 2,38,739
2	₹ 2,52,000	0·8264	₹ 2,08,253	0·8112	₹ 2,04,422
3	₹ 2,39,000	0·7513	₹ 1,79,561	0·7312	₹ 1,74,757
4	₹ 2,26,000	0·6830	₹ 1,54,358	0·6587	₹ 1,48,866
5	₹ 2,13,000	0·6209	₹ 1,32,251	0·5935	₹ 1,26,416
			₹ 9,15,335		₹ 8,93,200

By applying simple interpolation technique, we get

At 10%, total PV is ₹ 9,15,335

K_d ⟶ ⟵ ₹ 9,00,000

At 11%, total PV is ₹ 8,93,200

$$\frac{K_d-10}{11-10} = \frac{₹\ 9,00,000-₹\ 9,15,335}{₹\ 8,93,200-₹\ 9,15,335}\left[\frac{\text{Partial difference}}{\text{Total difference}}\right]$$

$$\text{or,}\quad \frac{K_d-10}{1} = \frac{-15,335}{-22,135}$$

$$\text{or,}\quad K_d-10 = 0\cdot6928$$

$$\therefore\quad K_d = 10\cdot69\%$$

Illustration 5.

Rajeev Ltd. issues 1,000, 15% debentures of face value of ₹ 100 each, redeemable at the end of 7 years. The debentures are issued at a discount of 5% and the flotation cost is estimated to be 1%. Find out the cost of capital of debentures given that the firm has 35% tax rate (including surcharge and education cess).

Solution :

Cost of capital of debentures after-tax (K_d) is given by.

$$K_d = \frac{I(1-t)+\frac{1}{n}(PD-ND)}{\frac{1}{2}(PD+ND)}$$

Where,

I = Annual interest payment i.e., 15% of ₹ 1,00,000 or ₹ 15, 000.

t = Tax rate i.e., 35%.

n = Number of years in which debenture is to be redeemed i.e., 7 years.

PD = Principal value at the time of redemption i.e., ₹ 1,00,000.

ND = Net cash proceeds at the time of issue of debentures i.e., ₹ 1,00,000 (face value) – 5% of ₹ 1,00,000 (discount) – 1% of ₹ 1,00,000 (flotation cost) or ₹ 94,000.

$$K_d = \frac{₹\,15{,}000(1-0.35) + \frac{1}{7}(₹\,1{,}00{,}000 - ₹\,94{,}000)}{\frac{1}{2}(₹\,1{,}00{,}000 + ₹\,94{,}000)}$$

$$= \frac{₹\,9{,}750 + ₹\,857{\cdot}1429}{₹\,97{,}000}$$

$$= 0{\cdot}10935 \text{ or } 10{\cdot}94\%$$

Illustration 8.

XYZ Ltd. issues 12% Debentures of face value ₹ 100 each at a discount of 3% and the flotation cost is estimated to be 2%. The debentures are redeemable after 10 years at a premium of 10%. Corporate tax rate is 40%. Calculate the cost of debt.

Solution :

Cost of capital of debentures after-tax (K_d) is given by,

$$K_d = \frac{I(1-t)+\frac{1}{n}(PD-ND)}{\frac{1}{2}(PD+ND)}$$

Where,

I = Annual interest payment i.e., ₹ 12 per debenture (here total interest obligation cannot be computed).

t = Tax rate i.e., 40%

n = Number of years in which debentures is to be redeemed i.e., 10, years.

PD = Principal value at the time of redemption i.e., ₹ 100 + (10% of ₹ 100) or ₹ 110 per debenture.

ND = Net cash proceeds at the time of issue i.e., ₹ 100 – 5% (i.e., 3% + 2%) of ₹ 100 or ₹ 95 per debenture.

$$K_d = \frac{₹\,12(1-0{\cdot}40)+\frac{1}{10}(₹\,110-₹\,95)}{\frac{1}{2}(₹110+₹95)}$$

$$= \frac{7{\cdot}20+1{\cdot}50}{102{\cdot}50}$$

$$= 0{\cdot}0849 \text{ or } 8{\cdot}49\%$$

4.5.2. Cost of Preference Share Capital

The rate of dividend is fixed in advance in case of preference shares at the time of their issue. Therefore, the dividend rate can be taken as its cost since it is this amount which the company intends to pay to its preference shareholders. As the rate of dividend is fixed, the calculation of cost

[pre]ference share capital is to some extent similar to the calculation of cost of debt. But it is [imp]ortant to note that preference dividend is *not a charge against profit like interest on debt ; it is an [appro]priation of profit*. It means, the preference dividend is paid after the tax had been paid by the [com]pany. Therefore, no adjustment is required for taxes while calculating the cost of preference [sha]re capital and thereby it is substantially greater than cost of debt.

[Pre]ference shares either be redeemable or irredeemable and, therefore, the cost of capital may also [be a]scertained accordingly.

Cost of capital of Irredeemable Preference Shares (K_p) :

[Th]e cost of irredeemable preference share capital is the rate of preference dividend (also called the [cou]pon rate) divided by net cash proceeds from the issue. Therefore,

$$K_p = \frac{D}{NP}$$

Where,

K_p = Cost of capital of preference shares.

D = Rate of dividend/Annual preference dividend.

NP = Net cash proceeds from issue of preference shares.

Illustration 7.

[AB]C Ltd. issues 1000, 8% irredeemable preference shares of the face value of ₹ 100 each. Flotation [co]sts are estimated at 4%. Find out the cost of capital of preference share, if it is —

(i) Issued at par value ;

(ii) Issued at 10% premium.

Solution :

Since the preference shares are irredeemable in nature, the cost of preference shares (K_p) may be computed as follows :

$$K_p = \frac{D}{NP}$$

Where,

D = Annual preference dividend.

NP = Net cash proceeds from issue of preference shares.

(i) *Issued at par :*

$$K_p = \frac{8\% \text{ of } ₹1{,}00{,}000}{₹1{,}00{,}000-(4\% \text{ of } ₹1{,}00{,}000)}$$

$$= \frac{₹\,8{,}000}{₹\,96{,}000}$$

$$= 0{\cdot}0833 \text{ or } 8{\cdot}33\%$$

(ii) *Issued at 10% premium :*

$$K_p = \frac{₹\,8{,}000}{₹1{,}00{,}000+(10\% \text{ of } ₹1{,}00{,}000)-(4\% \text{ of } ₹1{,}10{,}000)}$$

$$= \frac{₹\,8{,}000}{₹\,1{,}05{,}600}$$

$$= 0{\cdot}0758 \text{ or } 7{\cdot}58\%$$

Cost of capital of Redeemable Preference Shares (K_p) :

This may also be computed in two ways similar to those of cost of debt.

First method :

$$NP_0 = \frac{D_1}{(1+K_p)^1} + \frac{D_2}{(1+K_p)^2} + \frac{D_3}{(1+K_p)^3} + \cdots + \frac{D_n}{(1+K_p)^n} + \frac{PP_n}{(1+K_p)^n}$$

$$= \sum_{t=1}^{n} \frac{D_t}{(1+K_p)^t} + \frac{PP_n}{(1+K_p)^n}$$

Where,

NP_0 = Net cash proceeds from issue of preference share capital.

D = Annual preference dividend in periods 1, 2, 3, ..., n i.e., the time period 1, 2, and so on upto the year of maturity.

PP_n = Amount payable at the time of redemption.

K_p = Cost of preference share capital.

n = Number of years to redemption i.e., redemption period.

This method is also involves the *trial and error* approach and, therefore, a long procedure to calculate the cost of preference share capital. This method is very similar to the method of calculation of cost of redeemable debt. To avoid this, the alternative formula (second method) may be used as follows.

Second method :

$$K_p = \frac{D + \frac{1}{n}(PP - NP)}{\frac{1}{2}(PP + NP)}$$

Where,

K_p = Cost of redeemable preference shares.

D = Annual preference dividend/Rate of dividend.

n = Number of years in which preference shares are to be redeemed.

PP = Amount payable at the time of redemption.

NP = Net cash proceeds at the time of issue of preference share capital.

This is also very similar to the calculation of cost of redeemable debt except the tax adjustment i.e., $(1 - t)$ as preference dividend is payable out of profit after-tax and consequently there is *no tax shield to the company*. This is known as *short-cut method*.

Illustration 8.

PQR Ltd. has ₹ 100 preference share redeemable at a premium of 10% with 15 years maturity. The rate of dividend is 12%. Flotation cost is 5%. Sale price is ₹ 105. Calculate the cost of preference shares.

Solution :

The cost of capital of redeemable preference share (K_p) may be computed as follows :

$$K_p = \frac{D + \frac{1}{n}(PP - NP)}{\frac{1}{2}(PP + NP)}$$

Where,

D = Rate of dividend i.e., ₹ 12 per share.

n = Number of years to redemption i.e., 15 years.

PP = Amount payable at the time of redemption i.e., ₹ 100 + 10% of ₹ 100 = ₹ 110

NP = Net cash proceeds at the time of issue i.e., ₹ 100 + ₹ 5 (premium) – ₹ 5·25 (5% flotation cost on ₹ 105) = ₹ 99·75

$$K_p = \frac{₹\,12 + \frac{1}{15}(₹\,110 - ₹\,99{\cdot}75)}{\frac{1}{2}(₹\,110 + ₹\,99{\cdot}75)}$$

$$= \frac{12 + 0{\cdot}6833}{104{\cdot}875}$$

$$= \frac{12{\cdot}6833}{104{\cdot}875}$$

$$= 0{\cdot}1209 \text{ or } 12{\cdot}09\%$$

8.5.3. Cost of Equity Share Capital

Like other sources of capital, equity share capital also has a cost. Similar as in the case of debt and preference shares, the investors will invest the funds in the form of equity share capital of a company only if they expect a return from the company, which will compensate them for surrendering the funds as well as the risk undertaken. But determining the required rate of return on equity shares presents greater difficulties than the other form of capital. The ordinary or equity shareholders do not expect to receive any fixed, predetermined return like debentures or preference share capital. Rather, the shareholders receive the right to participate in sharing future earnings and cash dividends. To recognise these rights, the return on equity capital must take into account factors such as earnings, dividends, growth rate and market price of the equity shares. According to James C. Van Horne, *"Cost of equity shares may be defined as the minimum rate of return that the company must earn on the equity financed portion of an investment project in order to leave unchanged the market price of the existing stock"*. The cost of equity share capital can be computed with the help of following six possible approaches :

(i) Dividend / Price Ratio Approach or Dividend Yield Method :

In this approach, the cost of equity share capital is calculated on the basis of a required rate of return in terms of future dividends to be paid on equity shares for maintaining their present market price. In other words, the cost of equity capital will be that rate of expected future dividends which will maintain the present market price of equity shares. This method is based on the following assumptions :

(i) Market value of shares is directly related to the *future* dividends on the shares.

(ii) Future dividend per share is expected to be constant and the company is expected to earn at least this yield over a period of time.

Thus, the formula used to determine the cost of capital of equity shares (K_e) may be as follows :

$$K_e = \frac{DPS}{NPS} \text{ [in case of cost of new equity issue]}$$

or

$$= \frac{DPS}{MPS} \text{ [in case of cost of existing equity shares]}$$

Where,

DPS = Expected dividend per share.

NPS = Net proceeds per share.

MPS = Market price per share.

Limitations of Dividend Yield Method :

(i) This approach emphasises on the fact that the future dividend is expected to be constant. It does not consider any growth rate. But in reality, a shareholder expects the returns from his equity investment to grow over a period of time.

(ii) It may lead us to ignore the growth of capital appreciation of value of the share.

(iii) It does not consider the effect of future earnings or retained earnings which increase the rate of dividend on equity shares of the company.

Therefore, Dividend/Price Ratio Approach may not be adequate to deal with the problem of computing the cost of equity share capital.

Illustration 9.

Sun Ltd. issues 10,000 equity shares of ₹ 100 each at a premium of 10%. The company pays 5% of the issue price as underwriting commission. The equity shareholders expects the rate of dividend to 20%. Calculate the cost of new equity share capital.

Will it make any difference if the market price of equity share is ₹ 160, considering the existing equity share ?

Solution :

The cost of new equity share capital (K_e) is given by

$$K_e = \frac{DPS}{NPS}$$

Where,

DPS = Expected rate of dividend per share i.e., ₹ 20.

NPS = Net proceeds per share i.e., ₹ 100 + (10% of ₹ 100 as premium) − (5% of ₹ 110 as underwriting commission) = ₹ 104·50

$$\therefore K_e = \frac{₹\ 20}{₹\ 104·50}$$

$$= 0·1914 \text{ or } 19·14\%$$

In case of *existing equity shares*, market price is to be taken as basis for calculation of cost of equity capital as follows :

$$K_e = \frac{DPS}{MPS}$$

Where,

MPS = Market price per share i.e., ₹ 160

$$= \frac{₹\ 20}{₹\ 160}$$

$$= 0·125 \text{ or } 12·5\%$$

Dividend/Price plus Growth Approach or Dividend Growth Model :

In this approach the cost of equity share capital is determined on the basis of the expected dividend plus the expected rate of growth in dividend. The rate of growth in dividend is determined on the basis of the amount of dividends paid by the company for the last few years and assumed to be given under this approach. The method of computation of cost of equity capital (K_e) can be shown as follows :

$$K_e = \frac{DPS_1}{NPS} + g \quad \text{[in case of cost of new equity issue]}$$

or

$$= \frac{DPS_1}{MPS} + g \quad \text{[in case of cost of existing equity issue]}$$

Here $DPS_1 = DPS_0 (1 + g)$

Where,

DPS_1 = Expected dividend per share at the end of current year.

NPS = Net proceeds per share.

MPS = Market price per share.

g = Expected growth in rate of dividend.

DPS_0 = Previous year's dividend per share.

Illustration 10.

The current market price of an equity share of a company is ₹ 90. The expected dividend per share is ₹ 18. In case the dividends are expected to grow at the rate of 5%, calculate the cost of equity capital.

Solution :

The cost of equity capital (K_e) may be ascertained by using the following formula :

$$K_e = \frac{DPS_1}{MPS} + g$$

Where,

DPS_1 = Expected dividend per share at the end of current year i.e., ₹ 18.

MPS = Market price per share i.e., ₹ 90.

g = Expected growth rate of dividend i.e., 0·05.

$$\therefore K_e = \frac{₹\ 18}{₹\ 90} + 0·05$$

$$= 0·20 + 0·05 = 0·25 = 25\%.$$

Illustration 11.

The shares of Sunshine Ltd. are selling at ₹ 40 per share. The firm had paid dividend @ ₹ 2·50 per share last year. The estimated growth of the company is approximately 5% per year. Compute the cost of equity capital.

Solution :

The cost of equity capital (K_e) is given by —

$$K_e = \frac{DPS_0(1+g)}{MPS} + g$$

Where,

DPS_0 = Previous year's dividend per share i.e., ₹ 2·50.

MPS = Market price per share i.e. ₹ 40.

g = Expected growth in rate of dividend i.e., 0·05.

$$\therefore K_e = \frac{₹\,2{\cdot}50(1+0{\cdot}05)}{₹\,40} + 0{\cdot}05$$

$$= 0{\cdot}0656 + 0{\cdot}05$$

$$= 0{\cdot}1156 \text{ or } 11{\cdot}56\%$$

Illustration 12.

The share of Petronet Ltd. is presently traded at ₹ 50 and the company is expected to pay dividends of ₹ 2 per share with a growth rate expected at 5% per annum. It plans to raise fresh equity share capital at the market price. The flotation costs are expected to be ₹ 1·50 per share. Find out the cost of existing equity shares as well as the new equity given that the dividend rate and growth rate are uniform.

Solution :

Cost of existing equity share capital (K_e) :

$$K_e = \frac{DPS_1}{MPS} + g$$

Where,

DPS_1 = Expected dividend per share at the end of current year i.e., ₹ 2·00.

MPS = Market price per share i.e. ₹ 50.

g = Expected growth rate, i.e., 0·05.

$$\therefore K_e = \frac{₹\,2}{₹\,50} + 0{\cdot}05$$

$$= 0{\cdot}04 + 0{\cdot}05 = 0{\cdot}09 \text{ or } 9\%$$

Cost of new/fresh equity share capital (K_e) :

$$K_e = \frac{DPS_1}{NPS} + g$$

Where,

NPS = Net proceeds from fresh issue per share i.e., ₹ 50 – ₹ 1·50 (flotation expenses) or ₹ 48·50

$$\therefore K_e = \frac{₹\,2}{₹\,48{\cdot}50} + 0{\cdot}05$$

$$= 0{\cdot}0412 + 0{\cdot}05$$

$$= 0{\cdot}0912 \text{ or } 9{\cdot}12\%$$

Illustration 13.

... Ltd. has its shares of ₹ 100 each quoted on the stock exchange, the current market price per share is ₹ 240. The dividends per share over the last four years have been ₹ 12·00, ₹ 13·20, ₹ 14·50 and ₹ 16·00. Calculate the cost of equity shares. *[C.U. B.Com (H), 2012]*

Solution :

The dividends per share over the last four years are growing approximately @ 10% and are expected to continue to grow at this rate.

This can be shown as follows :

$$\frac{(₹13{\cdot}20-₹12{\cdot}00)}{₹12{\cdot}00} \times 100 = 10\% \; ; \; \frac{(₹14{\cdot}50-₹13{\cdot}20)}{₹13{\cdot}20} \times 100 = 9{\cdot}85\% \; ; \; \frac{(₹16{\cdot}00-₹14{\cdot}50)}{₹14{\cdot}50} \times 100 = 10{\cdot}34\%$$

$$\text{Simple Average} = \frac{10+9{\cdot}85+10{\cdot}34}{3} = 10{\cdot}06 \text{ or } 10\% \text{ (say)}$$

The cost of equity share capital (K_e) will be —

$$K_e = \frac{DPS_0(1+g)}{MPS} + g$$

Where,

DPS_0 = Last year's dividend per share i.e., ₹ 16·00.

MPS = Market price per share i.e., ₹ 240.

g = Expected growth rate of dividend i.e., 10% or 0·10.

$$\therefore K_e = \frac{₹\,16{\cdot}00(1+0{\cdot}10)}{240} + 0{\cdot}10$$

$$= 0{\cdot}0733 + 0{\cdot}10$$

$$= 0{\cdot}1733 \text{ or } 17{\cdot}33\%$$

Assumptions :

Dividend/Price (D/P) *plus* growth (*g*) Approach' is based on certain assumptions :

1. The current market price of the share is a function of future expected dividends.
2. The initial dividend, D_0 is > 0, i.e., the present dividend is positive.
3. The dividend pay-out ratio is constant.

Limitations :

This approach has also certain limitations :

1. It is very difficult to quantify the expectation of investor relating to dividends and growth in dividends.
2. It is almost impossible to predict the future growth pattern as this will be inconsistent and uneven.
3. Only historic growth can be used for prediction for future growth.
4. It ignores the impact of retained earnings on the dividend growth.

(3) **Earnings/Price Ratio Approach or Earning Yield Method :**

In this approach, earnings and not the dividends per share are compared to the current price per share to find out the cost of equity share capital. Advocates of this approach argued that earning yield approach is more useful than the dividend yield approach for following two reasons :

(i) This approach takes cognizance of the total earnings of the company whether they are distributed as dividend or retained. [since earnings = dividend distributed + retained earnings] ;

(ii) It is earnings and not dividends that are more relevant in determining market price of the equity shares.

The formula for calculating the cost of equity capital (K_e) according to this approach is as follows :

$$K_e = \frac{EPS}{NPS} \quad \text{[in case of cost of new equity issue]}$$

or

$$= \frac{EPS}{MPS} \quad \text{[in case of cost of existing equity shares]}$$

Where,

EPS = Earnings per share.

NPS = Net proceeds per share.

MPS = Market price per share.

Note : Some advocates, however, differ regarding the use of the term *EPS* and *MPS*. While some of them simply use the current earnings and current market price for determining the cost of capital, others recommend average earnings and average market price over the past few years.

Illustration 14.

Rallis Ltd. is considering an expenditure of ₹ 50 lakhs for expanding its operations. The necessary information is as follows :

Number of existing equity shares	4,00,000
	₹
After-tax Profit (*PAT*) available to equity shareholders for the year	80,00,000
Market price per share	120

Compute the cost of existing equity share capital and of new equity capital assuming that new shares will be issued at a price of ₹ 105 per share and the flotation costs of new issue will be ₹ 3 per share.

Solution :

Cost of existing equity share capital :

$$K_e = \frac{EPS}{MPS}$$

Where,

$$EPS = \text{Earnings per share i.e., } \frac{\text{Earnings available to equity shareholders}}{\text{No. of equity shares}}$$

$$= \frac{₹\,80,00,000}{4,00,000} = ₹\,20$$

MPS = Market price per share i.e., ₹ 120.

$$K_e = \frac{₹\,20}{₹\,120} = 0{\cdot}1667 \text{ or } 16{\cdot}67\%$$

Cost of new equity share capital :

$$K_e = \frac{EPS}{NPS}$$

Where,

NPS = Net proceeds per share i.e., ₹ 105 – ₹ 3 = ₹ 102

$$K_e = \frac{₹\,20}{₹\,102} = 0{\cdot}1961 \text{ or } 19{\cdot}61\%$$

Assumptions :

Earning per share is likely to remain constant in future and thus, there is no growth rate (g).

100% dividend pay-out ratio is there and, thereby, retention ratio is zero. It means, all the after-tax profits (*PAT*) are distributed as dividends among the shareholders.

Market price of the share is influenced only by earning per share.

Limitations :

Earning per share can not be expected to remain constant over a period of time.

Generally, all after-tax profits (*PAT*) are not distributed to the shareholders as dividends.

This approach ignores the factor relating to capital appreciation in the market value of shares.

Earnings/Price *plus* Growth Approach or Earning Growth Model :

According to this approach the cost of equity capital is based on the earnings and the growth rate.

$$K_e = \frac{EPS}{NPS} + g \quad \text{[in case of cost of new equity issue]}$$

or

$$= \frac{EPS}{MPS} + g \quad \text{[in case of cost of existing equity issue]}$$

Where,

K_e = Cost of equity capital.

EPS = Earnings per share.

NPS = Net proceeds per share.

MPS = Market price per share.

g = Growth rate in earnings.

Illustration 15.

If the data, as used in illustration 14 are taken into consideration and if the growth rate in earnings is given to be 5% p.a., find out the cost of equity capital.

Solution :

$$K_e = \frac{EPS}{MPS} + g$$

$$= \frac{₹\,20}{₹\,120} + 0{\cdot}05$$

$$= 0{\cdot}1667 + 0{\cdot}05 = 0{\cdot}2167 \text{ or } 21{\cdot}67\%$$

Realised Yield Approach :

In computation of cost of equity share capital, determining the expectations of the investors regarding future dividends and earnings which are being used in 'Dividend Yield Method' and 'Earnings

Yield Method" poses a great problem. Expected return is the function of future dividends and earnings, and are very difficult to estimate as both these variables are uncertain. To remove this problem, 'Realised Yield Approach' which takes into consideration the actual average rate of return, realised in the past few years as the expected return in the future. This is more logical because investors expect to receive in future at least what they received in the past. Therefore, according to this approach the past records in a given period regarding dividends alongwith the actual capital appreciation realised at the time of sale of share, should be considered to compute the cost of equity share capital. This approach is based on following assumptions :

Assumptions :

1. *The risks of the company remain the same over the period.*
2. *The past realised yield will be repeated in future.*
3. *The reinvestment opportunity rate of the shareholders is equal to the realised yield.*

(6) Capital Asset Pricing Model (CAPM) :

This model, developed by William F. Sharpe and John Linter in the 1960s, wants to show a relationship between the unavoidable risk and expected return from a security. This model takes into account not only the risk differential between common stocks and Government securities, but also the risk differential between the common stock of the firm and the average common stock of all firms or broad-based market portfolio.

This model is based on the following *assumptions* :

(a) Capital markets are highly efficient where investors are well-informed ;

(b) The cost of insolvency or bankruptcy is zero. If a firm fails, its assets can be sold for their economic values. No legal or selling costs are involved ;

(c) There are negligible restrictions on investment ;

(d) No investor is large enough to affect the market price of a stock ;

(e) Investors are in agreement about the likely performance and risk of individual securities, and their expectations are based on a common holding period (say, one year) ; and

(f) There are two types of investment opportunities. The first is a risk-free security and the return on this security over the holding period is known with certainty (Treasury bills are used as surrogate for the risk-free rate). The market portfolio of common stock gives the second investment opportunity.

This model is based on the risk-averse behaviour of an investor. When any asset-holder demands more expected return in order to take more risk then he is called a *risk-averter*.

According to the CAPM, the expected rate of return from any security is calculated as follows :

$$R_i = R_F + (R_M - R_F)\,\beta_i \text{ .... (1)}$$

Where,

R_i = The expected return on i-th security ;

R_F = The rate of return on a risk-free security like the Treasury Bill (i.e., Government bonds) ;

R_M = The expected rate of return on the market portfolio ; and

β_i = The risk associated with the i-th share relative to the stock market as a whole (called as *beta coefficient*), i.e.,

$$\beta_i = \frac{\text{risk of } i\text{-th security}}{\text{risk of stock market as a whole}}$$

Now, the cost of equity finance of any business firm can be determined on the basis of the CAPM.

Here, equation (1) can be restated as :

$$K_e = R_F + b_i\,(R_M - R_F) \text{ .... (2)}$$

Where,

K_e = The cost of equity finance.

K_e actually implies the minimum rate of return that a company must earn on the equity financed portion of its investments in order to leave the market price of its stock unchanged. Hence, it is the rate that investors expect the firm to earn on its equity after corporate taxes are paid. However, for the investors, it signifies their pre-tax return.

Let us suppose that

R_M = 18%, R_F = 10%

Now, if β_i = 1, then K_e = 10% + 1 (18% − 10%) = 18%

if β_i = 1·5, then K_e = 10% + 1·5 (18% − 10%) = 22%

and if β_i = 0·5, then K_e = 10% + 0·5 (18% − 10%) = 14%.

Thus, other things remaining the same, the cost of equity finance increases with an increase in the magnitude of *beta* coefficient and *vice versa*.

The CAPM shows that the variability of expected returns involved in holding any security is measured by the beta (b_i) coefficient. If b_i = 1, it implies that expected return from the i-th security exactly reflects the expected return for the security market as a whole. [It signifies that the excess returns on a stock vary proportionately with the excess returns for the market portfolio as a whole. Thus, the stock has the same systematic risk as the stock market as a whole]. On the other hand, if $b_i > 1$, it signifies that the expected return from i-th security varies more than proportionately with that of the market portfolio as a whole. Alternatively speaking, the i-th security has more systematic risk than the market as a whole. Investment in this type of security is called '*aggressive investment*'. Similarly, if $b_i < 1$, it means that the systematic risk of the i-th security is less than that of the security market as a whole. Investment in this type of security is often called a *defensive investment*.

Proof of CAPM :

Let us assume that a fraction (say, x) of the investor's wealth is invested in risky asset, and another fraction [say, $(1 - x)$] of his wealth is invested in risk-free asset.

Let M_s = Return on risky asset if state 's' occurs (s = 1, 2, ..., S),

R_F = Return on risk-free asset,

π_s = Probability that state 's' will occur and $\sum_{s=1}^{S} \pi_s = 1$

∴ Average value of return on risky asset will be —

$$R_M = \sum_{s=1}^{S} M_s \cdot \pi_s$$

Therefore, the expected rate of return on the portfolio of the investor will be —

$$R_x = \sum_{s=1}^{S} \{x M_s + (1 - x)\,R_F\}\pi_s$$

$$= x \sum_s M_s \pi_s + (1 - x) R_F \sum_s \pi_s$$

$$= x R_M + (1 - x)\,R_F \quad (\because \textstyle\sum_s \pi_s = 1 \text{ and } R_M = \sum_s M_s \pi_s)$$

The variance of the return of that portfolio will be –

$$\sigma_x^2 = \Sigma(xM_s + (1-x)R_F - R_x)^2\pi_s$$
$$= \Sigma[(xM_s + (1-x)R_F - xR_M - (1-x)R_F)]^2\pi_s$$
$$= \sum_s (xM_s - xR_M)^2\pi_s$$
$$= \sum_s x^2(M_s - R_M)^2\pi_s$$
$$= x^2\sum_s (M_s - R_M)^2\pi_s$$

$\therefore \quad \sigma_x^2 = x^2\sigma_M^2$ where $\sigma_M^2 = \sum_s (M_s - R_M)^2\pi_s$

or, $\quad \sigma_x = x\sigma_M$

or, $\quad x = \dfrac{\sigma_x}{\sigma_M}$

So, we get :

$$R_x = xR_M + (1-x)R_F$$
$$= \frac{\sigma_x}{\sigma_M}R_M + \left(1 - \frac{\sigma_x}{\sigma_M}\right)R_F$$
$$= R_F + \frac{\sigma_x}{\sigma_M}(R_M - R_F)$$
$$= R_F + \frac{(R_M - R_F)}{\sigma_M}\cdot\sigma_x$$

Here, $\dfrac{R_M - R_F}{\sigma_M}$ = Price of the risk

It is assumed that $R_M > R_F$.

When there are many risky assets, the riskiness of any single asset is measured by the β_i

Here, $\quad \beta_i = \dfrac{\text{Risk of the } i\text{–th security}}{\text{Risk of the stock market as a whole}(\sigma_M)}$

$\therefore \quad \beta_i \cdot \sigma_M$ = Total risk of the i-th security.

The cost of risk = Total risk × Price of risk

$$= \beta_i\,\sigma_M\,\frac{R_M - R_F}{\sigma_M}$$
$$= \beta_i(R_M - R_F)$$

This is known as risk adjustment.

At equilibrium, all assets must have similar risk-adjusted return.

$\therefore$ We get $R_i - \beta_i(R_M - R_F) = R_j - \beta_j(R_M - R_F)$

Let j = Risk-free asset ; $\therefore \beta_j = 0$

So, we get $R_i - \beta_i(R_M - R_F) = R_j = R_F$

or, $R_i = R_F + \beta_i(R_M - R_F)$

or, $K_e = R_F + \beta_i(R_M - R_F)$.

Illustration 16.

From the following information in respect of a steel company, calculate the cost of equity using the CAPM approach :

Risk-free rate of return 10%

Beta risk factor of the company is 0·80

Initial price of investment in equity shares of the company is ₹ 1000

Expected dividend at the year end is ₹ 120

Expected market price of equity shares at the year end is ₹ 1100.

Solution :

Cost of equity capital using CAPM approach is given by,

$$K_e = R_F + \beta_i(R_M - R_F)$$

where,

R_F = Risk-free rate of return i.e. 10% or 0·10

β_i = Beta co-efficient i.e. 0·80

R_M = Expected return on market portfolio i.e.

$$\frac{\text{Expected dividend} + \text{Capital appreciation}}{\text{Initial Investment}} \times 100$$

$$= \frac{₹\,120 + ₹\,100(\text{i.e. } ₹\,1100 - ₹\,1000)}{₹\,1000} \times 100$$

$$= 22\% \text{ or } 0{\cdot}22$$

$$\therefore \quad K_e = 0{\cdot}10 + 0{\cdot}80\,(0{\cdot}22 - 0{\cdot}10)$$
$$= 0{\cdot}196 \text{ or } 19{\cdot}6\%$$

8.4 Cost of Retained Earnings

Retained earnings are the funds accumulated over the years by keeping a part of the funds generated without distribution to the equity shareholders of the company. These undistributed portion of earnings provide a major source of financing expansion and diversification of business. Since retained earnings have not been raised from outside like debt, preference shares etc., many people feel that such retained earnings are absolutely cost free. But this is not true, because if earnings were not retained, they would have been distributed among the shareholders as dividends and would provide them some earnings. Therefore, when part of earnings retained by the company, the equity shareholders are forced to sacrifice dividends. The dividends sacrificed by the equity shareholders are, in fact, an opportunity cost which is associated with the retained earnings. The opportunity cost here represents the cost of sacrifice of the dividend income which the equity shareholders would have otherwise received immediately.

There are two methods for computing this opportunity cost.

I First method *(from the point of view of equity shareholders)* :

From the point of view of equity shareholders, any earnings retained by the company could have been profitably invested elsewhere by the equity shareholders themselves, had these been distributed to them. For example, if the shareholders could have invested the funds elsewhere, they could have got return of 12% (say). This 12% return has been foregone by the shareholders as the company not distributing the entire profits to them. Therefore, cost of retained earnings may be taken at 12%.

It is important to note that shareholders are required to pay personal income tax on their dividend income. But at present in India, dividend income is tax free in the hands of the recipient but there is brokerage and commission (flotation cost) for making investments. Therefore, the funds available with the shareholders are less even if there is 100% pay-out ratio. This makes cost of retained earnings slightly lower than the cost of equity capital. The cost of retained earnings (K_r) may be calculated as follows :

Cost of retained earnings (K_r) = K_e (1 – *t*) (1 – *B*)

Where,

K_e = Cost of equity share capital i.e., required rate of return by shareholders.

t = Shareholders' personal income tax rate.

B = Brokerage, commission etc. expressed as a percentage.

Illustration 17.

XYZ Ltd. is earning a net profit of ₹ 4,00,000 per annum. The shareholders' required rate of return is 12%. The shareholders of the company are assumed to be in 30% personal tax bracket. It is expected that the shareholders will have to incur 2% as brokerage on the after-tax dividends received by them. Assuming that the entire earnings are distributed to the shareholders. Calculate the cost of retained earnings of the company.

Solution :

The cost of retained earnings (K_r) is given by,

$$K_r = K_e\,(1 - t)\,(1 - B)$$

Where,

K_e = Required rate of return by the shareholders i.e., 12%.

t = Shareholders' personal income tax rate i.e., 30%

B = Cost of brokerage i.e., 2%

$\therefore$ K_r = 12% (1 – 0·30) (1 – 0·02)

= 12% × 0·70 × 0·98

= 8·23%

Notes : 1. The computation of cost of retained earnings as above, is based on the assumption that the company has 100% pay out ratio i.e., entire earnings are distributed to the shareholders and they can reinvest the amount elsewhere in similar type of securities carrying return of 12%. The details of which may be shown as follows :

	₹
Dividend receivable by the shareholders	4,00,000
Less : Shareholders' personal income tax @ 30%	(1,20,000)
After-tax dividends	2,80,000
Less : Cost of brokerage @ 2% of ₹ 2,80,000	(5,600)
Net amount available for investments	2,74,400
∴ Earning on re-investment by the shareholders (12% of 2,74,400)	32,928

However, if the company retains the entire earnings, no personal income tax and brokerage cost will be payable and the entire amount of ₹ 4,00,000 will be available for re-investment on which ₹ 32,928 must be earned. Therefore, cost of retained earnings would be as follows :

$$\frac{₹\ 32,928}{₹\ 4,00,000} \times 100 = 8·23\%$$

being lower than the cost of equity capital of 12% due to personal income tax and cost of brokerage.

2. The major difficulty in the above computation is to ascertain a single personal income tax rate of large number of shareholders with varying taxable incomes. A weighted average tax-rate may be calculated to avoid this problem.

Second method *(from the point of view of the company)* :

The alternative approach to determine the cost of retained earnings has suggested by **Prof. Solomon** and this is known as '*External yield criterion*'. According to this approach, the opportunity cost of retained earnings is the rate of return that could be earned by the company by investing the retained funds in another company instead of what would be obtained by the shareholders on investments elsewhere. Under this approach investments of retained earnings are assumed to be made by the company itself, therefore, the return would not be affected by varying personal income tax rates of large number of shareholders. Hence, cost of retained earnings (K_r) may be treated at par with cost of equity share capital (K_e) and represents an economically justifiable opportunity cost.

Computation of Overall Cost of Capital or Weighted Average Cost of Capital (*WACC*)

So far we are concerned with the determination of specific cost of finance coming from different sources like debt, preference share capital, equity share capital and retained earnings to a company. Once such computation is completed for the individual components of the capital structure of a company, we can calculate the Overall Cost of Capital or the Weighted Average Cost of Capital (WACC) for that company. This overall cost of capital is of utmost importance as this rate is to be used in various financial decisions such as the discount rate or the cut-off rate while evaluating the capital budgeting proposals of a firm. It may be defined as the rate of return that must be earned by the company in order to satisfy the requirements of the different investors. The overall cost of capital is thus, the minimum required rate of return on the assets of the company. This overall cost of capital should take into account the relative proportion or weights of different sources in the capital structure of the company and, therefore, this should be calculated by assigning weights to specific costs of capital in proportion of the various sources of funds to the total. As against simple concept, weighted average is useful due to the fact that the proportion of various sources of funds in the capital structure of the company are different. The computation of the weighted average cost of capital or overall cost of capital (K_o) involves the following steps :

1. Calculate after-tax cost of each specific sources of funds (i.e. cost of debt, cost of preference share capital, cost of equity share capital etc.).
2. Assignment of appropriate weights to specific costs. (i.e., book values of various sources, market values of various sources etc.) – to be discussed later on.
3. Multiply the cost of each source by its proportion or weights in capital structure.
4. Divide the total weighted cost by the total weights.

Illustration 18.

A company has the following capital structure and after-tax costs for the different sources of funds:

Source of funds	Amount (₹) (Book value)	Amount (₹) (Market value)	After-tax cost (%)
Equity share capital (paid up)	50,000	90,000 (including retained earnings)	15
Retained earnigs (Reserves)	10,000	—	15
Preference share capital	25,000	25,000	10
Debentures	15,000	15,000	8

You are required to calculate the weighted average cost of capital using (*a*) book value as weights and (*b*) market value as weights.

Solution :

(*a*) Calculation of Weighted Average Cost of Capital

(Book Value as weights)

Source of funds (1)	Amount (₹) (2)	Proportion/ Weights (3)	After-tax cost (%) (4)	Weighted cost (5) = (3) × (4)
Equity share capital	50,000	0·50 [50,000/1,00,000]	15	7·50
Retained earnings	10,000	0·10 [10,000/1,00,000]	15	1·50
Preference share capital	25,000	0·25 [25,000/1,00,000]	10	2·50
Debentures	15,000	0·15 [15,000/1,00,000]	8	1·20
Total	1,00,000	1·00		12.70

$\therefore$ Weighted Average Cost of Capital $(K_o) = \frac{12\cdot70}{1\cdot00}$

$= 12\cdot70\%$

An alternative method of computing the Weighted Average Cost of Capital is to compute the total cost of capital and then divide this by the total capital as follows :

Calculation of Weighted Average Cost of Capital

(Book Value as weights)

Source of funds (1)	Amount (₹) (2)	After-tax cost (%) (3)	Total after-tax cost (₹) (4) = (2) × (3)
Equity share capital	50,000	15	7,500
Retained earnings	10,000	15	1,500
Preference share capital	25,000	10	2,500
Debentures	15,000	8	1,200
Total	1,00,000		12,700

$\therefore$ Weighted Average Cost of Capital (K_o)

$$= \frac{₹\ 12,700}{₹\ 1,00,000} \times 100 = 12\cdot70\%$$

(*b*) Calculation of Weighted Average Cost of Capital

(Market Value as weights)

Source of funds (1)	Amount (₹) (2)	Proportion / Weights (3)	After-tax cost (%) (4)	Weighted cost (5) = (3) × (4)
Equity share capital (5/6)	75,000	0·5769	15	8·6535
Retained Earnings (1/6) [5 : 1]	15,000	0·1154	15	1·7310
Preference share capital	25,000	0·1923	10	1·9230
Debentures	15,000	0·1154	8	0·9232
Total	1,30,000	1·0000		13·2307

$\therefore$ Weighted Average Cost of Capital $(K_o) = \frac{13\cdot2307}{1\cdot0000} = 13\cdot23\%$

Notes : (1) The total market value of ₹ 90,000 has been bifurcated into Equity share capital and Retained earnings in the ratio of 50,000 : 10,000 or 5 : 1.

(2) The cost of Retained earnings is taken at par with Equity share capital assuming *External yield criterion* for the purpose — as the personal income-tax rates of the shareholder, and brokerage, commission etc. are not given.

Some important points :

(1) *Assignment of appropriate weights :*

The Weighted Average Cost of Capital can be computed by using the 'Book value' or the 'Market value' as weights. If there is a difference between market value and book value weights, the weighted average cost of capital would also differ. The market value weighted average cost is usually higher than it would be if the book value is used.

It is obvious from the above illustration that market value weighted average cost of capital (13.22%) is higher than the book value weighted average cost of capital (12.70%), because the maket value of equity share capital including retained earnings (₹ 90,000) is higher than its book value (₹ 50,000).

Merits and demerits of market value and book value weights :

The market value weights are sometimes preferred to the book value weights due to following reasons :

(a) *Market value represents the true expectations of the investors as they presumably reflect economic values.*

(b) *Market value weights are not influenced by the arbitrary accounting policy of the firm.*

(c) *It considers price level changes and, therefore, reflects current cost of capital. Because of this market value weights for calculating the cost of capital is theoretically more appealing.*

But it suffers from the following limitations :

(a) *Market value weights are operationally inconvenient as market values undergo frequent fluctuations. This will affect the overall cost of capital and, in turn decision criterion for investment.*

(b) *Market values of all sources of capital are not readily available.*

(c) *Use of market value tends to shift a greater importance towards the larger amounts of equity funds, particularly when additional financing is undertaken.*

(d) *When the shares are not listed in any stock exchange, the use of market value weights is impossible.*

Due to above limitations, it is desirable to use the book value weights. This method has the following advantages :

(a) *Book value weights are easily or readily available from published accounts.*

(b) *The capital structure targets are usually set in terms of book values.*

(c) *To evaluate the riskiness of the company, the book value debt-equity ratios are analysed by the investors.*

(d) *It is easier to evaluate the performance of a finance manager in procuring funds by comparing on the basis of book values.*

(e) *When the shares of a company are not listed in any stock exchange the use of book value weights is the only alternative.*

However, the book value weights system suffers from the following limitations :

(a) *It does not truly reflect the economic values.*

(b) *Book value weights may be based on arbitrary accounting policies followed to calculate retained earnings and value of assets.*

(c) *The book value weights system is not consistent with the definition of the overall cost of capital, which is defined as the minimum rate of return required to maintain the company's market value.*

(d) *Theoretically, it is very difficult to justify the use of book value weights.*

Selection of appropriate weights by using both the alternatives — book values and market values is an important aspect to calculate weighted average cost of capital. Both have their own commendable features. Market value weights are operationally inconvenient as compared to book value weights. However, market value weights are theoretically consistent and sound, and therefore a better indicator of firm's cost of capital, provided market value of various sources of capital are readily available and they seems to be stable.

*A third alternative may be used as **target weights** to calculate the overall cost of capital or weighted average cost of capital for evaluating the investment projects — known as **Marginal Weights**. The marginal weights refer to the proportions in which the firm wants or intends to raise funds from different sources. In other words, the proportions in which additional funds required to finance the investment proposals will be raised are known as marginal weights. Therefore in case of marginal weights, the firm in fact, calculates the actual weighted average cost of capital of the incremental funds. This method is based on the argument that the firm is concerned with the new or incremental capital rather than the capital raised in the past. (see Illustration 19)*

However, there are some shortcomings of the marginal weights system. One major limitation is that — this system ignores the long-term implications of the firm's new financing plans. A firm should give due attention to long-term implications while designing the firm's financing strategy. In the short-run, the firm may be tempted to raise funds only from cheaper sources and thereby accepting more and more proposals. However, in the long-run, when other sources will have to be considered, some of the projects which should have been accepted otherwise, will be rejected due to higher cost of capital.

Illustration 19.

The Polaris Ltd. has the following book values and market values of different sources of finance alongwith its specific costs :

Sources of finance	Amount (₹) (Book value)	Amount (₹) (Market value)	After-tax cost (%)
Equity fund	50,000	70,000	18
Preference share capital	20,000	20,000	15
Debentures/Long-term debt	30,000	30,000	8

You are required to calculate (i) Weighted Average Cost of Capital, using book and market value weights ; (ii) Weighted Average Cost of Capital, using marginal weights, if the company intends to raise the required funds 50% from long-term debt, 35% from preference share capital and 15% from retained earnings.

Solution :

(i) Calculation of Weighted Average Cost of Capital using Book Value and Market Value weights

Sources of finance	After-tax Cost (%)	Book value Amount (₹)	Book value Proportion / Weights	Book value Weighted cost	Market value Amount (₹)	Market value Proportion / Weights	Market value Weighted cost
Equity fund	18	50,000	0.50	9.00	70,000	0.5833	10.50
Preference shares	15	20,000	0.20	3.00	20,000	0.1667	2.50
Debentures /Long-term debt	8	30,000	0.30	2.40	30,000	0.2500	2.00
		1,00,000	1.00	14.40	1,20,000	1.00	15.00

∴ Weighted Average Cost of Capital (K_o)

Using book value weights → $\frac{14.40}{1.00} = 14.40\%$

Using market value weights → $\frac{15.00}{1.00} = 15.00\%$

(ii) **Calculation of Weighted Average Cost of Capital using Marginal Weights**

Sources of finance	After-tax cost (%)	Proportion / Weights	Weighted cost
Retained earnings	18	0.15	2.70
Preference shares	15	0.35	5.25
Long-term debt	8	0.50	4.00
		1.00	11.95

∴ Weighted Average Cost of Capital (K_o) by using Marginal Weights → $\frac{11.95}{1.00} = 11.95\%$

Note : The cost of Retained earnings is taken at par with equity share capital assuming external yield criterion for the purpose, as because the personal income-tax rates of the individual shareholder and brokerage, commission i.e., flotation cost are not given.

(2) **Rationale behind the use of after-tax Weighted Average Cost of Capital (WACC) :**

In financial decision making, the weighted average cost of capital may be calculated either before-tax or after-tax. However, it will always be more appropriate to measure the cost of capital on after-tax basis because of the following reasons :

(a) The weighted average cost of capital is calculated by combining the specific cost of various sources of long-term capital — like equity capital, preference capital, debt capital etc. The dividend on equity and preference shares are always paid out of after-tax profit. Therefore, these costs should have been computed after-tax basis. But cost of debt capital may be computed either before-tax or after-tax basis. If cost of debt capital is computed before-tax basis there will be inconsistency in respect of treatment of specific cost while computing WACC. So in order to maintain uniformity among the various sources of cost of capital, it is recommended that cost of debt capital should always be computed after-tax basis.

(b) For evaluating an investment proposal, a shareholder should compare the actual rate of return in the form of dividend obviously after-tax with the minimum expected rate of return. This minimum rate of return is nothing but the cost of capital. So to make the comparison more meaningful, the cost of capital should always be computed after-tax basis.

(c) In discounted cash flow technique of project evaluation, all cash inflows are discounted by the cost of capital. These cash inflows are computed after payment of tax. Therefore, if before-tax cost of capital is being used for discounting cash inflows it will obviously not be logical or rational.

(3) **Formula for Overall Cost of Capital/Weighted Average Cost of Capital :**

The overall cost of capital/weighted average cost of capital (K_o) may be computed by using the following formula :

$$K_o = K_eW_e + K_rW_r + K_pW_p + K_dW_d$$

Where,

K_e = Cost of equity capital.

K_r = Cost of retained earnings.

K_p = Cost of preference share capital.

K_d = Cost of debt (after-tax).

W_e = Proportion of equity share capital in capital structure.

W_r = Proportion of retained earnings in capital structure.

W_p = Proportion of preference share capital in capital structure.

W_d = Proportion of debt in capital structure.

Application :

Using the book value data from Illustration 18, the overall cost of capital (K_o) can be computed as follows :

$$\begin{aligned} K_o &= K_eW_e + K_rW_r + K_pW_p + K_dW_d \\ &= (15 \times 0.50) + (15 \times 0.10) + (10 \times 0.25) + (8 \times 0.15) \\ &= 7.50 + 1.50 + 2.50 + 1.20 \\ &= 12.70\% \end{aligned}$$

(4) **Reasons for change in calculation of Weighted Average Cost of Capital :**

It is very important to note that the weighted average cost of capital may change due to following reasons :

(a) Change in proportion of each sources of finance.

(b) Change in the specific cost of each sources of finance.

(c) Change in both.

6.6. Marginal Cost of Capital

Marginal cost of capital may be defined as the cost of raising an additional rupee of capital. The weighted average cost of capital is normally the overall cost of capital of existing funds of the firm, whereas marginal cost of capital is the weighted average cost of capital in respect of raising additional funds. If the additional fund is being raised by using more than one source, say a combination of debt and preference capital, then actual weighted average cost of capital (WACC) of the new financing mode is called the Weighted Marginal Cost of Capital (WMCC). In other words, marginal cost of capital is the weighted average cost of new capital calculated by using the marginal weights. As explained earlier, marginal weights represent the proportion of various sources of funds to be employed in raising additional funds. In case, a firm raises new capital funds in the same proportion as at present and at the same time specific cost of capital remain the same at present, then the Marginal Cost of Capital (WMCC) shall be equal to the Weighted Average Cost of Capital (WACC). But in practice, the proportion and/or the specific cost of capital may change for additional funds to be raised. Under this situation the marginal cost of capital obviously differs from weighted average cost of capital. This may be explained with the help of following illustration :

Illustration 20.

IKON Ltd. has the following capital structure along with after-tax cost of capital for the different sources of funds :

Sources of funds	Amount (₹)	Proportion/Weights	After-tax cost (%)
Equity Capital	3,00,000	0·60	12
Preference Capital	50,000	0·10	10
Debt/Loan Capital	1,50,000	0·30	8
	5,00,000	1·00	

(a) Calculate the weighted average cost of capital using book-value of weights.

(b) The firm intends to raise further ₹ 2,00,000 for the expansion of the project on the basis of existing proportion in its capital structure i.e., 60% by the issue of Equity Share Capital, 10% by the issue of Preference Share Capital and 30% by obtaining Debt Capital. The firm estimates that the cost of capital of additional funds of various sources will be the same as at present.

Calculate the Weighted Marginal Cost of Capital.

(c) If the firm wishes to raise further ₹ 2,00,000 on the basis of following proportion keeping specific cost of capital remain constant, calculate the Weighted Marginal Cost of Capital :

Sources of Funds	Amount
Equity Capital	1,00,000
Preference Capital	80,000
Debt/Loan Capital	20,000
	2,00,000

Solution :

(a) Calculation of Weighted Average Cost of Capital (WACC) using book-value as weights :

Sources of funds	Amount (₹)	Proportion/ Weights	After-tax cost (%)	Weighted cost
1. Equity Capital	3,00,000	0·60	12	7·20
2. Preference Capital	50,000	0·10	10	1·00
3. Debt/Loan Capital	1,50,000	0·30	8	2·40
	5,00,000	1·00		10·60

$\therefore$ WACC $(K_o) = \frac{10{\cdot}60}{1{\cdot}00} = 10{\cdot}60\%$

(b) Calculation of Weighted Marginal Cost of Capital (WMCC) :

Sources of funds	Amount (₹)	Proportion/ Weights	After-tax cost (%)	Weighted cost
1. Equity Capital	1,20,000	0·60	12	7·20
2. Preference Capital	20,000	0·10	10	1·00
3. Debt/Loan Capital	60,000	0·30	8	2·40
	2,00,000	1·00		10·60

$\therefore$ WMCC $= \frac{10{\cdot}60}{1{\cdot}00} = 10{\cdot}60\%$

Comment :

From the above calculation, it has been observed that the raising of additional fund of ₹ 2,00,000 has been made in the same proportion as existing (i.e., 60%, 10% and 30% by issue of equity capital, preference capital and debt capital) with the similar specific cost of capital. Therefore, WMCC shall be equal to the WACC.

(c) Calculation of Weighted Marginal Cost of Capital (WMCC) :

Sources of funds	Amount (₹)	Proportion/ Weights	After-tax cost (%)	Weighted cost
1. Equity Capital	1,00,000	0·50	12	6·00
2. Preference Capital	80,000	0·40	10	4·00
3. Debt/Loan Capital	20,000	0·10	8	0·80
	2,00,000	1·00		10·80

$\therefore$ WMCC $= \frac{10{\cdot}80}{1{\cdot}00} = 10{\cdot}80\%$

Comment :

It is evident from the above that WMCC (i.e., 10·80%) is different from the existing WACC (i.e., 10·60%). This is mainly due to the fact the firm raises additional capital of ₹ 2,00,000 in the different proportion as at present. Moreover, if there is any difference in specific cost of capital, WMCC and WACC may be different under such situations.

4.7. General Illustrations

A. Determination of Specific Cost of Capital

Illustration 21.

Various specific cost of capital :

Assuming that a company pays tax at a 35% rate, compute the after-tax cost of capital in the following cases :

(i) 12% debentures (perpetual) of ₹ 10,000, sold at a premium of 10% with no flotation costs.

(ii) 12% debentures (perpetual) of ₹ 10,000, sold at a premium of 10% with cost of flotation is 2% of the total issued amount.

(iii) 10-year, 15% debentures of ₹ 20,000, redeemable at par, with 2% flotation cost.

(iv) 10-year, 8% bond of ₹ 1000 sold at ₹ 950 less 4% flotation cost.

(v) The current market price of an equity share is ₹ 90 and the expected dividend at the end of current year is ₹ 4·50 with a growth rate of 5%.

(vi) The current market price of an equity share is ₹ 80. The company had paid dividend @ ₹ 5·00 per share in the previous year. Estimated growth rate is approximately 8% per year.

Solution :

(i) Cost of debenture (K_d) issued at a premium of 10% is given by,

$$K_d = \frac{I}{ND}(1-t)$$

Where,

I = Fixed annual interest i.e., 12% of ₹ 10,000 or ₹ 1,200

ND = Net cash proceeds from the issue of debt i.e., 110% of ₹ 10,000 or ₹ 11,000

t = tax rate i.e., 35%

$$\therefore K_d = \frac{₹\,1200}{₹\,11000}(1-0{\cdot}35) = 7{\cdot}09\%$$

(ii) $K_d = \frac{I}{ND}(1-t)$

Where, I = ₹ 1,200

ND = (110% of ₹ 10,000) – (2% of ₹ 11,000) = ₹ 10,780

t = 0·35

$$\therefore K_d = \frac{₹\,1200}{₹\,10{,}780}(1-0{\cdot}35)$$

$$= 7{\cdot}24\%$$

$$K_d = \frac{I(1-t)+\frac{1}{n}(PD-ND)}{\frac{1}{2}(PD+ND)}$$

Where, I = Annual interest i.e., 15% of ₹ 20,000 or ₹ 3,000.

t = Tax rate i.e., 0·35.

n = No. of years in which debenture is to be redeemed i.e., 10 years.

PD = Principal value at the time of redemption i.e., ₹ 20,000 (since at par).

ND = Net cash proceeds at the time of issue i.e., [₹ 20,000 – (2% of 20,000)] ₹ 19, 600.

$$\therefore K_d = \frac{₹\,3{,}000\,(1-0{\cdot}35)+\frac{1}{10}(₹\,20{,}000-₹\,19{,}600)}{\frac{1}{2}(₹\,20{,}000+₹\,19{,}600)}$$

$$= \frac{₹\,1{,}950+₹\,40}{₹\,19{,}800}$$

$$= 10{\cdot}05\%$$

(iv) $$K_d = \frac{I(1-t)+\frac{1}{n}(PD-ND)}{\frac{1}{2}(PD+ND)}$$

Where, I = ₹ 80

t = 0·35

n = 10 years

PD = ₹ 1000

ND = [₹ 950 – (4% of ₹ 1000)] i.e., ₹ 910

$$\therefore K_d = \frac{₹\,80\,(1-0{\cdot}35)+\frac{1}{10}(₹\,1{,}000-₹\,910)}{\frac{1}{2}(₹\,1{,}000+₹\,910)}$$

$$= \frac{52+9}{955}$$

$$= 6{\cdot}39\%$$

(v) The cost of equity share capital (K_e) is given by,

$$K_e = \frac{DPS_1}{MPS} + g$$

Where, DPS_1 = Expected dividend per share at the end of current year i.e., ₹ 4·50

MPS = Market price per share i.e., ₹ 90

g = Expected growth rate of dividend i.e., 0·05

$$\therefore K_e = \frac{₹\,4{\cdot}50}{₹\,90} + 0{\cdot}05$$

$$= 10\%$$

(vi) $K_e = \frac{DPS_0(1+g)}{MPS} + g$

Where, DPS_0 = Last year's dividend per share i.e., ₹ 5

MPS = Market price per share i.e., ₹ 80

g = Expected growth in rate of dividend i.e., 0·08

$\therefore \quad K_e = \frac{₹ 5(1+0.08)}{₹ 80} + 0.08$

= 14.75%

Illustration 22.

Cost of Preference Share Capital :

(a) A company raised preference share capital of ₹ 10,00,000 by the issue of 8% preference shares of ₹ 10 each. Find out the cost of preference share capital when it is issued at (i) 10% premium and, (ii) 10% discount.

(b) A company has 10% redeemable preference share of ₹ 100 each which are redeemable at the end of 10th year from the date of issue. The underwriting expenses are expected to 2%. Find out the cost of preference share capital.

Solution :

(a) Cost of preference share capital (K_p) is given by,

$$K_p = \frac{D}{NP}$$

Where, D = Annual preference dividend i.e., 8% of ₹ 10,00,000 or ₹ 80,000.

NP = Net cash proceeds from the issue of preference shares.

(i) *Issued at 10% premium*

$$K_p = \frac{₹ 80,000}{₹ 11,00,000}$$

$= 0.0727$ or 7·27%

(ii) *Issued at 10% discount*

$$K_p = \frac{₹ 80,000}{₹ 9,00,000}$$

$= 0.0888$ or 8·89%

(b) $$K_p = \frac{D + \frac{1}{n}(PP - NP)}{\frac{1}{2}(PP + NP)}$$

Where, D = Annual preference dividend/Rate of dividend i.e., ₹ 10.

n = No. of years in which preference shares are to be redeemed i.e., 10 years.

PP = Amount payable at the time of redemption i.e., ₹ 100.

NP = Net cash proceeds at the time of issue of preference share capital i.e., ₹ 98

$$K_p = \frac{₹ 10 + \frac{1}{10}(₹ 100 - ₹ 98)}{\frac{1}{2}(₹ 100 + ₹ 98)}$$

$= \frac{10 + 0.20}{99}$

$= 0.1030$ or 10·30%

Illustration 23.

Cost of Equity and Debt when income statement is given :

The following are the extracts from the financial statements of ABC Ltd.

(₹ lakhs)

Operating profit/*EBIT*	105
Less : Interest on Debenture	(33)
EBT	72
Less : Income tax @ 50%	36
EAT/Net profit	36
Equity share capital (share of ₹ 10 each)	200
Reserves and surplus	100
15% Non-convertible Debentures	220
	520

The market price per Equity share is ₹ 12 and per Debenture is ₹ 93·75.

(i) What is the earning per share (*EPS*) ?

(ii) What is the % of cost of capital to the company for the Debentures fund and the Equity ?

Solution :

(i) **Calculation of earnings per share (*EPS*) :**

$$EPS = \frac{\text{Earnings/Profit after tax}}{\text{No. of Equity Shares}}$$

$$= \frac{₹ 36,00,000}{20,00,000 \text{ Equity Shares}} = ₹ 1·80$$

(ii) (i) **Calculation of cost of debentures after-tax (K_d) :**

$$K_d = \frac{I}{ND}(1 - t)$$

Where, I = Annual interest i.e., ₹ 33,00,000.

ND = Net cash proceeds from the issue of debt i.e., 2,20,00,000 or total debenture fund.

t = Corporate tax rate i.e., 50%.

At Book Value

$$K_d = \frac{₹ 33,00,000}{₹ 2,20,00,000}(1 - 0.50)$$

$= 0.075$ or 7·5%

At Market Value

$$K_d = \frac{₹\ 33,00,000}{(93.75\% \text{ of } ₹\ 2,20,00,000)}(1-0.50)$$

$$= 0.08 \text{ or } 8\%$$

(b) (ii) **Calculation of cost of equity capital (Earning Yield Method) :**

$$K_e = \frac{EPS}{MPS}$$

Where, EPS = Earnings per share i.e., ₹ 1.80, as computed above

MPS = Market price per share i.e., ₹ 12

$$\therefore \quad K_e = \frac{₹\ 1.80}{₹\ 12}$$

$$= 0.15 \text{ or } 15\%$$

Illustration 24.

Cost of Equity with computation of Growth rate (g) :

Moonlight Ltd. intends to issue new equity shares of which the current market value is ₹ 150 per share. The flotation cost is estimated to be 4%. The dividends paid by the company during last 5 years are ₹ 10.50, ₹ 12, ₹ 15, ₹ 17, and ₹ 19. Find out the growth rate in dividends, cost of new equity shares and the cost of existing equity shares given that the same growth rate continues in future.

Solution :

(i) *Growth rate in dividends (g) :*

The dividend income of ₹ 10.50 has increased to ₹ 19 over a period of 4 years. Therefore, a cumulative growth rate of 1.81 (i.e., 19 ÷ 10.50) has been achieved on ₹ 10.50 over a period of 4 years.

In the *compound value (of ₹ 1) table*, in the 4 years row, a value of 1.81 may be found in 16% column. Therefore, the growth rate, g, in dividend rate may be taken as 16%.

Author's Note : For alternative calculation of growth rate in dividend please see next Illustration no. 27.

(ii) *Cost of new equity shares :*

$$K_e = \frac{DPS_1}{NPS} + g \text{ or } \frac{DPS_0(1+g)}{NPS} + g$$

Where, DPS_1 = Expected dividend per share at the end of current year i.e., ₹ 19

NPS = Net proceeds per share i.e., ₹ 150 − (4% of ₹ 150) or ₹ 144

g = 16% or 0.16

DPS_0 = Previous years dividend per share i.e., ₹ 19.

$$\therefore K_e = \frac{₹\ 19\ (1+0.16)}{₹\ 144} + 0.16$$

$$= 0.3131 \text{ or } 31.31\%$$

(iii) Cost of existing equity shares :

$$K_e = \frac{DPS_1}{MPS} + g = \frac{DPS_0(1+g)}{MPS} + g$$

$$= \frac{₹\ 19\ (1+0.16)}{₹\ 150} + 0.16 = 0.3069 \text{ or } 30.69\%.$$

Illustration 25.

Cost of Equity with computation of Growth rate (g) :

...k Ltd. furnishes the following particulars from which you have to compute the cost of equity capital under dividend growth model for both existing equity shares and new equity shares :

(i) Current market price per share : ₹ 220

(ii) Underwriting commission : 5%

(iii) Expected dividend per share : ₹ 12

(iv) Tax on dividend : 10%

(v) The company's past dividends per share for the last 6 years were as follows :

Year	Dividend per share (₹)
2013	6.50
2014	7.00
2015	8.00
2016	8.50
2017	10.00
2018	11.50

Solution :

(i) *Growth rate in dividends (g) :*

$$\text{Growth rate in 2014} = \frac{(₹\ 7.00 - ₹\ 6.50)}{₹\ 6.50} \times 100 = 7.6923\%$$

$$\text{Growth rate in 2015} = \frac{(₹\ 8.00 - ₹\ 7.00)}{₹\ 7.00} \times 100 = 14.2857\%$$

$$\text{Growth rate in 2016} = \frac{(₹\ 8.50 - ₹\ 8.00)}{₹\ 8.00} \times 100 = 6.2500\%$$

$$\text{Growth rate in 2017} = \frac{(₹\ 10.00 - ₹\ 8.50)}{₹\ 8.50} \times 100 = 17.6471\%$$

$$\text{Growth rate in 2018} = \frac{(₹\ 11.50 - ₹\ 10.00)}{₹\ 10.00} \times 100 = 15.0000\%$$

$$\therefore \text{Simple Average} = \frac{7.6923 + 14.2857 + 6.25 + 17.6471 + 15}{5}$$

$$= \frac{60.8751}{5} = 12.17502\% \text{ or } 0.1218$$

(ii) Cost of existing equity shares :

$$K_e = \frac{DPS_1(1+t)}{MPS} + g$$

Where, DPS_1 = Expected dividend per share at the end of current year i.e., ₹ 12

t = Corporate dividend tax i.e., 0·10

MPS = Market price per share i.e., ₹ 220

g = 0·1218

$$\therefore K_e = \frac{₹\,12(1+0\cdot10)}{₹\,220} + 0\cdot1218$$

$= 0\cdot1818$ or $18\cdot18\%$

(iii) Cost of new equity shares :

$$K_e = \frac{DPS_1(1+t)}{NPS} + g$$

Where, NPS = Net proceeds per share i.e., ₹ 220 – (5% of ₹ 220) or ₹ 209.

$$\therefore K_e = \frac{₹\,12(1+0\cdot10)}{₹\,209} + 0\cdot1218$$

$= 0\cdot184957$ or $18\cdot50\%$

Illustration 26.

Calculation of Market price of Equity share :

IPL Ltd.'s share is quoted in the market at ₹ 20 currently. The company pays a dividend of ₹ 1 per share and expected growth rate will be 5% per year.

Compute :

(a) The company's Equity Cost of Capital.

(b) If the anticipated growth rate is 6% p.a., calculate the indicated market price per share.

(c) If the company's cost of capital is 9%, and the anticipated growth rate is 4% p.a., calculate the indicated market price if the dividend of ₹ 1 per share is to be maintained.

Solution :

(a) Cost of Equity Capital (K_e) is given by,

$$K_e = \frac{DPS_1}{MPS} + g$$

Where, DPS_1 = Expected dividend per share i.e., ₹ 1.

MPS = Market price per share i.e., ₹ 20

g = Expected growth rate i.e., 0·05

$$\therefore K_e = \frac{₹\,1}{₹\,20} + 0\cdot05$$

$= 0\cdot10$

$= 10\%$

(b) Market price per equity share, if the growth rate is anticipated at 6% :

We have, $K_e = \frac{DPS_1}{MPS} + g$

or, $K_e - g = \frac{DPS_1}{MPS}$

or, $MPS = \frac{DPS_1}{K_e - g}$

Here, DPS_1 = ₹ 1 ; K_e = 10% or 0·10 ; g = 0·06

$\therefore MPS = \frac{₹\,1}{0\cdot10-0\cdot06} = \frac{₹\,1}{0\cdot04}$

= ₹ 25.

(c) Again we have,

$MPS = \frac{DPS_1}{K_e - g}$

or, $MPS = \frac{₹\,1}{0\cdot09-0\cdot04} = \frac{₹\,1}{0\cdot05}$

= ₹ 20

Illustration 27.

Cost of Retained Earnings :

Oxford Ltd. has an annual profit of ₹ 5,00,000 and the required rate of return of the shareholders is 10%. It is further expected that the shareholders will have to incur 4% brokerage cost of the dividends received and invested by them for making new investments. Find out the cost of retained earnings to the firm given that the tax rate applicable to shareholders is 30%.

Solution :

The cost of retained earnings (K_r) may be computed with the help of following formula :

$$K_r = K_e(1-t)(1-B)$$

Where, K_e = Cost of equity share capital i.e., required rate of return by the shareholders i.e., 10

t = Shareholders' personal income tax rate i.e., 30%.

B = Cost of brokerage of the shareholders i.e., 4%.

$\therefore$ K_r = 10 (1 – 0·30) (1 – 0·04)

= 10 × 0·70 × 0·96

= 6·72%.

The cost of retained earnings may also be computed as follows :

	₹
Dividend receivable by the shareholders	5,00,000
Less : Shareholders' personal income tax @ 30%	(1,50,000)
After tax dividends	3,50,000
Less : Cost of brokerage @ 4% of ₹ 3,50,000	(14,000)
Net amount available for investments	3,36,000
∴ Earnings on re-investment by the shareholders (10% of ₹ 3,36,000)	33,600

$$\therefore K_r = \frac{₹\ 33,600}{₹\ 5,00,000} \times 100$$
$$= 6.72\%$$

Note : It is based on the assumption that the dividend income is taxable in the hands of the shareholders. But in India at present the dividend income is tax free in the hands of the shareholders u/s 10(34) of the Income Tax Act, 1961.

Illustration 28.

Cost of Equity by applying CAPM method :

From the following information, determine the cost of equity capital using CAPM approach :

(i) Risk free rate of return 10%.

(ii) Beta co-efficient, β_i of the company is 1·20.

(iii) Return on market portfolio is 15%.

What would be the cost of equity if β_i rises to 1·80 ?

Solution :

Cost of Equity Capital using CAPM approach is given by,

$$K_e = R_F + \beta_i (R_M - R_F)$$

Where, R_F = Risk-free rate of return i.e., 10% or 0·10

β_i = Beta co-efficient i.e., 1·20

R_M = Return on market portfolio i.e., 15% or 0·15

$$\therefore K_e = 0.10 + 1.20\,(0.15 - 0.10)$$
$$= 0.10 + 0.06 = 0.16 \text{ or } 16\%$$

If β_i rises to 1·80,

$$K_e = 0.10 + 1.80\,(0.15 - 0.10)$$
$$= 0.10 + 0.09$$
$$= 0.19 \text{ or } 19\%$$

5. Determination of Weighted Average Cost of Capital or Overall Cost of Capital

Illustration 29.

C Ltd. has the following capital structure :

	Book value (₹)	Market value (₹)
Equity share capital :		
25,000 shares @ ₹ 10 each	2,50,000	3,75,000
12% Preference share capital :		
500 shares @ ₹ 100 each	50,000	50,000
Reserves and Surplus :		
General reserve	1,50,000	—
14% Debentures :		
1,500 debentures @ ₹ 100 each	1,50,000	1,50,000

The expected dividend per share is ₹ 1·50 with expected growth rate of 8%. Preference shares are redeemable after 5 years at par whereas debentures are redeemable after 6 years at a premium of 5%. The tax rate for the company is 30%.

You are required to compute the specific cost of capital of different sources of funds and also compute the weighted average cost of capital using market value as weights.

Solution :

Calculation of Specific Cost of Capital :

(1) *For Equity Share Capital (K_e) :*

$$K_e = \frac{DPS_1}{MPS} + g$$

Where, DPS_1 = Expected dividend per share with a growth rate of 8% i.e., ₹ 1·50.

MPS = Market price per share i.e., ₹ 3,75,000 ÷ 25,000 or ₹ 15.

g = Expected growth rate i.e., 8%.

$$\therefore K_e = \frac{₹\ 1.50}{₹\ 15} + 0.08$$
$$= 0.18 \text{ or } 18\%$$

(2) *For Preference Share Capital (K_p) (redeemable) :*

$$K_p = \frac{D + \frac{1}{n}(PP - NP)}{\frac{1}{2}(PP + NP)}$$

Where, D = Dividend per share i.e., ₹ 12.

n = Redemption period i.e., 5 years.

PP = Amount payable at the time of redemption i.e., ₹ 100.

NP = Net cash proceeds at the time of issue of shares i.e., ₹ 100.

$$\therefore K_p = \frac{₹\,12 + \frac{1}{5}(₹\,100 - ₹\,100)}{\frac{1}{2}(₹\,100 + ₹\,100)}$$

$$\text{or, } K_p = \frac{₹\,12 + 0}{₹\,100} = 12\%$$

It is important to note that, in this case PP = NP and, hence we can alternatively use the formula for irredeemable preference share for computing the value of K_p.

Alternatively,

$$K_p = \frac{D}{NP} = \frac{₹\,12}{100} = 12\%$$

(3) *For 14% Debentures (K_d) (redeemable) :*

$$K_d = \frac{I(1-t) + \frac{1}{n}(PD - ND)}{\frac{1}{2}(PD + ND)}$$

Where, I = Interest rate i.e., 14.

n = Redemption period i.e., 6 years.

PD = Principal value at the time of redemption i.e., ₹ 105.

ND = Net cash proceeds at the time of issue i.e., ₹ 100.

t = tax rate i.e., 35%.

$$\therefore K_d = \frac{₹\,14\,(1-0{\cdot}35) + \frac{1}{6}(₹\,105 - ₹\,100)}{\frac{1}{2}(₹\,105 + ₹\,100)}$$

$$= \frac{9{\cdot}10 + 0{\cdot}83}{102{\cdot}50}$$

$$= 0{\cdot}0969 \text{ or } 9{\cdot}69\%$$

Calculation of Weighted *Average* Cost of Capital using Market Value Weights

Sources of funds	Amount (Market value) (₹)	Proportion/ Weights	After-tax cost (%)	Weighted cost
Equity share capital	2,34,375	0·4076	18·00	7·3368
Reserves & surplus	1,40,625	0·2445	18·00	4·4010
Preference share capital	50,000	0·0870	12·00	1·0440
Debentures	1,50,000	0·2609	9·69	2·5281
	5,75,000	1·0000		15·3099

$\therefore$ *Weighted Average Cost of Capital* (K_o)

$$= \frac{15{\cdot}3099}{1{\cdot}0000} \text{ or } 15{\cdot}31\%$$

Notes: 1. The total market value of equity of ₹ 3,75,000 has been bifurcated into Equity share capital and Reserves and surplus in the ratio of their book values i.e., 2,50,000 : 1,50,000.

2. The cost of Reserves and surplus/Retained earnings is taken at par with Equity share capital assuming external yield criterion for the purpose.

Illustration 30.

Calculate weighted average cost of capital from the following information :

(i) **Capital structure of AB Ltd.**

	₹ '000
Equity capital : Shares of ₹ 10 each fully paid	100
Reserves (General)	50
Long-term debt	100
	250

(ii) Market price per share of AB Ltd. is ₹ 60 and Earnings per share is ₹ 6. Expected growth rate in earnings is 5% p.a.

(iii) Cost of debt (before tax) : 12% p.a.

(iv) Applicable corporate tax : 40%.

(v) Use market values as weights and show your workings.

Solution :

Calculation of Specific Cost of Capital :

1. *For Equity Share Capital (K_e)*

$$K_e = \frac{EPS}{MPS} + g$$

Where, EPS = Earnings per share i.e. ₹ 6.

MPS = Market price per share i.e., ₹ 60.

g = Expected growth rate in earnings i.e. 5%.

$$\therefore K_e = \frac{₹\,6}{₹\,60} + 0{\cdot}05$$

$$= 0{\cdot}10 + 0{\cdot}05$$

$$= 0{\cdot}15 \text{ or } 15\%$$

2. *For Reserves (General) (K_r) :*

$K_r = K_e = 15\%$

Cost of Reserves is taken at par with equity share capital assuming *external yield criterion* for the purpose

3. *For Long-term Debt (K_d) :*

$$K_d = K_i(1-t)$$

Where, K_i = Cost of debt before tax i.e. 12%

t = corporate tax rate i.e., 0·40

$$\therefore K_d = 12(1 - 0{\cdot}40)$$

$$= 7{\cdot}20\,\%$$

Calculation of Weighted Average Cost of Capital (K_o) using Market Values as weights

Source of Capital	Amount (Market value) (₹)	Proportion/ Weights	After-tax Cost (%)	Weighted Cost
1. Equity Share Capital (10/15) (See note below)	4,00,000	0·5714	15·00	8·5710
2. Reserves (General) (5/15) (See note below)	2,00,000	0·2857	15·00	4·2855
3. Long-term Debt	1,00,000	0·1429	7·20	1·0289
	7,00,000	1·00		13·8854

$$\therefore K_o = \frac{13 \cdot 8854}{1 \cdot 00} \text{ or } 13 \cdot 89\%$$

Note : The total market value of Equity of ₹ 6,00,000 (i.e., 10,000 shares @ ₹ 60 per share) has been bifurcated into Equity share capital and Reserves in the ratio of their book values i.e 1,00,000 : 50,000 or 10 : 5.

Illustration 31.

Calculate weighted average cost of capital from the following information :

(i) **Capital structure of Hindustan Ltd.**

	₹ '000
Equity share capital (shares of ₹ 10 each fully paid)	200
Reserves and Surplus	100
14% Bond (before tax)	600
Total	900

(ii) Market price per share of H Ltd. is ₹ 90 and dividend per share is ₹ 13·50. Expected growth rate in dividend is 5% p.a.

(iii) Corporate tax rate : 40%.

What is the weighted average cost of capital of Hindustan Ltd.? Use market values as weights and show your workings.

Solution :

Calculation of Specific Cost of Capital :

1. *For Equity Share Capital (K_e) :*

$$K_e = \frac{DPS_1}{MPS} + g$$

Where, DPS_1 = Dividend per share at the end of current year i.e., ₹ 13·50.

MPS = Market price per share i.e. ₹ 90.

g = Growth rate in dividend i.e. 5%.

$$\therefore K_e = \frac{13 \cdot 50}{90} + 0 \cdot 05$$

$$= 0 \cdot 15 + 0 \cdot 05$$

$$= 0 \cdot 20 \text{ or } 20\%$$

2. *For Reserves and Surplus (K_r) :*

$K_r = K_e = 20\%$

Cost of Reserves and surplus is taken at par with Equity share capital assuming external yield criterion for the purpose.

3. *For 14% Bond (K_d) :*

$K_d = K_i(1 - t)$

Where, K_i = Cost of bond before tax i.e. 14%

$\therefore K_d = 14(1 - 0 \cdot 40)$

$= 8 \cdot 40\%$

Calculation of Weighted Average Cost of Capital (K_o) using Market Value as weights

Source of capital	Amount (market value) (₹)	Proportion/ Weights	After-tax cost (%)	Weighted cost
1. Equity share capital (2/3) (See note below)	12,00,000	0·50	20·00	10·00
2. Reserves and surplus (1/3) (See note below)	6,00,000	0·25	20·00	5·00
3. Bond	6,00,000	0·25	8·40	2·10
	24,00,000	1·00		17·10

$$\therefore K_o = \frac{17 \cdot 10}{1 \cdot 00} \text{ or } 17 \cdot 10\%$$

Note : The total market value of Equity of ₹ 18,00,000 (i.e., 20,000 share @ ₹ 90 per share) has been bifurcated into Equity share capital and Reserves in the ratio of their book values of 2,00,000 : 1,00,000 or 2 : 1.

Illustration 32.

The Capital Structure and cost of capital of a company are given below :

Source	Book Value ₹ lakhs	After-tax Cost of Capital (%)
Equity	200	16
Retained Earnings	200	?
Debentures	400	7
Total	800	

Equity shares represent shares of ₹ 10 each. The current market value of each share is ₹ 80 and the corporate tax rate is 40%.

(i) Compute weighted average cost of capital of the company using both book values and market values as weights.

(ii) How would you account for the difference, if any, in the average cost of capital under (i) above ?

Solution :

Calculation of Specific Cost of Capital :

1. *For Equity Capital (K_e) :*

   K_e = 16% (given)

2. *For Retained Earnings (K_r) :*

   $K_r = K_e$ = 16%

   Cost of Retained earnings is taken at par with equity capital assuming *external yield criterion* for the purpose.

3. *For Debentures (K_d) :*

   K_d = 7% (given)

Calculation of Weighted Average Cost of Capital (K_o) using Book Value and Market Value as weights

Source of Capital	After-tax cost (%)	Book Value Weights			Market Value Weights		
		Amount	Weights	Weighted cost	Amount	Weights	Weighted cost
1. Equity Capital	16·00	2,00,00,000	0·25	4·00	8,00,00,000 (see note below)	0·40	6·40
2. Retained Earnings	16·00	2,00,00,000	0·25	4·00	8,00,00,000 (see note below)	0·40	6·40
3. Debentures	7·00	4,00,00,000	0·50	3·50	4,00,00,000	0·20	1·40
		8,00,00,000	1·00	11·50	20,00,00,000	1·00	14·20

∴ *Weighted Average Cost of Capital (K_o) using*

— Book Value Weights = $\frac{11 \cdot 50}{1 \cdot 00}$ or 11·50%

— Market Value Weights = $\frac{14 \cdot 20}{1 \cdot 00}$ or 14·20%

Note : The total market value of Equity of ₹ 16,00,00,000 (i.e., 20,00,000 shares @ ₹ 80 per share) has been bifurcated into Equity Share Capital and Retained Earnings in the ratio of their book values i.e., ₹ 2,00,00,000 : ₹ 2,00,00,000 or 1 : 1.

(ii) It has been observed that the calculation of weighted average cost of capital using market value is higher than that using book value. The reason being that the market value of equity shares is considerably greater than their book value. Therefore, it provides higher specific cost of capital and given greater emphasis to this source of finance.

In general, overall cost of capital based on market value will be greater than the overall cost of capital based on book values. However, this is not the rule.

LIST OF FORMULAE

	Source of Finance	Formula
	DEBT	
1.	Before-tax cost of Perpetual/Irredeemable Debt	$K_i = \frac{I}{ND}$ or I
2.	After-tax cost of Perpetual/Irredeemable Debt	$K_d = \frac{I}{ND}(1-t)$ or $I(1-t)$
3.	Before-tax cost of Redeemable Debt	$K_i = \frac{I + \frac{1}{n}(PD - ND)}{\frac{1}{2}(PD + ND)}$
4.	After-tax cost of Redeemable Debt	$K_d = \frac{I(1-t) + \frac{1}{n}(PD - ND)}{\frac{1}{2}(PD + ND)}$
	PREFERENCE SHARE CAPITAL	
5.	Cost of Irredeemable Preference Share Capital	$K_p = \frac{D}{NP}$
6.	Cost of Redeemable Preference Share Capital	$K_p = \frac{D + \frac{1}{n}(PP - NP)}{\frac{1}{2}(PP + NP)}$
	EQUITY SHARE CAPITAL	
7.	Cost of Equity Share Capital — Dividend Yield Method	$K_e = \frac{DPS}{NPS}$ OR $\frac{DPS}{MPS}$
8.	Cost of Equity Share Capital — Dividend Growth Model	$K_e = \frac{DPS_1}{NPS} + g$ OR $\frac{DPS_1}{MPS} + g$ $DPS_1 = DPS_0(1+g)$
9.	Cost of Equity Share Capital — Earning Yield Method	$K_e = \frac{EPS}{NPS}$ OR $\frac{EPS}{MPS}$
10.	Cost of Equity Share Capital — Earning Growth Model	$K_e = \frac{EPS}{NPS} + g$ OR $\frac{EPS}{MPS} + g$
11.	Cost of Equity Share Capital — CAPM Model	$K_e = R_F + \beta_i (R_M - R_F)$
12.	COST OF RETAINED EARNINGS	$K_r = K_e(1-t)(1-B)$
13.	OVERALL COST OF CAPITAL / WEIGHTED AVERAGE COST OF CAPITAL	$K_o = K_e W_e + K_r W_r + K_p W_p + K_d W_d$

Summary

From the view point of an investor, the term 'Cost of Capital' denotes the minimum required rate of return that an investment project should earn to cover its cost of raising funds. This concept is very important in financial management since it helps in

(i) Capital budgeting decisions,

(ii) Capital structure decisions,

(iii) Evaluating financial performance of top management,

(iv) Inventory management policy,

(v) Dividend policy etc.

The Cost of Capital is classified into the following categories :

(a) Future Cost and historical cost,

(b) Specific cost and composite cost,

(c) Explicit Cost and implicit cost,

(d) Average and marginal cost etc.

The computation of the cost of capital involves two steps :

1. Computation of specific costs of various sources of capital.
2. Computation of weighted average cost of capital.

While calculating the cost of debt, we require the following information :

(a) Net Cash proceeds from the issue,

(b) Periodic interest payment and tax shield,

(c) Repayments of principal,

(d) Nature of debt (i.e., whether it is irredeemable debt or redeemable debt).

In a similar fashion, the cost of Irredeemable and redeemable preference share capital can be calculated.

The cost of equity capital can be computed on the basis of :

(a) The Dividend Yield Method,

(b) The Dividend Growth Model,

(c) The Earning Yield Method,

(d) The Earning Growth Model,

(e) The Realised Yield Model, and

(f) The Capital Asset Pricing Model (CAPM).

Again, the weighted average cost of capital can be computed by using the 'Book Value' or 'Market Value' as weights. In financial decision making, the weighted average cost of capital may be calculated either before-tax or after-tax. However, calculation on after-tax basis seems to be more appropriate.

Assignment

Objective Type Questions

1. State whether each of the following statement is 'True' or 'False':

(i) The Cost of Capital is the minimum rate of return which the company must pay to the suppliers of capital.

(ii) Cost of each component of capital is termed as specific cost.

(iii) Retained earnings have no cost to the firm.

(iv) The explicit cost of any source of capital is the discount rate which equates the present value of cash outflows.

(v) Composite cost refers to the cost of equity and preference share capital.

(vi) Book value weights are theoretically consistent and sound as compared to market value weights.

(vii) Marginal cost is the weighted average cost of the new funds raised by the firm.

(viii) Weighted average cost of capital may change due to a change in the proportion of each sources of finance.

[Answer : (i) True ; (ii) True ; (iii) False ; (iv) True ; (v) False ; (vi) False ; (vii) True ; (viii) True.]

Short Answer Type Questions

1. What is Cost of Capital ? (See Section 6.2.)
2. What are the uses of cost of capital ? (See Section 6.3.)
3. What is meant by Future Cost and Historical Cost ? (See Section 6.4.)
4. What is meant by Explicit Cost and Implicit Cost ? (See Section 6.4.)
5. What is the difference between Specific Cost and Composite Cost ? (See Section 6.4.)
6. Write a short note on Cost of Preference Share Capital. (See Subsection 6.5.2.)
7. Write a short note on Cost of Retained Earnings. (See Subsection 6.5.4.)
8. Write a short note on Marginal Weights. (See Subsection 6.5.4.)
9. Name the various methods of computing Cost of Equity Capital. (See Subsection 6.5.3.)
10. How will you determine the cost of equity share capital in a growth company.

Essay Type Questions

1. (i) Define Cost of Capital and explain its significance. (See Sections 6.2 and 6.3.)
   (ii) What weights can you take for computing Overall Cost of Capital ? (See Subsection 6.5.4.)
2. What is meant by cost of capital for a firm and what relevance does it have in decision making ? How is it calculated with different types of sources of capital fund ? (See Sections 6.2, 6.3 and 6.5)
3. State in brief the weights that you would take into consideration for computing weighted average cost of capital. Why market value weights are considered superior to the book value weights ? (See Subsection 6.5.4.)
4. Explain briefly the various methods of computing cost of equity capital. Which of them do you consider most appropriate and why ? (See Subsection 6.5.3.)
5. What is the relevance of cost of capital in capital budgeting and capital structure planning decisions ? (See Section 6.3.)
6. Write a note on different types of cost of capital. (See Section 6.5. & Subsections 6.5.1-6.5.4.)
7. (i) "Cost of capital is used as a decision criterion." Do you agree ? (See Section 6.3.)
   (ii) What weights would you suggest for computing weighted average cost of capital ? (See Subsection 6.5.4.)
8. What are the steps involved in calculating a firm's weighted average cost of capital ? (See Section 6.5.)
9. What is cost of capital ? Examine the rationale behind the use of after-tax weighted average cost of capital. (See Section 6.2. & Subsection 6.5.4.)
10. How do you calculate cost of equity using dividend growth model and capital asset pricing model ? (See Subsection 6.5.3.)
11. What do you understand by specific cost of capital ? Explain how you would compute specific costs in respect of (i) retained earnings and (ii) debt capital. (See Section 6.4. and Subsections 6.5.1. & 6.5.4.)
12. How can you determine the cost of equity capital in a growth company ? (See Subsection 6.5.3.)

13. "Cost of Capital is used by a company as a minimum benchmark for its yield." — Comment.
(See Section
14. What do you mean by Marginal Cost of Capital ?
(See Section
15. Explain Capital Asset Pricing Model (CAPM) used for measuring cost of equity capital and examine its rationality.
(See Subsection

Practical Problems

1. Calculate the cost of capital in the following cases :
   (i) A Ltd. issues 13% debentures of face value ₹ 100 each and realises ₹ 95 per debenture. The debentures are redeemable after 10 years at a premium of 10%.
   (ii) Y Ltd. issues 12% preference shares of face value ₹ 100 each and realises ₹ 92 per share. The shares are repayable after 12 years at par.
   Note : Both the companies are paying income-tax @ 40%.
   [Answer : (i) 9·07% ; (ii) 13·20%]
2. XYZ Ltd. issues 15% debentures of face value of ₹ 100 each, redeemable at the end of 7 years. The debentures are issued at a discount of 5% and the flotation cost is estimated to be 1%. Find out the cost of debentures given that the firm has 40% tax rate.
   [Answer : 10·16%]
3. Sun Ltd. has ₹ 100 preference share redeemable at a premium of 10% with 15 years maturity. The coupon rate is 15%. Flotation cost is 5%. Sale price is ₹ 95. Calculate the Cost of Preference Shares.
   [Answer : 16·33%]
4. Moon Ltd. issued 10,000 equity shares of ₹ 10 each at a premium of ₹ 2 each. The company has incurred issue expenses of ₹ 4,000. The equity shareholders expects the rate of dividend to be 18% p.a. Calculate the cost of equity share capital.
   Will your answer be different if the current market price of share is ₹ 22 ?
   [Answer : 15·52% ; 8·18%]
5. Sur Ltd. plans to issue 1000 new shares of ₹ 100 each at par. The flotation costs are expected to be 4% of the share price. The company pays a dividend of ₹ 10 per share initially and the growth in dividends is expected to be 5%. Compute the cost of new issue of equity shares.
   If the current market price of an equity share is ₹ 150, calculate the cost of existing equity share capital.
   [Answer : 15·42% ; 11·67%]
6. The shares of a company are being sold at ₹ 60 per share and it had paid a dividend of ₹ 4 per share last year. The investor's market expects a growth rate of 5% per year.
   (a) Compute the company's equity cost of capital ;
   (b) If the anticipated growth is 7% p.a., calculate the indicated market price per share.
   [Answer : (a) 12% ; (b) ₹ 85·60 *Hint* : (a) $K_e = \frac{DPS_0(1+g)}{MPS} + g$]
7. The shares of Essar Ltd. are being currently sold at ₹ 20 per share. It has just paid a dividend of ₹ 2 for the last year. The profits of the company are expected to show a growth of 10% p.a. and the company maintains a 100% pay-out ratio. Determine the cost of equity capital of the company.
   What should be the expected current price of the share if the growth rate is (i) 8% or (ii) 12% ?
   [Answer : 21% ; ₹ 16·61, ₹ 24·88]
8. Dark Ltd. has 50,000 equity shares of ₹ 10 each and its current market value is ₹ 40 each. The after-tax profit of the company for the year ended 31st March, 2004 is ₹ 9,60,000. Calculate the cost of capital based on price/earning method.
   [Answer : 48%]

9. From the following capital structure and other related information compute cost of equity share capital :

Sources of Capital		Amount (₹)
Equity Share Capital (shares of ₹ 10 each)		8,00,000
9% Preference Share Capital (shares of ₹ 100 each)		1,80,000
8% Debentures (Debentures of ₹ 100 each)		5,00,000
	Total	14,80,000
Operating Profit/EBIT		5,00,000
Corporate tax rate		30%
Market value of each equity share		₹ 20

[Answer : $K_e = \frac{EPS}{MPS} = \frac{₹\ 3.80}{₹\ 20} = 0.19$ or 19%]

10. X is a shareholder in ABC Company Ltd. Although earnings for the ABC Ltd. have varried considerably, X has determined that the long-run average dividend for the firm have been ₹ 2 per share. He expects a similar pattern to prevail in the future. Given the volatility of the ABC's dividends, X has decided that a minimum rate of 20% should be earned on his share. What price would X be willing to pay for the ABC's share ?
    [Answer : Market Price (per share) = ₹ 10]
11. Bharat Ltd. is considering the issue of new equity shares of the face value of ₹ 100 each at ₹ 125 each. The cost of flotation per share is estimated to be ₹ 3. Dividends paid per share by the company on the existing equity shares for the last 5 years are : ₹ 10·25, ₹ 11·70, ₹ 12·65, ₹ 13·15 and ₹ 14·47. The company has a fixed dividend pay-out ratio. The expected dividend on the new shares at the end of the first year is ₹ 15·25.
    Determine (a) the cost of existing shares, and (b) the cost of new shares of the company.
    [Answer : (a) 21·2% ; (b) 21·5%
    Hint : calculate the growth rate (g) first and it is 9%]
12. Cipla Ltd. is earning a net profit of ₹ 25,00,000 per annum. The shareholders' expected rate of return is 15%. The marginal tax rate is 30%. Investment of the retained earnings in new shares involves brokerage cost of 3%. Assuming that the entire earnings are distributed to the shareholders, determine the cost of retained earnings.
    [Answer : 10·19%]
13. The following information are available from the Balance Sheet of a company :

Equity Share Capital — 20,000 shares of ₹ 10 each	₹ 2,00,000
Reserves and Surplus	₹ 1,30,000
8% Debentures	₹ 1,70,000

The rate of tax for the company is 50%. Current level of Equity Dividend is 12%.
Calculate the weighted average cost of capital using the above figures.
[Answer : 9·28%]

14. MVC company has assets of ₹ 1,60,000 which have been financed with ₹ 52,000 of debt and ₹ 90,000 of equity capital and a general reserve of ₹ 18,000. The Company's total profits after interest and taxes for the year ended 31st March 2005, are ₹ 13,500. It pays 8% interest on borrowed funds and is in the 50% tax bracket. It has 900 equity shares of ₹ 100 each selling at a market price of ₹ 120 per share. What is the weighted average cost of capital ?
    [Answer : WACC(K_o) (using market value weights) = 9·74%]
15. Your company is considering an investment proposal at a cost of ₹ 100 crores. The various sources from which the same can be financed and their relative specific costs are given below :

(i)	Equity :	₹ 50 crores at a cost of capital of 15%.
(ii)	Debentures :	₹ 40 crores at 13% (before tax).

(iii) ₹ 10 crores may be financed from Retained Earnings.

Assuming a corporate tax rate of 50%, determine the minimum acceptable rate of return based on the overall cost of capital of the project.

[Answer : 11·60%]

16. AB Ltd. has assets of ₹ 5,00,000 which have been financed by ₹ 3,00,000 of debt and ₹ 1,50,000 of equity (share of ₹ 100 each) and a general reserve of ₹ 50,000. The firm's *EBIT* (earnings before interest and taxes) for the year ended 31st March 2005 are ₹ 45,000. It pays 10% interest on debt capital and is in the 40% tax bracket. The company's share are selling at a market price of ₹ 200 each. Compute weighted average cost of capital using market values as weights.

[Answer : 4·5%]

17. Excel Industries Ltd. has assets of ₹ 1,60,000 which have been financed with ₹ 52,000 of debt and ₹ 90,000 of equity and a general reserve of ₹ 18,000. The firm's total profits after interest and taxes for the year ended 31st March, 2009 were ₹ 13,500. It pays 8% interest on borrowed funds and is in the 50% tax bracket. It has 900 equity shares of ₹ 100 each selling at a market price of ₹ 120 per share. What is the weighted average cost of capital ?

[Answer : K_o **= 9·74%]**

18. S company has the following capital structure on 1st July, 2016 :

	₹
Equity shares (4,00,000)	80,00,000
10% Preference Shares	20,00,000
14% Debentures	60,00,000
	1,60,00,000

The share of the company currently sells for ₹ 25. It is expected that the company will pay a dividend of ₹ 2 per share which will grow at 7% for ever. Assume a 30% tax rate.

You are required to compute a weighted average cost of capital on existing capital structure.

[Answer : K_o = 12·425% ; K_e = 15% ; K_p = 10 ; K_d = 9·8%]

19. From the following information in respect of a company for the year ended 31.12.16, calculate weighted average cost of capital taking market values as weights.

(i) Capital Structure :

	₹ in Lakhs
Equity (shares of ₹ 100 each)	300
Retained Earnings	300
11% Convertible Debentures	300
12% Institutional Loan	300
Total	1,200

(ii) Current market price per share : ₹ 200. Corporate tax is 40%.

(iii) Current dividend per share is ₹ 12. Tax on dividend is 10%. Future growth rate in dividend may be taken as a proxy of the average of annual growth rates. The company's past dividends per share were as follows :

Year	Dividend per share (₹)
2011	6·50
2012	7·00
2013	8·00
2014	8·50
2015	10·00

[Answer : K_o = 13·02% ; K_e = 18·07% ; K_r = 18·07% ; K_d = 6·60% ;

K_{IL} = 7·2% ; Average growth rate = 11·47%.

$$K_e = \frac{DPS_1(1+t)}{MPS} + g = \frac{₹12(1+0·10)}{200} + 0·1147 = 18·07\%]$$

20. Calculate weighted average cost of capital from the following particulars of BHEL Ltd.

(i) 50,000 Equity shares of ₹ 10 each. Present dividend per share is ₹ 100 and market price per share is ₹ 900. The average growth rate in dividend is 10%.

(ii) Retained Earnings — ₹ 5,00,000.

(iii) 10% ₹ 3,00,000 preference shares of ₹ 100 each issued at ₹ 95 each.

(iv) 3,000, 10% Debentures of ₹ 100 each issued at a premium of 5%, redeemable after 5 years.

(v) 13% Term-loan of ₹ 4,00,000. The company received the entire proceeds of the loan.

Assume the BHEL Ltd. is in a 50% tax bracket and it uses book values as weights. If BHEL Ltd. wants to undertake a new project what would be the minimum acceptable rate of return ?

[Answer : K_o = 18·47% ; K_r = 20% ; K_p = 10·53% ; K_d = 3·90% ; K_{tl} = 6·5%]

21. You are given the following particulars with respect to a firm for the year just ended :

Sources	Amount ₹ / lakhs	After-tax Cost of Capital
Equity share capital	200	18
Retained earnings	100	?
Long-term debt	200	?
Total	500	

The corporate tax rate is 40% and the average cost of capital of the firm is 11·88%. Determine cost of retained earnings (K_r) and cost of debt (K_d) (after-tax and before tax). Make assumption where necessary.

[Answer : K_r = 15%

K_d = (after-tax) = 7·2%

K_d = (before tax) = 12%]

22. The average cost of capital of a firm is 15·36%. The following particulars are ascertained from its books of accounts :

(i) Equity Share Capital : 40,000 shares of ₹ 10 each, cost of equity is 20%.

(ii) Retained Earnings : ₹ 2,00,000

(iii) Debentures : 4000 at ₹ 100 each issued at par.

Calculate cost of retained earnings (K_r) and cost of debentures (K_d) (after-tax and before tax), if the corporate tax is 40%. Make assumption where necessary.

[Answer : K_r = 20%

K_d (after-tax) = 8·4%

K_d (before tax) = 14%]

23. A company supplied the following information to you and requested to compute cost of capital based on book values as well as market values.

Source of Finance	Book Value ₹	Market Value ₹	After Tax Cost (%)
Equity Capital	10,00,000	15,00,000	12
Long-term Debt	8,00,000	7,50,000	7
Short-term Debt	2,00,000	2,00,000	4
Total	20,00,000	24,50,000	

[Answer : Overall cost of Capital (K_o)
— using Book value as weights = 9·20%
— using market value as weights = 9·8167%]

24. The following is the capital structure of Simons Company Ltd. as on 31.12.2016 :

Equity Shares : 10,000 shares (of ₹ 100 each)	₹ 10,00,000
10% Preference Shares (of ₹ 100 each)	₹ 4,00,000
12% Debentures	₹ 6,00,000
	₹ 20,00,000

The market price of the company's share is ₹ 110 and it is expected that a dividend of ₹ 10 per share would be declared after 1 year. The dividend growth rate is 6%.

(i) If the company is in the 50% tax bracket, compute the weighted average cost of capital.

(ii) Assuming that in order to finance an expansion plan, the company intends to borrow a fund of ₹ 10 lacs bearing 14% rate of interest, what will be the company's revised weighted average cost of capital ? This financing decision is expected to increase dividend from ₹ 10 to ₹ 12 per share. However, the market price of equity share is expected to decline from ₹ 110 to ₹ 105 per share.

[Answer. : K_e = 15·09% ; K_p = 10% ; K_d = 6%; K_o = 11·34% ; revised K_e = 17·42% ; revised K_o = 10·67%]

25. The following information are provided for CAP Ltd.:

(i) Present Capital structure at Book Value

	₹
Debentures (₹ 100 per debenture)	8,00,000
Preference Shares (₹ 100 per share)	2,00,000
Equity Shares (₹ 10 per share)	10,00,000

All the above securities are traded in the capital market and the current ruling market prices are :

	₹
Debentures (per debenture)	110
Preference Shares (per share)	120
Equity Shares (per share)	22

Anticipated external financing opportunities are :

(i) ₹ 100 per Debenture redeemable at par ; 10 year maturity, 13% coupon rate, 4% flotation costs, sale price ₹ 100.

(ii) ₹ 100 Preference Share redeemable at par ; 10 year maturity, 14% dividend rate, 5% flotation costs, sale price ₹ 100.

(iii) Equity Shares : ₹ 2 per share flotation costs, sale price ₹ 22, dividend expected on the equity share at the end of the year ₹ 2 per share.

(iv) The anticipated growth rate in dividends is 7% and the company has the practice of paying all its earning in the form of dividends.

(v) The corporate Tax rate applicable is 30%.

You are required to determine the weighted average cost of capital of the company using market weights from the above information.

[Answer : K_e = 17%, K_p = 14·87%, K_d = 8·49%, K_o = 14·697%]

7

Capital Structure

CONTENTS

7.1. Introduction

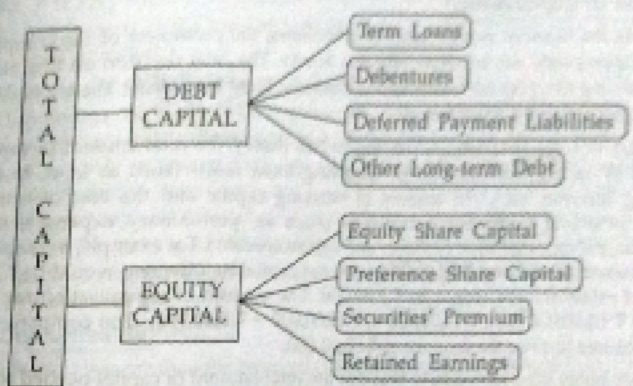
One of the important objectives of a business firm is to maximise the value of the firm, i.e., the value of equity shares of the firm. Keeping this objective in view, the business firm should select a capital structure, i.e., a mixture of debt and equity capital, that would be appropriate in achieving the said objective. Thus, in the arena of financial management, the decision regarding the capital structure of a business firm has an important bearing upon the value of the firm. As a result, it is expected that the financial managers would select such a capital structure that maximises the value of the firm. Capital structure of a business firm can affect the value of the firm through its impact either on the expected income or the cost of capital or both of the concerned business firm. It implies that the earnings available to the shareholders are affected by the capital structure decision and hence, this decision affects the value of the firm.

It is also true that the debt-equity ratio or the leverage of a business firm influences the cost of capital and, hence, the value of the firm. Thus, the desired capital structure of a business firm should be based on a minimum cost of capital and a maximum possible value of the firm (i.e., the value of the equities of the firm). So, a notion of optimum capital structure has developed. It refers to the capital structure where the overall cost of capital is minimum or the value of the firm is maximum.

7.2. Meaning of Capital Structure

Capital structure of a company generally implies different components of capital and their proportions. If we view capital from the liability side of the balance sheet of a company, it would include both equity and debt capital. While equity includes ordinary shares, preference shares and retained earnings, the debt capital comprises bonds or debentures, long-term borrowings and other long-term liabilities of the company. *When the details of these sources of capital are represented by way of their respective quantities and proportion, we call it the capital structure of a company.* According to **John J. Hampton,** *'Capital structure is the combination of debt and equity securities that comprises a firm's financing of its assets'.*

Gerstenberg, however, defines capital structure as follows : *"Capital structure of a company refers to the composition or make-up of its capitalisation and it includes all long-term capital resources, viz., loans, reserves, shares and bonds."*



7.3. Capitalisation and Capital Structure

In the above mentioned definition, the term 'Capitalisation' and 'Capital Structure' do not mean the same thing. While capitalisation refers to the total amount of securities issued by a company, capital structure refers to the kinds of securities and their proportions which make up capitalisation. In a common parlance, capitalisation refers to total amount of capital employed in a business. Gerstenberg defines capitalisation as *"Capitalisation comprises ownership capital which includes capital stocks and surplus in whatever form it may appear and borrowed capital which consists of bonds or similar evidence of long-term debt".* In this connection, it is important to note that the term capitalisation is relevant only in case of joint stock companies.

The term 'Capitalisation' however defined by various financial scholars in different ways, but the basic theme of all those definitions remains almost same.

According to **Bonneville and Dewey,** *"Capitalisation refers to the balance sheet values of stocks and bonds outstanding".* **Pearson Hunt** define *"the term Capitalisation is used to mean the total of funds raised on a long-term basis, whether debt, preferred equity or common equity".* Again according to **A. S. Dewing,** *"the term Capitalisation or the valuation of the capital includes the capital stock and debts".*

On the basis of the above definitions, it can be said that, the term capitalisation is the sum total of Share Capital, Reserves and Surplus, Debentures and other Long Term Borrowings. However, some modern scholars of this subject like **Walker** and **Banghen** prefer to include short-term loan and trade creditors within the term of capitalisation. Since the term capitalisation indicates the quantitative aspect of aggregate capital of a business firm, it helps us in pointing out whether the firm is *over-capitalised* or *under-capitalised* (discussed later on). Thus, capitalisation is concerned with the quantitative aspect of the financial planning of an enterprise. But capital structure is concerned with the qualitative aspects of financial planning. Let us take an example, if a firm wants to procure total capital of ₹ 50,00,000 by issuing equity shares, preference shares and debentures in the ratio of 5 : 3 : 2, the total capitalisation of the firm would be ₹ 50,00,000 and the proportion of equity share capital of ₹ 25,00,000 preference share capital of ₹ 15,00,000 and debentures of ₹ 10,00,000 would be

the capital structure of the firm. Hence, it can be said that, capitalisation is the total amount of capital procured from long-term sources and capital structure is the respective proportion of various sources of capital collected from long-term sources. The need of capitalisation arises not only at the time of incorporation or promotion of a company but may also arise as a going concern after promotion and during the life time of a corporation.

7.3.1. Theories of Capitalisation

After identifying the financial requirements of a business, the promoters of the company have to determine the appropriate mix between debt and equity. The final decision on this matter will be made by considering two popular capitalisation theories. They are (i) Cost Theory of Capitalisation and (ii) Earnings Theory of Capitalisation.

(i) **Cost Theory of Capitalisation :** According to this theory, the total amount of capitalisation is calculated by taking the total cost of acquiring fixed assets (such as land, building, plant, machinery, furniture etc.), the amount of working capital and the cost of establishing the business including promotional expenses (such as, preliminary expenses, underwriting commission, expenses on issue of shares and debentures etc.). For example, a company estimates that fixed assets would cost ₹ 40,00,000, working capital requirement would be ₹ 15,00,000 and the cost of establishment would be ₹ 5,00,000. The amount of capitalisation for the company would be ₹ 60,00,000 (i.e., ₹ 40,00,000 + ₹ 15,00,000 + ₹ 5,00,000). The company issues shares and debentures to raise the amount of ₹ 60,00,000.

Cost theory helps the promoters to find out the total amount of capital needed for establishing the business provided the costs of assets are to be ascertained accurately. This theory is also suitable for determining the financial requirements or the amount of capitalisation of a newly promoted corporation. It enables the promoter to know the total initial amount of capital which they should raise.

However, the cost theory has not been considered an efficient tool on the following grounds :

(a) It takes into consideration only the cost of assets and not the future earning capacity of the investments.

(b) It is very difficult to ascertain the correct cost of promotion and establishment.

(c) Cost theory is not satisfactory in case of a growing concern whose earnings keep on changing whereas the amount of capitalisation remains constant.

(d) Cost theory does not consider the price level changes.

(e) This theory remain silent when the assets becomes obsolete or might have been purchased at inflated price.

(ii) **Earnings Theory of Capitalisation :** According to this theory, the capitalisation of a company depends upon its earnings and the expected fair rate of return on its capital invested. Thus, the value of capitalisation is equal to the capitalised value of the estimated earnings and the process of capitalisation begins with the estimation of future earnings. In case of a new company, it will have to estimate the average future annual earnings and the normal earnings rate which is prevalent in the same industry. The approach of this theory is the best method of capitalisation for the existing companies. It may not be suitable for new companies, as the estimation of earnings is fairly risky and difficult task.

For example, if a company is making profit of ₹ 3,00,000 per annum and the fair rate of return is 10%, the capitalisation of the company will be as follows :

$$\text{Capitalisation} = \frac{₹\ 3{,}00{,}000}{10\%} = ₹\ 30{,}00{,}000$$

A comparison of actual value of capitalisation with this value will show whether the company is fairly capitalised, over capitalised or under capitalised.

Earnings theory of capitalisation seems to be logical because it correlates the value of the company with the amount of capitalisation directly with its earnings capacity. Earnings theory acts as a check on the costs of establishing new companies.

This theory also suffers from the following limitations :

(i) In real life, it is very difficult to estimate expected income and capitalisation rate.

(ii) A mistake committed at the time of estimating the earnings will be directly influencing the amount of capitalisation.

Relationship between Cost Theory and Earnings Theory : Both these theories have their own merits and limitations. As there is no contradiction between them, both are complementary to each other. Cost theory of capitalisation should be applied for determining the amount of capitalisation for a newly promoted company, as it will be difficult to estimate future earnings for such company. As earnings provide a better basis of capitalisation of an established company, a newly promoted company should gradually move to the earning theory of capitalisation in later years. This will save the company from the evils of both under and over capitalisation.

Hence, on the basis of cost theory and earnings theory, we can determine the required or proper capitalisation of a business firm.

7.3.2. Over-Capitalisation

If actual capitalisation is more than the required capitalisation, the firm is said to be over capitalised. In other words, over-capitalisation refers to the situation where earnings of a company do not justify the amount of capital invested in the business. Let us take an example, suppose a company earns ₹ 4,00,000 and the expected normal rate of return is 10%, then the capitalisation of the company on the basis of its earnings would be ₹ 40,00,000 (i.e., $\frac{₹\ 4{,}00{,}000}{10\%}$). But suppose the actual capital employed of this company is ₹ 50,00,000. Then we can say that the company is over-capitalised to the extent of ₹ 10,00,000 (i.e., ₹ 50,00,000 – ₹ 40,00,000). The new rate of earnings would be 8% (i.e., $\frac{₹\ 4{,}00{,}000}{₹\ 50{,}00{,}000} \times 100$) which is less than the normal rate of return (i.e., 10%). This situation arises when a company raises more capital that what is justified by its actual earnings. This leads to the inability of the company to pay fair rate of return in the form of dividend and interest on its shares and debentures respectively.

According to **C.W. Gerstenberg,** *"a company is over-capitalised when its earnings are not large enough to yield a fair return on the amount of stock and bonds that have been issued, or when the amount of securities outstanding exceeds the current value of the assets"*. In the words of **Bonneville, Dewey and Kelly,** *"When a business is unable to earn fair rate on its outstanding securities, it is over-capitalised"*.

It may be noted that over-capitalisation does not necessarily mean abundance or excess of capital. Abundance of capital may be one of the reasons of over-capitalisation, but it is not the only reason. On the contrary, an over-capitalised company may be short of capital. Over-capitalisation arises when the existing capital of a firm is not effectively utilised thus causing a constant decline in earnings. This leads to the inability of the company to pay fair rate of dividend and interest on shares and debentures respectively and the resultant fall in the market value of its shares. If a company has been unable to earn a fair rate of return on its capital, and the market value of its share is lower than the book value over a fairly long period of time, it is over-capitalised.

Causes of Over-capitalisation : There are many factors which account for the situation of over-capitalisation of a company. Following are some of the important causes of over-capitalisation :

(i) Over-capitalisation occurs if a company raises excessive capital than what it can utilise effectively.

(ii) Borrowing large amount of capital at a rate of interest higher than the rate of earnings of the company.

(iii) Payment of excessive amounts for acquisition of goodwill and fixed assets under inflationary conditions. The gap between the book value and the real worth of assets may account for over-capitalisation.

(iv) High promotional expenses and excessive preliminary expenses may lead to over-capitalisation.

(v) Providing inadequate depreciation results in over-capitalisation as it leaves insufficient provision for replacement of assets.

(vi) When a company prefers to follow a liberal dividend policy, it would find it difficult to replace its wornout assets. The company is compelled to resort to costly borrowing which adversely affects its earning capacity. Over a long period of time the combined effect of these factors leads the company to over-capitalisation.

(vii) Rigorous taxation policy of the Government also results in over-capitalisation since this would result in decline of net earnings for the shareholders.

(viii) Under-estimation of the capitalisation rate or over-estimation of earnings will lead to over-capitalisation.

(ix) Time lag between installation and production.

(x) When there is idle capacity and idle funds.

Evils of Over-capitalisation : There are number of harmful effects of over-capitalisation not only to the company and its shareholders but also the society as a whole. Following are the evil effects of over-capitalisation :

On Company	On Shareholders	On Society
1. Decrease in the value of goodwill. 2. Possible difficulty of raising new capital funds. 3. Financial institutions hesitate in providing loans. 4. For earning a better return, the company may increase the prices of the product but due to competition it may not able to sell the product. 5. Temptation for the manipulation of accounts to cover up the deficiency of the decreased earnings and to present a reasonable figure of profits. 6. Demand for liquidation by the money lenders and creditors due to inability to pay the principal and interest. 7. Injury to credit-worthiness.	1. Decrease in the value of share price. 2. Low return on their investments in the form of low dividends on account of low earnings of the company. 3. Shares have small value as collateral security. 4. Low-priced shares are subject to speculative gambling, the real investors have to suffer on account of this manipulation. 5. At the time of re-organisation and reduction of its capital in order to write off accumulated loss, the shareholders are the worst sufferers by reducing the face value of shares.	1. To increase earnings, over-capitalised company reduces the quality and increases the price of products. Hence, consumers have to suffer to pay high prices for poor quality products. 2. Over-capitalisation leads to retrenchment and reduction in wages and salaries for the workers. 3. Tight financial position will affect the morale of the workers and industrial peace. 4. An over-capitalised company either misutilised or under utilised its resources. 5. Over-capitalisation may vitiate the overall investment environment of capital market. 6. It may cause a failure and the failure of the firm may bring about an unhealthy economic situation.

Remedies or Corrective Steps for Over-capitalisation : The following remedial measures may be taken to overcome the situation of over-capitalisation :

(i) Reduction in debt by repayment or redemption.

(ii) Try to reduce interest rate on debentures and bonds.

(iii) Try to reduce cost, so that the profits are improved.

(iv) Redemption of preference shares carrying high rate of dividend.

(v) Reduction in face value as well as the number of equity shares.

(vi) Profit should be ploughed back by suspending the distribution of dividends for few years.

(vii) Try to make management more efficient and to curb excessive expenditure.

The above methods individually or a combinations of one or more methods may be adopted to reduce the situation of over-capitalisation.

7.3.3. Under-Capitalisation

Under-capitalisation is the reverse of over-capitalisation. If the actual capitalisation falls short of the required capitalisation, the firm is said to be under-capitalised. A company is under-capitalised when its earnings are exceptionally high in relation to other similar firms in the industry, or, when it has very small capital to carry on its activities, or, when the real value are more than the book value of its assets.

In the words of **Gerstenberg**, *"A company may be under-capitalised when the rate of profits it is making on the total capital is exceptionally high in relation to the return enjoyed by similarly situated companies in the same industry, or when it has too little capital with which to conduct its business."*

Both over-capitalisation and under-capitalisation are evils. Hence, both the situations are to be avoided. Ultimately every finance manager should aim at fair or proper capitalisation. The condition of under-capitalisation is not as serious as that of over-capitalisation and its remedies are much easily applied.

Causes of Under-capitalisation : The causes of under-capitalisation are :

(i) Under-estimation of initial earnings, as a result the actual earnings may be much higher than those expected.

(ii) When the future capital requirements are under-estimated by the promoters, the amount of capitalisation will be low due to inadequacy of capital.

(iii) When a company is promoted during the period of recession, it may acquire assets at cheaper prices. As soon as the recession is over, such a company becomes under-capitalised when its earnings increases which results in increasing the real value of the assets of the company.

(iv) Because of conservative dividend policy, a company may retain the earnings which results availability of large funds for financial development and expansion. This improves the higher earnings and results in conditions of under-capitalisation.

(v) Efficient management exploiting every possibility to increase the rate of return as compared to the companies in the same industry.

(vi) Low promotional expenses make the company under-capitalised.

(vii) Low tax burden a symtom of under-capitalisation.

(viii) Creation of secret reserves in the form of considerable apprecation in the value of fixed assets not brought into accounts may cause under-capitalisation.

Evils of Under-capitalisation : Like over-capitalisation, under-capitalisation has also many harmful effects on the company and its owners as well as the society as a whole. The principal drawbacks of under-capitalisation are as follows :

(i) High earnings of the under-capitalised companies attract new competitors to enter the business.

(ii) High rate of earnings may induce the employees to demand for higher remuneration and other welfare facilities which may lead to labour unrest. Moreover, the dissatisfaction of the workers probably reduce their efficiency and productivity.

(iii) Due to high earnings, the consumers may feel that they are being cheated by over charging prices for its products.

(iv) High rate of earnings and dividend leads to high market price of the shares of under-capitalised companies. It encourages management to manipulate the share values.

(v) Government generally keeps a watchful eye on under-capitalised companies which earn extraordinary profits. This may lead to more Government control and higher taxation.

(vi) Under-capitalised companies may seek additional long-term funds at a high rate of interest due to inadequacy of capital.

(vii) Due to excess profits, huge retained earnings and long-term debt financing, under-capitalisation leads a company to over-capitalisation in the long-run.

(viii) A high rate of earnings per share result in an increase in market price and the company may be tempted to raise new capital and hence stock may not enjoy the high market price in the long-run.

Remedies or Corrective Steps for Under-capitalisation : The condition of under-capitalisation is not as serious as that of over-capitalisation and its remedies are much easily applied. The situation of under-capitalisation may be corrected by taking the following measures :

(i) The shares may be splitted into shares of small denomination to increase the number of shares. With this split in shares, the rate of earnings will not be changed, but the earnings per share will be substantially decreased.

(ii) Issue of bonus shares is perhaps the most commonly used and effective method for correcting under-capitalisation. This will reduce both the dividend per share and the average rate of earnings.

(iii) The shareholders may be given shares of higher par value in exchange for their existing holding. This would reduce the rate of earnings per share. This method is, however, seldom used, partly because it would not improve the marketability factor.

(iv) Try to make management more efficient to make a proper or fair capitalisation.

7.4. Capital Structure and Financial Structure

The financial structure of a company generally refers to entire liabilities, i.e., both short-term and long-term liabilities of the company. According to **Nemmers** and **Grunewald**, *"Financial structure refers to all the financial resources marshalled by the firm, short as well as long-term, and all forms of debt as well as equity."* Thus, the financial structure is composed of specified proportions of short-term debt, long-term debt and shareholders' funds. When short-term borrowing are omitted from the list, the remaining claims represent the capital structure. However, some authors on financial management consider capital structure in a broader sense so as to include even the short-term debt. So, according to them, there is no distinction between financial structure and capital structure.

7.5. Importance of Capital Structure

One of the crucial problems of any business firm is to make arrangement or planning for the financing of firm's assets. In fact, there should be a prudent decision for fixing up a proper mix of debt and equity capital in financing the firm's assets. Thus, the most crucial decision of any company is concerned with the formulation of an appropriate capital structure.

(1) **Maximisation of return :** Proper designing of a capital structure obviously helps the management of any company to maximise its return on equity capital. In fact, higher profitability

of the company would mean higher return to the shareholders in the form of higher dividend payments. Generally, an increase in the proportion of the debt capital in the capital structure (i.e., an increase in the debt-equity ratio) implies greater amount of interest payments to the bond/debentureholders. Thus, the company should be very much confident about getting a steady return on its capital so that it can easily meet its liability of interest payments. The debt capital can be regarded as the best source of capital so long as the rate of profits on total capital before interest and tax becomes higher than the interest rate of debt capital. Since interest payment on debt capital is a deductible expenditure for income-tax calculations, debt capital (to finance long-term capital requirements) seems to be the cheapest source of capital. For instance, if the interest rate on debt capital is 8% p.a. and the tax rate is 30%, then the real cost of debt would be only 5·6% [= 8% – (30% of 8%)]. As a result, the rate of return to the equity holders will rise. However, higher debt-equity ratio also increases the financial risk of the company because of the fixed contractual obligations on the part of the company to pay the interest on debt capital. Thus, an excessive reliance on debt endangers the very survival of a company. On the other hand, a conservative policy (i.e., depending less on debt capital) may deprive a company of its advantages in terms of the opportunities to magnify the rate of return to the shareholders of the company.

All these factors signify the importance of an optimal capital structure which aims at maximising the return on capital.

(2) **Minimisation of the cost of capital :** The primary objective of any business firm is to maximise the shareholders' wealth through the minimisation of the average cost of capital. Proper financial planning regarding the composition of debt and equity capital (or a capital structure) becomes important at this juncture since the average cost of capital of any company can be brought down to its minimum only through a judiciously planned capital structure.

(3) **Minimisation of risks :** Any business firm is subject to various business risks such as sudden increase in its operating costs, an increase in tax payments, higher costs of borrowing debt capital, falling prices of the products sold by the firm etc. A sound capital structure acts as a shield against such business risks. These risks can be minimised through suitable adjustments in different components of the capital structure.

(4) **Increasing the value of a company :** The total market values of the shares and bonds of a company determine the value of that company. A company with an inappropriate capital structure or debt-equity ratio suffers from financial distress and it would fail to attract investors in its favour. As a result, the market prices of the shares of that company will decline. It leads to a fall in the market value of its securities and hence, a fall in the value of the firm. So, it becomes clear that the value of a company/firm cannot be maximised without a proper capital structure.

(5) **Liquidity :** Formulation of an appropriate capital structure has significant impact on the liquidity of a firm in the form of (a) payment of interest on debt, (b) repayment of debt, (c) payment of preference dividend, and (d) redemption of preference share capital. But the requirement of liquidity should be justified with the cash availability from operations of the firm.

(6) **Financing the long-term development plans of a firm :** The company which fails to design its capital structure in a pre-planned and judicious manner, often faces the difficulties in raising funds on favourable terms in the long-run to finance its developmental plans. Thus, the present capital structure has to be designed in the light of a targeted future capital structure that would support the long-run expansion or growth programmes of the business firm.

(7) **Full utilisation of the available capital :** An ideal capital structure also enables a company to make full utilisation of its available capital. Such a capital structure can establish proper co-ordination between the quantum of capital and the financial requirements of the business.

A balanced capital structure helps a company to avoid either a state of over-capitalisation or (of) under-capitalisation.

(6) **Preservation of control :** The attitude of the management towards preservation of control over the company will have an important impact on the capital structure. In case the funds are raised through the issue of equity shares, the control of the existing shareholders is diluted. Hence, the company might raise the additional funds by way of fixed interest and dividend bearing securities who do not have any voting right. If the management is more answerable to the existing shareholders regarding the performance vis-a-vis the improvement in EPS, the only mode of finance left for the company is to raise finance by way of borrowing.

7.6. Factors Influencing the Planning of a Capital Structure

The financial planners in any business firm have to plan such a pattern of capital structure that would serve the interests of the owners of that firm. Accordingly, the capital structure should be chosen in such a way that it minimises the cost of capital and maximises the value of the stocks (or the value of the owners' capital). So, generally speaking, the capital structure decision is primarily governed by the goal of "wealth maximisation."

While choosing a suitable capital structure for any company, the financial planners of a company should take into account some fundamental principles (or the determining factors) in this regard. These principles are often militant to each other (i.e., there may be a trade off between two different principles). Thus, it becomes difficult to satisfy all such principles at the same time. A prudent finance manager tries to maintain a balance by giving proper weightage to each of those factors which determine the capital structure of a company. We shall first identify some of the guiding principles regarding the capital structure decisions of any business firm.

7.7. Guiding Principles of Capital Structure Decisions

The guiding principles of capital structure decisions of any business firm can be classified into five broad heads, viz., the cost principle, the risk principle, the control principle, the flexibility principle and the timing principle.

(1) **The Cost Principle :** According to this principle, the capital structure of a business firm is said to be an ideal one when it tends to minimise the cost of capital and maximise the Earning Per Share (EPS). We have already mentioned that debt capital is cheaper than equity capital because : (a) cost of debt is limited and the bond holders do not have any claim upon the superior profits of the business firm. The rate of interest on bonds is usually much less than the dividend rate; and (b) interest on debt capital is deductible for income tax purposes (and hence it helps in raising the earnings of the firm after tax payment). However, no such deduction is allowed for dividends payable on equity capital. As a result, the effective interest burden which a firm ultimately bears becomes less than the actual interest rate. So, the use of debt capital can reduce the cost of capital incurred by the firm.

(2) **The Risk Principle :** This principle places greater reliance on common stock for financing the capital requirements of a business firm to minimise the risk element in the capital structure. The interest payment obligations of the firm to the bond holders entails a risk element. If the income of the firm declines unexpectedly to a very low level then the debt obligations (having legal bindings) cannot be met by the firm out of its current income. Hence, too much dependence on debt capital may prove to be highly risky for the firm. Similarly, if the firm issues a large volume of preferred stock, residual owners may be left with little or insignificant income after meeting the fixed dividend obligations to the holders of the preferred stock. These situations may lead to a fall in the share values and share prices of the firm in the stock market. As a result, the common stockholders would suffer a capital loss.

...the risk principle suggests that the firm should put more emphasis on common stock in its ...al structure since the common stock neither entails fixed charges nor the issuer is under legal ...ation to pay dividends to the common stockholders. However, this strategy may lead to a fall ...the EPS of the common stockholders. The effect of the change in debt-equity mix on the EPS of a ...can be shown with the help of a simple example as given below :

$$\text{Here EPS} = \frac{\text{Net profit available to equity holders}}{\text{Number of ordinary shares outstanding}}$$

...ample : A company, say, X Ltd. has a share capital of ₹ 2,00,000 divided into shares of ₹ 20 each. ...ndertakes an expansion programme which involves an investment of another ₹ 1,00,000. The ...ncial managers of the company have suggested the following three alternatives to raise this ...nd :

(a) Issuing 5,000 equity shares of ₹ 20 each,

(b) Issuing 5,000 preference shares (carrying a fixed dividend @ 12%) of ₹ 20 each,

(c) Issuing bonds worth ₹ 1,00,000 bearing an interest rate of 10%.

It is assumed that the present EBIT (Earning Before Interest and Tax payment) of the company is ₹ 1,00,000 p.a. and it remains same even after the expansion of the firm. It is also assumed that the company has to pay 35% tax on its EBIT.

Statement showing calculation of EPS before expansion and after expansion under alternate capital structure.

Particulars	Present Capital Structure i.e., Before Expansion(All Equity)	Alternative Capital Structure		
		All Equity (a)	Equity + Preference Shares (b)	Equity + Bonds (c)
	₹	₹	₹	₹
(1) EBIT	1,00,000	1,00,000	1,00,000	1,00,000
(2) Less : Interest	—	—	—	10,000
(3) EBT	1,00,000	1,00,000	1,00,000	90,000
(4) Less : Tax @ 35%	35,000	35,000	35,000	31,500
(5) EAT	65,000	65,000	65,000	58,500
(6) Preference Dividend	—	—	12,000	—
(7) Earnings available to equity shareholders /Equity Earnings	65,000	65,000	53,000	58,500
(8) Number of Equity Shares	10,000	15,000	10,000	10,000
(9) Earnings Per Share (EPS) [(7) ÷ (8)]	₹ 6·50	₹ 4·33	₹ 5·30	₹ 5·85
(10) Decrease in EPS as against the initial amount	—	(–) ₹ 2·17	(–) ₹ 1·20	(–) ₹ 0·65

This example shows that the dilution of EPS has been the least when the additional funds are raised through debt capital (i.e., through the issue of bonds). It also shows that the dilution of EPS is maximum when the same amount of additional fund is arranged by issuing only equity shares.

(3) **The Control Principle :** This principle suggests that while designing the capital structure of a company, the financial planners of the company should see that the control of the residual owners of the company remains undisturbed. If the additional capital requirement is financed

through common stock, the already existing equity shareholders would lose its control over the affairs of the company because the new shareholders would share the control with the previous shareholders (i.e., the voting rights of the previous shareholders would be reduced). Thus, if the management does not want to disturb the control of the present shareholders over the company, it should raise the additional fund through the issue of bonds (since the bondholders do not have any voting rights in the company).

(4) **The Flexibility Principle** : According to this principle, the financial planners of a company should design such a capital structure that would help them in manoeuvring the sources of funds in accordance with the changing needs for funds. Thus, the management should try to avoid even the cheaper sources of loan when the terms and conditions for availing of such loan restricts the ability of the company to procure additional loan in future. Again, if a company depends too much on debt capital and mortgages all of its fixed assets to secure the presently outstanding debt, it may find it difficult to obtain additional loan (even if the market condition is favourable in availing of the debt capital). So, it cripples the manoeuvrability of the company to finance the needs for additional capital. Thus, the flexibility principles shows that a company, for the sake of its manoeuvrability over the financial sources, should not depend too much on debt capital.

(5) **The Timing Principle** : This principle suggests that the financial managers of a company, while designing the capital structure, should also take into account the opportune moment of time at which funds should be raised from the market by issuing equities or bonds. During the periods of an allround expansion in business activities (or a situation of economic boom), investors have a strong desire to invest in securities. So, at that time period, a business firm can easily raise its required funds from the market by selling equity shares. However, at times of economic depression, the investors are supposed to be risk-averters. So, at that time period the company should issue bonds to raise its required fund (since the investors would get a stipulated interest rate on the face value of those bonds).

7.8. Factors Determining the Capital Structure

Now, we shall identify some of the factors which determine the capital structure of a company.

A. Security Characteristics :

The characteristics of securities affect the capital structure of a company. This can be classified under four broad heads :

(1) **Ownership rights** : In any new business firm the capital requirements can be financed by either of the following means :

(a) Exclusively by the equity stock, or

(b) By a combination of equity and preferred stocks, or

(c) By the combinations of bonds, equity and preferred stocks.

The ownership rights of the new security holders would depend on the type of new securities issued by the firm. The creditors or the bondholders exercise no ownership control over the firm. The preference shareholders may or may not have ownership rights (depending on whether the stock is voting or non-voting). However, the equity share-holders having voting rights, possess the ownership rights over the company.

If the existing shareholders are reluctant to share their ownership rights with the new investors, the business firm then puts more emphasis on debt financing or preferred stock financing.

(2) **Repayment requirements** : The common stock (or the common equity shares) involves no repayment requirements. However, debt financing involves a given contractual repayment obligation (i.e., repayment of principal amount alongwith the accrued interest) for the business

firm. Though the preferred stock has no maturity date, it has a '*call feature*' that allows its repayment. Thus, if a business firm does not like any specific repayment obligation, the proportion of common or preferred stock increases in its capital structure (i.e., the proportion of debt capital would fall).

(3) **Claims on assets** : In the event of a liquidation of the business firm, the bondholders possess the first claim on the assets of that firm. In this ranking, the claims of the preference shareholders are placed at the second position. The common shareholders have the residual claim. If the business firm does not want to given new investors a priority claim on the assets of the firm, common stock would be prominent in the capital structure.

(4) **Claim on profits** : The bondholders of a business firm have no claim upon the profits of the firm. However, they have a legally enforceable right to the payment of an interest at stipulated rates. Thus, interest must be paid to the bondholders regardless of the level of profits earned by the firm. The preference shareholders, however, have the first right to establish their claims on the profits of the firm (upto a specified limit). The common shareholders, on the other hand, have the absolute right to share in the profits of the firm.

Thus, if a firm has an intention to restrict the claims of the new investors upon the profits of the firm, then it would prefer higher proportions of debt or preferred stock in its capital structure.

B. Internal Factors :

Some internal factors within the domain of a business firm also determine the capital structure of the firm.

(1) **The characteristics of a company** : The size and credit standing of any company determine its internal characteristics. Generally, a small-sized company relies to a great extent upon the owner's funds for meeting its fund requirements. It finds it difficult to obtain long-term debts because of its limited creditworthiness. From the view point of the investors, small-sized business firms are considered to be more risky compared to the large-sized firms. Thus, large-sized firms with sound credit standing, find it easier to finance their capital requirements from different sources of their choice.

(2) **Stability of earnings** : The capital structure of any business enterprise also depends on the stability of earnings. If a company has stable earnings, it can afford to raise funds through sources involving fixed charges (i.e., by issuing bonds and preferred stocks). However, the companies who are not assured of such stable earnings, depend on internal sources or common stocks to meet their financial requirements.

(3) **Degree of risk** : The degree of financial risks involved in financing the capital requirements of a company has also an important bearing upon the capital structure of that company. We have already discussed that greater volume of debt capital would mean higher statutory obligations in terms of interest payments. Thus, the chance of inability to meet these debt obligations on the part of a company creates higher risk element. In fact, if a firm raises more debt, the chances of its cash insolvency also rise to a great extent. Again, higher proportion of debt in the capital structure of the company also increases the *risk of variability in the expected earnings available to the equity shareholders* (after meeting the debt obligations). Again, the preferred stocks of the company involve relatively lower risk compared to that in case of bonds/debentures. This is because of the fact that the fixed dividends on such preferred stocks are to be paid only if the company earns a profit. However, *the common stocks are least risky from the view point of a company* because the company may not declare dividend and it does not require to repay equity share capital except on its liquidation.

Thus, the attitude of a company towards risk-aversion gets reflected in its capital structure.

(8) **Cost of capital :** The costs of debt and equity capital also determine the capital structure of any company. The profitability and earnings of a company are affected by the cost of capital. Thus, the financial managers of the company should design the capital structure in such way that it minimises the overall cost of capital. The earnings of the company should be sufficient to meet its cost of capital.

(9) **Surrendering operational control :** The attitude of the existing shareholders of a company towards retaining their operational control over the company also determines its capital structure. If the present shareholders do not want to surrender their control over the company, greater proportion of funds are raised by the issuance of 'non-voting right securities' such as bonds and preference shares (However, in some cases, the company may be unable to sell bonds without agreeing to allow the bondholders to exercise certain operational control, e.g. selecting a member of the Board of Directors if interest payments are not made in due time).

The control principle of capital structure decisions gets higher weightage in private limited companies where ownership is closely held in few hands. However, in case of public limited companies, this control principle is not so much important because the large number shareholders of these companies are so widely scattered that it becomes difficult to them to organise in order to seize the control.

(10) **Matching fluctuating needs against short-term sources :** Sometimes a business firm may require additional fund to conduct the business only during the festival season, say, during September-November in the Eastern region of India. The firm generally wants to avoid long-term financing to meet such short-lived fluctuating needs since it can be easily matched against short-term loans from commercial banks. In such cases, the capital structure remains unaffected.

(11) **Attitude of the management :** The attitude of the management towards the same risk differs from one business firm to another depending upon their motivations, managerial skills, decision making capabilities etc. The capital structure of a company also depends upon whether the management takes a conservative or aggressive attitude towards the financial risks of the company.

(12) **Trading on equity :** When any business enterprise employs borrowed capital including preferred stock to raise the rate of return on equity shares, it is said to be *trading on equity*. If the rate of return on the total capital employed by the company (i.e., long-term borrowing plus shareholder's funds) is higher than the fixed interest on its borrowed capital or the dividend on preference shares, then the equity shareholders get an advantage in the form of additional dividend. Thus, trading on equity would imply a favourable financial leverage in the capital structure of the company.

(13) **Age of a company :** Younger companies which are yet to acquire goodwill and reputation, find it difficult to raise its required capital from the market. Hence, the capital structure of such companies generally indicates greater proportion of equities. These companies should give more weightage to the flexibility or the manoeuvrability principle in raising its funds to ascertain its future growth possibilities. However, the established old companies, having widespread reputation in the market, remains at a comfortable position in raising their required funds from the sources of their choice. The capital structure of these companies are relatively more leveraged compared to those younger companies.

(14) **Comparison with the leverage ratios of other firms in the industry :** The debt-equity ratios or the leverage ratios of different firms (having similar business risks) in an industry should centre around a certain standard. Hence, while designing the capital structure, the financial planners of a firm should compare its leverage ratio with that standard ratio. However, there may be some exceptions (i.e., some firms may be more conservative or more aggressive risk takers than the average number of firms in the industry).

External Factors :

Factors which are beyond the control of a company, and influence the capital structure of the company should be regarded as external factors affecting the capital structure of a company. The external factors are noted below :

(1) **General level of business activity :** The general level of business activity may either show a depressionary condition or a boom condition. Any business enterprise should take into account the timing principle of designing the capital structure at this stage. We have already noted that during the period of economic depression, a company should emphasise on raising its required funds through the issue of bonds (since the investors would prefer to have a given interest income to avoid risks). However, during the boom period, a company can easily raise its required funds by selling equity shares.

(2) **Nature of the industry within which the firm operates :** Sometimes an industry producing any consumer durable (such as coloured T.V., refrigerators, washing machines etc.) is subject to wide fluctuations in its sales. Thus, the firms operating within such an industry, are subject to higher business risks and hence, higher operating leverage. Such firms, while designing their capital structure, should maintain a low degree of financial leverage. So, the proportion of debt capital should be lowered in their capital structure.

On the other hand, the industries which produce essential and non-durable consumer goods such as food articles or inexpensive consumer items (such as paper clips, match boxes etc.) are not subject to such wide fluctuations. As a result, any firm operating within such industry does not run the risk of default in meeting their commitments (or interest payments and other fixed charges). These firms can afford to maintain higher proportions of debt capital in their capital structure.

(3) **Level of interest rates :** Theoretically speaking, the supply and demand forces in the bond market lead to a fluctuation in the interest rates on bonds of different maturity. If the interest rates remain at high levels, firms may avoid debt financing and switch over to equity securities.

(4) **Level of stock prices :** If the stock prices are depressed in the stock markets, a company may not prefer to raise its required funds through the issue of equity shares. On the other hand, a bullish trend in the stock market may induce the company to raise relatively large amount of fund by issuing equity shares. Hence, the condition of the capital market also influences the capital structure of a company.

(5) **Nature and kind of investors :** The psychological parameters of different types of investors and their varied preference patterns also influence the capital structure of any company. If most of the investors are risk-averters in any particular industry, the most of the companies within that industry should depend on debt financing in raising their required funds. However, if the investors are risk-lovers, the firms find it easier to raise their funds by issuing equity shares.

(6) **Present statutory provisions and rules :** The capital structure of any company is also determined by the statutory provisions and rules prevailing in the country. The finance manager has to take into account all such legal aspects while designing the capital structure of a company. For instance, under the Income Tax Laws, dividend on shares is not deductible but interest paid on the debt capital is considered as deductible. Hence, the provisions in the corporate tax laws in any country play a crucial role in determining the capital structure of a company. Similarly, the Rules and Regulations framed by the Stock Exchange Board of India (SEBI) also affect the capital issues policy and hence, the capital structure of different companies.

(7) **Government policies :** The monetary and fiscal policies of the Government also affect the capital structure of a company. If the Government follows a liberal financial policy by allowing almost free entry of foreign institutional investors in the domestic capital market, it would be easier for the domestic companies to raise their required funds by issuing equity shares.

The above factors may be shown below through a chart :

Factors determining the Capital Structure

A. Security Characteristics

1. Ownership rights.
2. Repayment requirements.
3. Claims on assets.
4. Claims on profits.

B. Internal Factors

5. The characteristics of a company.
6. Stability of earnings.
7. Degree of risk.
8. Cost of capital.
9. Surrendering operational control.
10. Matching fluctuating needs against short-term sources.
11. Attitude of the management.
12. Trading on equity.
13. Age of a company.
14. Comparison with the leverage ratios of other firms in the industry.

C. External Factors

15. General level of business activity.
16. Nature of the industry within which the firm operates.
17. Level of interest rates.
18. Level of stock prices.
19. Nature and kind of investors.
20. Present salutory provisions and rules.
21. Government policies.

Chart - 1

7.9. Measurement of Capital Structure

Capital structure refers to the sources of financing for a company. It also refers to the relationship between various long-term sources of financing such as Debentures, Preference Share Capital and Equity Share Capital including reserves and surplus. A company's financial stability and risk of insolvency depend on its financing sources and the types and amounts of various assets it owns. Financing the firm's assets is a very crucial problem in every business and as a general rule there should be a proper mix of debt and equity capital in financing the firm's assets. Preparation of a *common-size statement* of the liabilities and equity section of the balance sheet and computation of some ratios are primarily the measures of the risk of a company's capital structure. This section discuss only two important ratios which are commonly used to measure the degree of financial risk vis-a-vis the relationship between the debt and equity components of the firm's capital structure.

(1) **Debt-Equity Ratio/Liabilities-Proprietorship Ratio :** This ratio show how much of the firm's assets are financed by debt and equity and provide important information about the prospects for future financing. This ratio, also known as *External-Internal Equity Ratio*, is calculated to measure the relative claims of outsiders and the owners against the firm's assets. It may be worked out as follows :

$$\text{Debt-Equity Ratio} = \frac{\text{Long - term Debt}}{\text{Shareholders' Fund/Net Worth}}$$

If this ratio is 2 : 1, it indicates that long-term fund is twice that of the shareholders' fund. A higher proportion of debt would be risky since loans carry with them the obligation to pay interest and dividend at a fixed rate which may become difficult if profit is reduced. Therefore, it does not indicate sound financial health for the company. This is a case of *under-capitalisation*, solely dependent on external funds and obviously displaying its weak financial strength.

Again, a very low *debt-equity ratio* signifies predominance of *equity shareholders' funds* or owners' funds over external funds. This is an indication of *over capitalisation* of the business. Though this situation indicates that the interests of *outsiders* are safe and the firm need not worry about their payment, but the firm is unable to use low-cost outsiders' funds to magnify their earnings. Therefore, it will bring inadequate return on owners' funds because of the absence of *trading on equity*.

(2) **Capital Gearing Ratio :** Capital Gearing Ratio indicates the relationship between fixed interest and/or dividend bearing securities and equity shareholders' funds. This may be expressed as follows :

$$\text{Capital Gearing Ratio} = \frac{\text{Fixed interest and / or dividend bearing securities}}{\text{Equity shareholders' fund}}$$

$$= \frac{\text{Preference Share Capital + Debentures + Other Borrowed Fund}}{\text{Equity Share Capital + Reserves + Surplus - Losses}}$$

This ratio also indicates the degree of vulnerability of earnings available to equity shareholders. Gearing should be maintained in such a way that the company is able to maintain a steady rate of dividend in favour of equity shareholders. Therefore, the term 'gear' which is generally used to control the speed of a motor car, is used to measure the financial risk involved in the capital structure of a company in terms of (i) long-term slovency and (ii) return to equity shareholders.

Using high gear in case of a motor car means increasing the speed and using low gear means bringing down the speed. From this point of view, there can be three types of capital structure of a company :

(a) *Highly Geared* : A company has highly geared capital structure when the fixed interest and/or dividend bearing securities are proportionately larger than the equity shareholders' funds i.e., the ratio is more than 1 : 1.

(b) *Low Geared* : The Capital structure is low geared when the equity shareholders' funds are proportionately larger than the fixed interest and/or dividend bearing securities i.e., the ratio is less than 1 : 1.

(c) *Evenly Geared* : It is evenly geared when these two components are more or less equal i.e., the ratio is almost 1 : 1.

A highly geared capital structure means greater dependence of the firm on its debt and preference share capital. This situation provides a high degree of financial risk in terms of redemption of capital amount within a stipulated time period as well as payment of fixed rate of interest on debt regardless of profit and payment of fixed rate of preference dividend, but subject to availability of profit. Therefore, this is not safe in terms of long-term solvency of a firm.

However, a highly geared company may provide higher return to the equity shareholders during prosperous years. It means, if the rate of return on total fund or capital is more than the average rate of interest on debt and the dividend on preference shares, a highly geared ratio may be considered as blessing to the equity shareholders. This is because in a high profit year, the profit left after paying a fixed rate of interest and preference dividend would be adequate to declare a very high rate of dividend to the small proportion of equity shareholders. Thus, in a favourable situation, a highly geared capital structure is able to take the full advantage of

trading on equity. On the contrary, in a low profit year, the profit earned would be just sufficient to pay interest and preference dividend and thereby equity shareholders being left with very little profit for distribution or nothing at all. Therefore, under this circumstances, a highly geared capital structure will be a curse to the equity shareholdrs. Hence, it is said that in a highly geared capital structure, the equity shareholders live between feast and fast. Similarly, in a low geared capital structure, by virtue of the predominence of equity shares, there is hardly any scope for *trading on equity*. Now the question is, *Is there any optimum level of gearing in terms of capital structure ?*

The answer is, it is not possible to specify an optimum level of gearing for companies, but the general principle is, gearing should be low in those firms where demand is volatile and profits are subject to fluctuation. Therefore, the management needs to aim at maintaining a balance between high geared and low geared capital with specific reference to profitability, financial leverage on EPS, rate of interest and preference dividend, income tax rate and other financial risks involved in the business.

It is quite evident from the above discussion, that there is a close similarity between the debt-equity ratio and the capital gearing ratio. But a basic difference should be noted. In the case of debt-equity ratio, the classification was outsiders' funds against members' funds while in case of capital gearing ratio, the classfication is based on the return on the funds.

7.10. Optimum Capital Structure

The combination of debt and equity that leads to the maximum value of the firm, is referred to as the optimum capital stock. Alternatively speaking, the optimum capital structure is attained when the market value per equity share becomes maximum. At this stage, the cost of capital becomes minimum and the market price per share is maximum. Thus, if the act of borrowing helps a company in increasing the value of its shares in stock exchange, than such borrowings assist the company in moving towards that optimum capital structure. However, if such borrowings result in a fall in the market value of the equity shares of the company, then the act of borrowing is said to move the firm away from its optimum capital structure.

Thus, the objective of any business firm should be to choose such a debt-equity mix in its capital structure that maximises the value of the firm. The advantages of such an optimum capital structure are twofold : (a) the cost of capital would be minimised and that, in turn, would raise the ability of the firm to locate new wealth-creating investment opportunities, and (b) it also helps in boosting up the overall growth rate of an economy since several firms within the economy get the opportunity of productive investment.

In reality, however, it is very difficult to determine the optimum capital structure of a firm. In fact, some financial analysts are of the opinion that the debt-equity mix in the capital structure has no impact on the shareholder's wealth, and hence, the concept of an optimum capital structure is irrelevant from the view point of any company. However, another group of financial analysts strongly support the close interlinkage between the debt-equity mix in the capital structure (or the leverage) and the value of the firm. In order to identify that optimum debt-equity mix, the finance managers should be conversant with the basic theories underlying the capital structure of corporate firms. We shall discuss these theories in our next section.

7.10.1. Features of an Optimum Capital Structure

We have already indicated that the capital structure of a firm is said to be optimum when the cost of capital is minimum and the market price of its share is maximum. Though this capital structure would vary from one firm to another depending upon the expectation of the investors, type of the firm, financing policy of the firm etc., we can identify some of the features of an optimum capital structure as follows :

(1) **Minimisation of the cost of capital :** The average cost of capital of a firm remains at its minimum when its capital structure attains the optimum level. Thus, the weighted average cost of raising the debt and equity capital would be minimum at this stage. This weighted average cost of capital is computed by assigning proper weightage to the cost of debt and cost of equity in proportion to their contribution to the company's total capital.

(2) **Maintenance of proper debt-equity mix :** The finacial managers choose such a debt-equity mix that maintains the capital structure at its optimum level. In fact, debt is a cheaper source of finance because of tax advantage related to the deductibility of interest payments. But too much dependence on debt capital raises the risk of default in meeting the fixed charges. So, optimum capital structure involves such a debt-equity mix that leads to minimum average cost of raising capital.

(3) **Maximisation of the value of the firm :** The optimum capital structure is attained when the market value of the equity shares of the firm or the value of the firm is at its maximum. According to **Ezra Solomon**, 'the optimum capital structure maximises the value of the company and hence, the wealth of its owners.'

(4) **Maximisation of the market price of equity shares :** The market price of the equity shares of the firm becomes maximum when its capital structure remains at its optimum level.

(5) **Maximum possible use of financial leverage :** In any firm, if the return on investment is higher than the fixed cost of funds, the firm should prefer to raise funds carrying fixed interest charges (e.g., debentures, loans and preference share capital). At the optimum capital structure, the use of such debt capital or the use of leverage reaches at its maximum possible level. This results in higher returns for equity shareholders.

(6) **Greater advantage of tax leverage :** When the firm uses debt capital, it can save considerable amount in payment of tax since interest is allowed as a deductible expenses in the computation of corporate taxes. As a result, the effective cost of debt is reduced. This is called tax leverage. An optimum capital structure gets the advantage of such tax leverage.

(7) **Avoidance of higher financial risks :** A firm should also avoid excessive financial risks involved in the use of higher debt capital. This step is required for maintaining an optimum capital structure. In fact, if the shareholders perceive higher risks involved in such debt financing, the market price of equity share will fall. As a result, the optimum capital structure cannot be maintained.

(8) **Maintenance of adequate liquidity :** The chances of default in meeting the debt obligations can be minimised by any firm through keeping adequate amount of liquid assets in its capital structure. All current assets do not possess same degree of liquidity. Similarly, all current liabilities do not require immediate payments. Hence, adequate amount of cash and other liquid assets should be kept with the firm for maintaining an optimum capital structure.

(9) **Maintenance of finacial stability :** The capital structure of a firm should be such that it can generate adequate and stable cash inflows in excess of its cash-outflows. Thus, an optimum capital structure also assures the financial stability of a firm.

(10) **Flexibility of the capital structure :** The capital structure should be very flexible at its optimum level. Thus, the capital structure should be capable of making necessary adjustments in accordance with changing needs of the firm. Such a capital structure enables the firm to raise additional capital from different sources without much delay and difficulty. The preference shares and convertible debentures which can be redeemed at the discretion of the firm offer highest flexibility in its capital structure.

(11) **Conservatism in capital structure :** The capital structure has to be flexible no doubt, but some degree of conservatism should also be maintained. It implies that the current ratio or the debt-equity ratio has to be maintained at its ideal level even if there is a need for changing the components of capital structure.

(12) **Minimum dilution of control :** If additional funds of a firm are raised through the issue of equity shares, the control of the existing shareholders over the firm gets diluted. If additional funds are raised by way of fixed interest-bearing debt and fixed dividend-bearing preference share capital, then the existing shareholders' control is not diluted. Therefore, the capital structure is supposed to be at its optimum level when such dilution of control is minimum.

(13) **Profitability of the firm :** An optimum capital structure not only aims at maximising shareholders' wealth but also the profitability of a firm. Thus, it aims at minimising the costs of raising necessary funds ; and the funds so raised from different sources are invested in such a way that the profitability of the firm reaches at its maximum level.

(14) **Maintaining the capacity of debt repayments :** The optimum capital structure also assures the capacity of the firm in repaying its debt obligations. The use of debt capital should be such that the firm finds no difficulty in repaying the loan amount alongwith the interest charges within the stipulated time period. Otherwise, the solvency of the firm would be at a jeopardy.

Several finacial analysts, however, are of the opinion that the capital structure of a firm has no bearing upon the value of a firm. Hence, there cannot be any optimum capital structure. According to **Myers**, *'the search for optimal capital structure is like the search for Truth or Wisdom : you will never completely attain either goal.'* It implies that the discovery of an optimum capital structure, if any, requires the identification of a large number of factors which influence the capital structure of different firms in different ways. Thus, the capital structure which is considered to be optimum for one firm may not be optimum for the other. Hence, the financial managers often strive for attaining a 'sound' or 'appropriate' capital structure.

Features of Optimum Capital Structure

- Minimisation of the cost of capital
- Maintenance of proper debt-equity mix
- Maximisation of the value of a firm
- Maximisation of the market price of equity shares
- Maximum possible use of financial leverage
- Greater advantage of tax leverage
- Avoidance of higher financial risks
- Maintenance of adequate liquidity
- Maintenance of financial stability
- Flexibility of the capital structure
- Conservatism in capital structure
- Minimum dilution of control
- Profitability of the firm
- Maintaining the capacity of debt repayments

1.11. Capital Structure Theories

There are four major theories which explain the relationship between the capital structure, cost of capital and the value of a firm. These are :

(1) Net Income (NI) Approach,
(2) Net Operating Income (NOI) Approach,
(3) Modigliani-Miller (M-M) Approach, and
(4) Traditional Approach.

Assumptions :

Before presenting those theories, we make some general assumptions to avoid complexities and represent those theories in simple forms :

(1) A business firm employs only two types of capital : debt and equity stock. There is no preferred stock.

(2) There are no corporate taxes (this assumption has been removed later).

(3) The firm pays 100% of its earnings as dividend, i.e., dividend pay-out ratio is 100. Thus, there are no retained earnings.

(4) The total assets of the firm are given and they remain unchanged. Thus, the investment decisions are assumed to remain unchanged.

(5) The total financing of the firm is also assumed to remain constant. Thus, the firm can change its degree of leverage (or the capital structure) either by selling the equity shares and use the proceeds to redeem the debentures, or by raising more debt and reduce the equity capital.

(6) The operating earnings (the Earning Before Interest and Tax or the EBIT) of the firm are not expected to grow.

(7) The business risk of a firm is also assumed to remain constant and it remains independent of the capital structure and financial risks.

(8) All the investors have similar expectations regarding the future variability of EBIT for any given firm (i.e., they have the same subjective probability distribution of the future expected EBIT for a given firm).

(9) The business firm is supposed to have a perpetual life.

1.11.1. Net Income (NI) Approach

According to the Net Income Approach, as suggested by **David Durand**, the capital structure decisions have an important bearing upon the valuation of the firm. Alternatively speaking, a change in the capital structure results in a corresponding change in the overall cost of capital as well as the value of the firm. Thus, the firm can influence its value by changing the proportion of debt capital in the debt-equity mix of its capital structure.

According to this approach, higher financial leverage or the higher debt content in the capital structure leads to a reduction in the weighted average cost of capital of the firm. As a result, the returns available to the shareholders will increase. The increased returns to the shareholders would, in turn, increase the total value of the equity and hence, it would lead to an increase in the value of the firm. Following the similar argument, we can say that a fall in the financial leverage of a firm would cause an increase in the overall cost of capital and hence, it ultimately leads to a fall in the value of the firm.

This approach is based on the following assumptions :

(a) There are no corporate taxes (as we have already stated).

(b) The cost of debt capital is less than the cost of equity capital i.e., $K_d < K_e$

(c) Any change in the financial leverage or the debt content in the capital structure does not alter the risk perception of the investors, and

(d) The cost of debt capital and the cost of equity capital will remain unchanged irrespective of any change in the debt-equity mix of the firm.

The value of the firm, on the basis of this approach, can be ascertained as follows :

$$V = S + D$$

where, V = Value of the firm,

S = Market Value of the Equity,

D = Market Value of the Debt.

Again, the **market value of the equity capital(s)** can be expressed as follows :

$$S = \frac{E_e}{K_e}$$

where, E_e = Earnings available for equity shareholders or, Equity Earnings,

K_e = Cost of equity capital(s) or the equity capitalisation rate.

Here, $K_e = \frac{DPS_1}{MPS} + g$,

where DPS_1 = Expected Dividend at the end of the first year,

MPS = Current market price of the equity share, and

g = Expected growth rate of dividend payments.

Since we have assumed that a firm pays 100% of its earnings as dividend, so the percentage of its retained earnings will be zero.

In our case, $g = br$, where b = retention rate,

and r = rate of return on equity shares.

But, on the basis of our previous assumption, $b = 0$ and therefore, $g = 0$.

$$\therefore \quad K_e = \frac{DPS_1}{MPS}$$

In operational terms, we can say that $DPS_1 = EPS_1$,

where EPS_1 = Earning Per Share (EPS) at the end of the first year.

$$\therefore \quad K_e = \frac{EPS_1}{MPS}$$

If N = Number of outstanding equity shares, then

we can write $K_e = \frac{EPS_1.N}{MPS.N} = \frac{E_e}{S}$

$$= \frac{\text{Earnings available to the equity shareholders or, Equity Earnings}}{\text{Total market value of equity shares}}$$

$$= \frac{EBIT - I}{S} \quad [\because \text{ There is no corporate tax}]$$

where $EBIT$ = Earnings before Interest and Tax Payments, and

I = Total interest payments.

The **market value of debt capital** (D) can be estimated as follows :

$D = \frac{I}{K_d}$, where K_d = the cost of debt capital, and

I = total interest payments.

$$\therefore \quad K_d = \frac{I}{D}$$

The overall cost of capital (K_o) can now be estimated as the weighted average of the costs of debt and equity capital.

$$\therefore \quad K_o = W_1 K_d + W_2 K_e$$

where W_1 = Proportion of market value of debt capital in the total value of the firm (i.e., the relative weight of the debt capital), and

W_2 = Proportion of market value of equity capital in the total value of the firm (i.e., the relative weight of the equity capital).

$$\therefore \quad K_o = \left(\frac{D}{V}\right)K_d + \left(\frac{S}{V}\right)K_e = \left[\frac{D}{D+S}\right]K_d + \left[\frac{S}{D+S}\right]K_e$$

$$= \frac{DK_d + SK_e}{D+S}$$

$$= \frac{I + E_e}{V} = \frac{EBIT}{V} \quad [\because DK_d = I \text{ and } SK_e = E_e]$$

$$\therefore \quad \text{Total value of the firm } (V) = \frac{EBIT}{K_o}$$

Illustration 1.

P Ltd. has operating income of ₹ 1,00,000 and its cost of equity is 10% and cost of debt is 6%. The amount of debt capital is ₹ 5,00,000.

(a) What is the value of the firm ? Find out the overall cost of capital (K_o).

(b) What is the value of the firm and corresponding overall cost of capital if the amount of debt capital increases to ₹ 7,00,000.

Solution :

(a) **Computation of the value of the firm (V) and overall cost of capital (K_o) under Net Income (NI) Approach.**

	₹
Operating Income (*EBIT*)	1,00,000
Less : Interest on debt capital (*I*) [6% of ₹ 5,00,000]	30,000
Earnings available to equity shareholders/ Equity Earnings (E_e)	70,000
Cost of equity/Equity capitalisation rate (K_e)	10%
∴ Market value of Equity Capital (S) = $\frac{E_e}{K_e} = \frac{₹\ 70,000}{10\%}$	₹ 7,00,000
Market value of Debt Capital (*D*) = $\frac{I}{K_d} = \frac{₹\ 30,000}{6\%}$	₹ 5,00,000
∴ Value of the firm [*V* = (*S* + *D*)]	₹ 12,00,000

Overall Cost of Capital/Overall Capitalisation Rate (K_o)

$$\frac{EBIT}{V} = \frac{₹\ 1,00,000}{₹\ 12,00,000} = 0{\cdot}08333 \text{ or } 8{\cdot}33\%$$

Alternatively, overall cost of capital may be computed as follows :

$K_o = W_1 K_d + W_2 K_e$, where $W_1 = \frac{D}{V}$ and $W_2 = \frac{S}{V}$

$$\therefore \quad K_o = \left(\frac{D}{V}\right)K_d + \left(\frac{S}{V}\right)K_e$$

$$= \left(\frac{₹\ 5,00,000}{₹\ 12,00,000}\right) \times 0{\cdot}06 + \left(\frac{₹\ 7,00,000}{₹\ 12,00,000}\right) \times 0{\cdot}10$$

$$= 0{\cdot}0250 + 0{\cdot}0583$$

$$= 0{\cdot}0833 \text{ or } 8{\cdot}33\%$$

(b) **Computation of the value of the firm (V) and overall cost of capital (K_o) when debt capital increases to ₹ 7,00,000.**

	₹
Operating Income (*EBIT*)	1,00,000
Less : Interest on debt capital (*I*) [6% of ₹ 7,00,000]	42,000
Equity Earnings (E_e)	58,000
Equity capitalisation rate (K_e)	10%
Market value of Equity Capital (S) = $\frac{E_e}{K_e} = \frac{₹\ 58,000}{10\%}$	₹ 5,80,000
Market value of Debt Capital (*D*) = $\frac{I}{K_d} = \frac{₹\ 42,000}{6\%}$	₹ 7,00,000
Value of the firm [*V* = (*S* + *D*)]	₹ 12,80,000

Overall Cost of Capital (K_o) $= \frac{EBIT}{V}$

$$= \frac{₹\ 1,00,000}{₹\ 12,80,000}$$

$$= 0{\cdot}0781 \text{ or } 7{\cdot}81\%$$

Therefore, the essence of Net Income approach is that a firm can minimise its overall cost of capital (from 8·33% to 7·81% in the above illustration) and increase its value (from ₹ 12,00,000 to ₹ 12,80,000) by increasing the proportionate use of debt capital (from ₹ 5,00,000 to ₹ 7,00,000) in the overall capital structure. Thus higher the leverage, higher the total value of the firm. So optimum capital structure will be the one having 100% debt financing which would result in achieving the lowest overall cost of capital. But in reality this is not possible. So, appropriateness in capital structure is essential rather than of its optimality. Thus the desirable structure, according to this approach, should be, the highest possible leverage leading to maximisation of the value of the firm and minimisation of overall cost of capital. The reduction of overall cost of capital with more and more use of debt capital and increase in the value of the firm is only possible when assumptions in respect of NI approach (as mentioned earlier) are held valid.

Thus, according to the Net Income approach, a capital structure is a said to be optimum when K_o is the lowest and hence, '*V*' attains its maximum possible value. At this stage, the market price per share would be maximum.

Again, if the degree of leverage $= \frac{D}{V} = 1$

$$\Rightarrow D = V$$

$$\Rightarrow S = 0$$

$$(\because V = D + S)$$

∴ In this case, $K_o = K_d$

$$\left[\because K_o = \left(\frac{D}{V}\right)K_d + \left(\frac{S}{V}\right)K_e\right]$$

On the other hand, if the firm uses no debt, i.e.,

if the financial leverage $= \frac{D}{V} = 0$, then $K_o = \left(\frac{S}{V}\right)K_e = K_e$ [$\because S = V$]

These relationships have been indicated in Fig.-1. In Fig.-1, we have measured the degree of leverage along the horizontal axis and the cost of capital (i.e., the percentage rates of K_d, K_o and K_e) along the vertical axis. The K_e and K_d curves remain parallel to the horizontal axis since we have assumed that K_e and K_d remain independent of the degree of leverage.

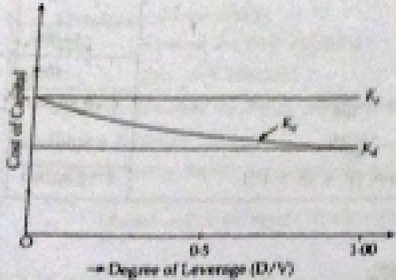


Fig. 1

We also observe that when $\frac{D}{V} = 0$, the vertical intercepts of the K_e and K_o curves become same (i.e., $K_o = K_e$ when $\frac{D}{V} = 0$). The K_o curve slopes downwards because the overall cost of capital declines with an increase in the degree of leverage. When $\frac{D}{V}$ approaches to the value 1, K_o tends to become equal to K_d. At this point K_o becomes minimum. Thus, the Net Income approach indicates that if a firm employ 100% debt capital (and no equity capital) then the overall cost of capital would be minimum and the value of the firm would become maximum.

Criticisms of Net Income Approach :

The principal drawback of the Net Income approach is that it assumes constancy of cost of debt capital and the cost of equity capital. Empirical observations indicate that with the introduction of debt capital, the cost of equity tends to rise and, after a certain stage of leverage, the cost of debt capital also starts rising. Moreover, the general assumption of the distribution of entire earnings of a firm as dividend or the similarity of risk perception among all investors have no practical feasibility.

7.11.2. Net Operating Income (NOI) Approach

The *Net Operating Income (NOI) approach* towards the designing of capital structure was also suggested by *Durand*. But this approach is diametrically opposite to the NI approach which we have already discussed. According to this approach, the value of a firm is not at all affected by the changes in its capital structure. This theory indicates that the market price of shares and the overall cost of capital would be independent of the degree of financial leverage of a firm. Hence, this theory suggests that the capital structure decisions in any firm are irrelevant and hence, there remains no such thing as optimum capital structure. Any capital structure can be considered as an optimum capital structure for a firm.

This theory is based on the following *assumptions* :

(1) The overall cost of capital (K_o) remains constant for all degrees of financial leverage.

The investors see the firm as a whole and thus capitalises the total earnings of the firm to determine the value of the firm as a whole (i.e. the split between the debt and equity capital is not relevant here).

The value of equity (S) is a residual value and it is determined by deducting the total value of debt (D) from the value of the firm (V). $\therefore S = V - D$.

The cost of debt capital (K_d) also remains constant at all degrees of financial leverage, and $K_d < K_e$.

The cost of equity capital (K_e) or the equity capitalisation rate increases with an increase in the degree of financial leverage. Greater use of debt capital having a low cost increases the financial risk of the equity shareholders. Hence, to compensate that risk, the shareholders would expect higher rates of return on their investment. This will cause an increase in the cost of equity capital or the equity capitalisation rate. Thus, the advantage of debt is set off exactly by an increase in the cost of equity capital.

There are no corporate taxes.

According to this approach the value of a firm can be determined by capitalising the EBIT/Operating Income at Overall Cost of Capital (K_o) as follows :

$$V = \frac{EBIT}{K_o}$$

Again, the cost of equity capital can be estimated as follows :

$$K_e = \frac{EBIT - I}{S}$$

$$= \frac{EBIT - I}{V - D} \quad [\because S = V - D]$$

The relationship between K_e and the degree of financial leverage $\left(\frac{D}{S}\right)$ is shown as follows :

$$K_e = K_o + (K_o - K_d)\left(\frac{D}{S}\right)$$

Since the values of K_d and K_o remain constant, the value of K_e rises with an increase in the value of $\frac{D}{S}$.

Proof : We know that $K_o = \left(\frac{D}{V}\right)K_d + \left(\frac{S}{V}\right)K_e$

$$\text{or, } K_e = \frac{K_o - K_d\left(\frac{D}{V}\right)}{\frac{S}{V}}$$

Again, we know that $V = D + S$

$$\therefore \frac{S}{V} = \frac{S}{D+S} = 1 - \frac{D}{D+S} \left[\because \frac{D}{D+S} + \frac{S}{D+S} = 1\right]$$

$$\therefore K_e = K_o - K_d\left[\frac{D}{D+S}\right] / 1 - \frac{D}{D+S}$$

$= $ [illegible]

$= $ [illegible]

$= $ [illegible]

$= K_o + (K_o - K_d)\left(\frac{D}{S}\right)$

Illustration 2.

P Ltd. has operating profit of ₹ 1,00,000 and its overall cost of capital is 10% and cost of debt capital is 6%. The company has employed debt capital of ₹ 5,00,000.

(a) Compute the value of equity capital and cost of equity capital under Net Operating Income (NOI) approach.

(b) What will be the implication for increase in the debt capital from ₹ 5,00,000 to ₹ 7,00,000.

Solution :

(a) Computation of Value of Equity Capital and Cost of Equity Capital (K_e) under Net Operating Income (NOI) approach

We know that, $V = D + S$

or, $S = V - D$

where, V = Value of the firm

D = Value of the debt capital

S = Value of Equity capital

Value of the firm $(V) = \frac{EBIT}{K_o} = \frac{₹1,00,000}{10\%}$	=	₹ 10,00,000
Less : Value of the Debt Capital $(D) = \frac{I}{K_d} = \frac{(6\% \text{ of } ₹5,00,000)}{6\%}$	=	₹ 5,00,000
Value of Equity Capital (S)		₹ 5,00,000

Now, Cost of Equity Capital $(K_e) = \frac{EBIT - I}{S}$

$= \frac{(₹1,00,000 - ₹30,000)}{₹5,00,000}$

$= \frac{₹70,000}{₹5,00,000}$

$= 0{\cdot}14$ or 14%

Alternatively,

Cost of Equity Capital may be computed as follows :

$K_e = K_o + (K_o - K_d)\left(\frac{D}{S}\right)$

$= 0{\cdot}10 + (0{\cdot}10 - 0{\cdot}06)\left(\frac{₹5,00,000}{₹5,00,000}\right)$

$= 0{\cdot}10 + 0{\cdot}04$

$= 0{\cdot}14$ or 14%

Verification of NOI approach by calculating K_o of the firm.

$K_o = K_d\left(\frac{D}{V}\right) + K_e\left(\frac{S}{V}\right)$

$= 0{\cdot}06\left(\frac{5,00,000}{10,00,000}\right) + 0{\cdot}14\left(\frac{5,00,000}{10,00,000}\right)$

$= 0{\cdot}03 + 0{\cdot}07$

$= 0{\cdot}10$ or 10%.

(b)

Value of the firm $(V) = \frac{EBIT}{K_o} = \frac{₹1,00,000}{10\%}$	=	₹ 10,00,000
Less : Value of Debt Capital $(D) = \frac{I}{K_d} = \frac{(6\% \text{ of } ₹7,00,000)}{6\%}$	=	₹ 7,00,000
Value of Equity Capital (S)		₹ 3,00,000

Cost of Equity Capital $(K_e) = \frac{EBIT - I}{S}$

$= \frac{(₹1,00,000 - ₹42,000)}{₹3,00,000} = 0{\cdot}1933$ or 19·33%.

Therefore, the essence of Net Operating Income approach is that the market value of the firm (V) remains the same (₹ 10,00,000 in the above illustration) irrespective of the method of financing i.e. it is not affected by the use of debt capital. Again, since the values of K_d and K_o remain constant, the equity capitalisation rate/cost of equity (K_e) increases (from 14% to 19·33%) with the increase in debt capital (from ₹ 5,00,000 to ₹7,00,000) in the total capital structure.

The NOI approach may be represented with the help of a diagram (Fig.-2) based on the data used in illustration 2 above.

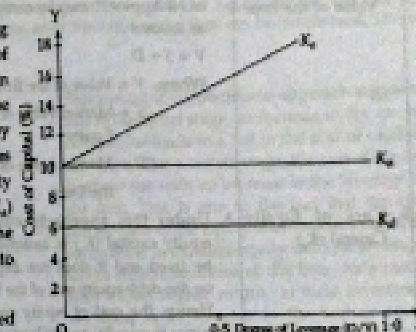


Fig.-2

In Fig.-2. We have measured the degree of leverage along the horizontal axis and the cost of capital (i.e. the percentage rates of K_d, K_e and K_o) along the vertical axis. The K_d and K_o curves seem parallel to X-axis since we have assumed that K_d and K_o remain independent of the degree of leverage. But if the degree of leverage $\left(\frac{D}{V}\right)$ increases, the cost of equity (K_e) increases continuously.

Criticisms of Net Operating Income Approach :

This theory has also been criticised on the following grounds :

(a) This approach presumes that the benefits from the use of cheaper debt capital will be just off set by the increase in the cost of equity. Therefore, the value of the firm will remain unchanged. But this seems to be an absurd proposition and is unlikely to happen in reality.

(b) Under this approach, change in the capital structure of a firm does not affect the market value of the firm and every capital structure is the optimum capital structure, provided there are no corporate taxes. However, when the existence of taxes are assumed, the optimum capital structure can be achieved by maximising the debt mix in the capital structure of a firm.

(c) According to this approach, there will be no optimum capital structure of any firm. If this is true, there will be no need of any financial plan for any firm.

Difference between Net Income (NI) Approach and Net Operating Income (NOI) Approach :

Following are the important points of difference between NI Approach and NOI Approach :

Point of difference	Net Income (NI) Approach	Net Operating Income (NOI) Approach
1. Value of the firm	1. According to this approach, value of the firm depends on capital structure. It means, the firm can affect its value by changing the debt proportion in the overall capital structure.	1. In this case value of the firm does not depend on capital structure. It means, capital structure is irrelevant and does not affect the value of the firm.
2. Computation of Value of the firm	2. The value of the firm on the basis of NI Approach may be computed as follows : $V = S + D$ Where, V = Value of the firm, S = Market value of the Equity, D = Market value of the Debt	2. Accordingly to NOI Approach, the value of the firm can be determined as follows : $V = \frac{EBIT}{K_0}$ Where, $EBIT$ = Earnings before Interest and Tax, K_o = Overall Cost of Capital
3. Cost of Equity Capital (K_e)	3. Under this approach, cost of equity capital (K_e) is assumed to be fixed and it does not depend on the debt-equity mix of the firm. Hence, the cost of equity capital curve is parallel to the horizontal axis.	3. Under this approach, Cost of equity capital (K_e) is not fixed, it increases with an increase in the degree of financial leverage. Hence, the cost of equity capital curve becomes upward sloping.
Overall Cost of Capital (K_o)	4. According to this approach, if we increase the degree of financial leverage i.e., if we increase cheaper debt in the capital structure, the overall cost of capital (K_o) decreases and K_o curve is downward slopping.	4. Accordingly to this approach, the overall cost of Capital (K_o) of the firm is constant for all degrees of financial leverage and K_o curve is parallel to the horizontal axis.
Risk Perception of Investors	5. Any change in the financial leverage or the debt content in the capital structure does not alter the risk perception of the investors.	5. The use of more and more debt in the capital structure increase the risk of the shareholders.
Optimum Capital Structure	6. According to this approach, every firm may have an optimum capital structure in case of one having 100% debt financing which would result in achieving the lowest K_o. But in reality, this is not possible. Hence, appropriateness is better than its optimality.	6. According to this approach, there will be no optimum capital structure of any firm.

4.3 Traditional Approach

The traditional view of capital structure theory (which has been popularised by Ezra Solomon) is a compromise between the two extreme views regarding the relationship between cost of capital, leverage and value of a firm. As the NI and the NOI approach hold extreme views regarding the relationship between cost of capital, leverage and the value of the firm, the traditional approach lies a midway between NI and NOI approach.

The approach suggests that through a judicious use of both debt and equity capital, the cost of capital of a firm can be minimised and consequently the value of the firm can be maximised. There are two variants of this approach.

First Variant :

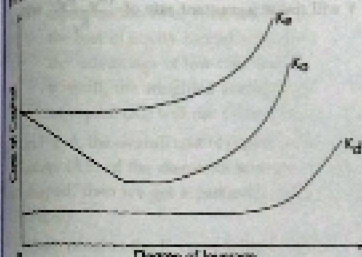


Fig.-4

This variant of the traditional approach suggests that at the initial stage, an increase in the use of debt capital leads to a fall in the cost of capital. However, after a certain stage, the cost of capital tends to rise with an increase in the leverage of the firm. This is due to the fact that the risk perceptions of the equity shareholders rise with an increase in the debt-equity ratio (or leverage) in the capital structure of the firm. As a result, they expect higher return on their investment resulting in an increase in the cost of equity capital. Now, if the proportion of debt capital is increased still further, the risk perception of the

bond or debenture holders would also increase, and consequently, the cost of debt capital will rise. Thus, the average cost of capital (or the overall cost of capital) will increase at a higher rate with the simultaneous increase in the costs of debt capital and equity capital of the firm. The proposition of the first variant of the traditional approach can be shown with the help of a simple diagram (Fig.-3).

▪ Second Variant :

The second variant of the traditional approach has divided the impact of the degree of leverage on the cost of capital in three stages.

(a) **First Stage :** At the initial stage, the cost of equity capital remains constant or rises slightly with an increase in debt capital. Again, at this stage, the cost of debt capital also remains constant or rises negligibly since the market views the use of debt as a reasonable policy. As a result, the overall cost of capital will fall (or the value of the firm increases) with an increase in debt-equity ratio.

This can be shown as follows :

The value the firm $(V) = S + D$ = Market value of equity capital + Market value of debt capital

$$= \frac{EBIT - I}{K_e} + \frac{I}{K_d}$$

where, I = Interest on debt capital

$= K_d D$;

$EBIT$ = Earning Before Interest and Tax payments.

Since, there is no corporate tax (by assumption), therefore,

$EBIT - I$ = Earnings available to the equity shareholders/Equity earnings.

Here, K_e = Cost of equity capital and it is assumed to remain constant within an acceptable limit of debt.

$$\therefore \text{ We get, } \quad V = \frac{EBIT - K_d D}{K_e} + \frac{K_d D}{K_d}$$

$$\text{or, } \quad V = \frac{EBIT - K_d D}{K_e} + D$$

$$\text{or, } \quad V = \frac{EBIT}{K_e} + \frac{D(K_e - K_d)}{K_e}$$

Thus, so long as K_e and K_d remain unchanged, V will rise at a constant rate of $\frac{(K_e - K_d)}{K_e}$ with an increase in D (i.e., the debt capital).

Again, we know that, $V = \frac{EBIT}{K_o}$

$$\text{or, } K_o = \frac{EBIT}{V}$$

$$\therefore V = \frac{EBIT}{K_e} + \frac{D(K_e - K_d)}{K_e}$$

$$\text{or, } 1 = \frac{EBIT}{V} \cdot \frac{1}{K_e} + \frac{D}{V} \cdot \frac{(K_e - K_d)}{K_e}$$

$$= \frac{1}{K_e}\left\{K_o + \frac{D}{V}(K_e - K_d)\right\}$$

$$\text{or, } \quad K_e = K_o + \frac{D}{V}(K_e - K_d)$$

$$\text{or, } \quad K_o = K_e - \frac{D}{V}(K_e - K_d)$$

This result shows that when $K_e > K_d$ and K_e & K_d remain unchanged then the average cost of capital (K_o) will decline with an increase in leverage $\left(\frac{D}{V}\right)$.

(b) **Second Stage :** After a certain degree of leverage, the cost of equity capital will tend to rise because of the increased risk perception of the equity shareholders. At this stage, the increase in the cost of equity (due to added financial risk arising out of higher leverage) will just offset the benefit of using cheaper debt capital. It will continue upto a certain range of the degree of leverage. Hence, within this range of the degree of leverage, the average cost of capital will remain unchanged. The average cost of capital will be minimum and hence, the value of the firm will be maximum. So, that range of the degree of leverage would be regarded as the optimum degree of leverage.

(c) **Third Stage :** If the degree of leverage is increased beyond that range (as shown in the second stage), the risk perception of the debt holders will also rise. At the same time, the cost of equity capital will rise at higher pace because the equity stockholders perceive a high degree of financial risk and hence, demand a higher equity capitalisation rate. Thus, the rise in the cost of equity capital will offset the advantage of low-cost debt. As a result, the weighted average cost of capital (K_o) will rise (This is shown in Fig.-4 &-5).

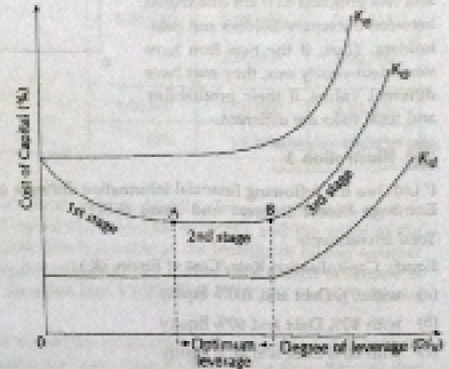


Fig.-4

In Fig.-4, the overall cost of capital curve becomes saucer-shaped. It shows that within a particular range (AB) of the degree of leverage, K_o reaches at its minimum. However, if the K_o curve is U-shaped, then we get a particular degree of leverage $(D/V)^*$, at which the K_o becomes minimum. (Fig.-5)

Thus, whether the K_o curve is horizontal or U-shaped is not very much pertinent from the theoretical point of view. The relevant theoretical issue is whether K_o declines with an increase in the degree of

leverage or not. The supporters of the traditional approach are of the opinion that K_o declines with an increase in the debt-equity mix in the capital structure of a firm upto certain stage.

- **Criticisms of the Traditional Approach :**

Many financial analysts are of the opinion that the value of a firm depends upon :

(i) the profitability or the net operating income of a firm, and

(ii) the risk component attached to it.

The debt-equity planning of a firm can neither change the profitability nor the risk attached to it. The total profits and risks of the firms are distributed between the debt holders and the equity stakeholders. Thus, the debt-equity planning can simply change the way in which the profit and risk attached to it are distributed between the equity-holders and debt-holders. Thus, if the two firm have same debt-equity mix, they may have different values if their profitability and total risks are different.

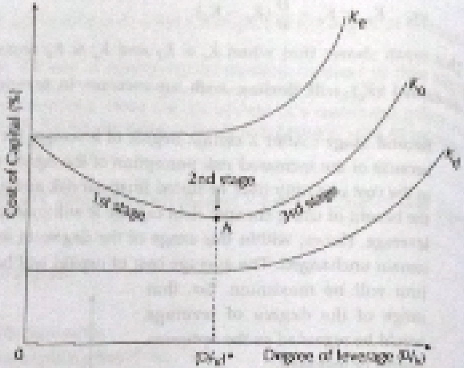


Fig.-5

Illustration 3.

P Ltd. has the following financial information during a given period :

Earnings before Interest and Taxes (EBIT)	₹	1,00,000
Total Investments	₹	5,00,000

Equity Capitalisation Rate/Cost of Equity (K_e) :

(a) with 0% Debt and 100% Equity	15%
(b) with 40% Debt and 60% Equity	16%
(c) with 60% Debt and 40% Equity	18%

The firm can raise debt of ₹ 2,00,000 at 10% rate of interest and ₹ 3,00,000 at 12% rate of interest. Determine the market value of the firm (V) and average cost of capital or overall capitalisation rate (K_o) under Traditional Approach.

Solution :

Computation of Value of the firm (V) and Overall Cost of Capital (K_o) under Traditional Approach

Particulars	(a) 0% Debt	(b) 40% Debt	(c) 60% Debt
Total Investments :			
Debt (₹)	—	2,00,000	3,00,000
Equity (₹)	5,00,000	3,00,000	2,00,000
	5,00,000	5,00,000	5,00,000
EBIT (₹) 1,00,000	1,00,000	1,00,000	1,00,000
Less : Interest on Debt (I) ₹	—	20,000	36,000
Earnings available to equity share holders/Equity earnings (E_e) (₹)	1,00,000	80,000	64,000
Equity Capitalisation Rate (K_e)	15%	16%	18%
Market value of Equity (S) $\left[\frac{E_e}{K_e}\right]$ (₹)	6,66,667	5,00,000	3,55,556
Market value of Debt (D) (₹)	—	2,00,000	3,00,000
Market value of the firm [$V = S + D$] (₹)	6,66,667	7,00,000	6,55,556
Overall Cost of Capital (K_o) = $\left[\frac{EBIT}{V}\right]$	15%	14·29%	15·25%

Comment :

It is clear from the above illustration that with the increase in debt (i.e., leverage) from 0% to 40%, the firm is able to reduce its overall cost of capital (K_o) from 15% to 14·29% and the value of the firm (V) increases from ₹ 6,66,667 to ₹ 7,00,000. This is possible as the benefits of raising cheaper debt are available and the K_e does not rise significantly. However, if more debt is used to finance in place of equity (60%), the value of the firm decreases from ₹ 7,00,000 to ₹ 6,55,556 and K_o increases from 14·29% to 15·25%.

Therefore, it shows that upto a certain point a firm can, by increasing the proportion of debt in its capital structure, reduce overall cost of capital and raise market value of the firm. Beyond that point, further introduction of debt will cause the overall cost of capital to rise and market value of the firm to fall. Thus, by a judicious mix of debt and equity, the firm can minimise its overall cost of capital and maximise the value of the firm.

7.11.4. The Modigliani-Miller Hypothesis

F. Modigliani and M.H. Miller, in their article titled "The Cost of Capital, Corporation Finance and the Theory of Investment" (1958), had developed a theoretical view point regarding the capital structure. The Modigliani-Miller (M-M) hypothesis is identical with the net operating income approach which we have already discussed. The M-M hypothesis shows that in the absence of corporate taxes, the change in the capital structure or the degree of leverage of a firm will have no impact upon the firm's cost of capital and its market value.

Assumptions of the M-M hypothesis :

The M-M hypothesis is based on the following assumptions :

(a) **The capital markets are perfect.** This assumption implies that

(i) the investors are free to buy and sell securities ;

(ii) the transaction costs involved in buying and selling the securities are absent (i.e., there remains no brokerage costs, commission etc.) ;

(iii) the investors are rational in their behaviour ;

(iv) information is perfect, i.e., each investor has the same information which is readily available to him without any cost ;

(v) securities are infinitely divisible ; and

(vi) investors can also borrow without any restriction.

(b) **Investors have same expectations regarding firm's net operating income.** Thus, in evaluating the value of a firm, the investors possess same expectations regarding the EBIT of a firm.

(c) **Business risks are similar for all firms within similar operating environment.** If the expected earning of some firms have identical risk characteristics, they can be grouped into a *homogeneous risk class.*

(d) **The risk of investors is defined in terms of the variability of the net operating income (NOI).** This risk depends on the random fluctuations of the expected NOI and the deviation of the actual value of NOI from its estimated value.

(e) **The dividend payout is hundred per cent.** The firms distribute all net earnings to the shareholders as dividends.

(f) **There remains no corporate tax.** The original formulation of the M-M hypothesis assumes the absence of corporate taxes. (However, this assumption has been removed later by M-M and will be discussed in a separate section). The propositions of the M-M theory are stated as follows :

Proposition I :

This proposition indicates that the value of a firm (V) and its overall cost of capital (K_o) are independent of its capital structure.

Here, $V = S + D = \dfrac{EBIT}{K_o}$

or, $K_o = \dfrac{EBIT}{V} = \left(\dfrac{D}{V}\right)K_d + \left(\dfrac{S}{V}\right)K_e$

where, V = Value of the firm,

S = Market value of equity capital,

D = Market value of debt capital,

K_d = Cost of debt capital,

K_e = Cost of equity capital,

K_o = Weighted average cost of capital.

Here, if both EBIT and K_o are independent of the capital structure of a firm, then V remains unaffected by the capital structure or the debt-equity mix of a firm. This relation is shown in Fig.-6. The M-M hypothesis states that two identical firms, except for their degree of leverage, cannot have different market values or different cost of capital. Any difference in their market values is restored through a process of *arbitrage*. Here, arbitrage refers to an act of buying a security at lower prices in one market, and selling it at a higher price in another market. Let us assume that the degree of leverage in firm A is higher than that of firm B. Let us assume that the market value of firm A is higher than that of firm B. As a result, the investors in firm A will sell their shares and purchase the shares of firm B. By doing so, the investors of firm B will be able to obtain same return without any increase in financial risk. This arbitrage process continues until the market price of the shares of firm A falls and that of firm B rises enough to make the total values of those two firm identical. This can be explained with the help of the following illustration :

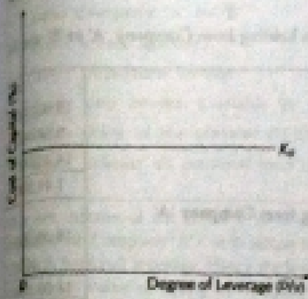


Fig.-6

Illustration 4.

The following are the data regarding two companies 'A' and 'B' belonging to the same risk class :

	Company A	Company B
Number of ordinary shares	90,000	1,50,000
Market price per share	₹ 120	₹ 100
6% Debentures	₹ 60,00,000	—
Profit before interest	₹ 18,00,000	₹ 18,00,000

All profits after debenture interest are distributed as dividends.

Explain how under M-M Approach an investor holding 10 per cent of shares in Company A will be better off in switching over his holding to Company B.

Solution :

According to Modigliani and Miller, two identical firms in all respect except for the difference in the pattern of financing cannot have different market values as the arbitrage process will drive the total values of the two firms together. In case of two identical firms having different market values due to difference in the financing pattern, switching mechanism of arbitrage process will take place and the investors will employ 'Personal leverage' as against the 'Corporate leverage'. It means, investor will borrow additional funds equal to his proportionate share in the levered firm's debt on his personal account and introduce leverage in the capital structure of the unlevered firm. This procedure will be worked out as follows in the above illustration :

1. The investor will sell 10% of share of the levered Company 'A' in the market for ₹ 10,80,000 [(10% of 90,000 shares) × ₹ 120 per share]
2. The investor will now raise a personal loan of ₹ 6,00,000 at 6% (10% of ₹ 60,00,000) to substitute personal leverage (or home-made leverage) for corporate leverage as Company 'B' does not have any debentures in its capital structure.
3. Total amount available in the hands of the investor is ₹ 16,80,000 (₹ 10,80,000 + ₹ 6,00,000).
4. The investor will by 10% of shares in Company 'B' for ₹ 15,00,000 (15,000 shares @ ₹ 100 each) from the available fund of ₹ 16,80,000 and therefore, having a surplus fund of ₹ 1,80,000 (i.e., ₹ 16,80,000 – ₹ 15,00,000) in his hand.
5. The income position of the investor by switching his holding from Company 'A' to 'B' can be shown as follows :

Existing Income in Company 'A'	₹
Profit before interest	18,00,000
Less : Debenture Interest (6% of ₹ 60,00,000)	3,60,000
Profit after interest available for dividend	14,40,000
10% share of the investor (10% of ₹ 14,40,000)	1,44,000
Income in Company 'B' after switching his holding from Company 'A'	
Profit before interest	18,00,000
Less : Debenture Interest	NIL
Profit after interest available for dividend	18,00,000
10% share of the investor (10% of ₹ 18,00,000)	1,80,000
Less : Interest on personal loan (6% of ₹ 6,00,000)	36,000
	1,44,000

Thus, by switching his holding from Company 'A' to 'B', the income in the hands of the investor in both the firms are equal (i.e., ₹ 1,44,000) but there is an unutilised amount of fund of ₹ 1,80,000. This unutilised amount can be invested in any profitable opportunity and therefore, the total income of the investor will be increased. Here risk is the same as before as it is assumed that the personal leverage (i.e. home made leverage) is a perfect substitute of the corporate leverage. This arbitrage process will continue till it is possible to reduce the investment outlays and get the same return. Beyond this point, shifting from Company 'A' to 'B' or arbitrage will not be identical. This point is known as the equilibrium point. At this point, the total value of the firm should be identical and the overall cost of capital (K_o) must be the same. If the amount of investment exceeds the equilibrium point, total income of the investor will be decreased by the arbitrage process. Therefore, the investor will be better off by selling his holding in the levered Company 'A' and buying the shares of the unlevered Company 'B' resulting the same income and having capital funds of ₹ 1,80,000 with him, which he can invest elsewhere.

Alternatively, the investor can buy 16,800 shares @ ₹ 100 per share in Company 'B' from the available fund of ₹ 16,80,000 (i.e., ₹ 10,80,000 + ₹ 6,00,000) and hence the investor will have 11.2% of shares in company 'B' $\left(\frac{16,800}{1,50,000} \times 100 = 11.2\%\right)$

The investor will gain by shifting his holding from Company 'A' to 'B' as follows :

Existing Income in Company 'A'	₹
Profit before interest	18,00,000
Less : Debenture Interest (6% of ₹ 60,00,000)	3,60,000
Profit after interest available for dividend	14,40,000
10% share of the investor (10% of ₹ 14,40,000)	1,44,000
Income in Company 'B' after shifting his holding from Company 'A' to 'B'	
Profit before interest	18,00,000
Less : Debenture Interest	NIL
Profit after interest available for dividend	18,00,000
11.2% share of the investor (11.2% of ₹ 18,00,000)	2,01,600
Less : Interest on personal loan (6% of 6,00,000)	36,000
	1,65,600

The net income of the investor in Company 'B' of ₹ 1,65,600 is higher than a net income of ₹ 1,44,000 in Company 'A' due to selling the shares of Company 'A' and buy the shares in Company 'B' with personal leverage. Hence the leverage ratio is the same in both the cases. With this action, the market value of equity of Company 'A' tends to decline and the market value of equity of Company 'B' tends to rise. This arbitrage process continues until the net market values of both the firms become identical. As a result of this the cost of capital for both the firms is the same.

Proposition II :

In this proposition, the M-M theory defines the cost of equity. It states that for any firm in a given risk class, the cost of equity (K_e) is equal to the constant average cost of capital (K_o) plus a premium for the financial risk. The financial risk, in turn, is equal to the debt-equity ratio times the spread between the constant overall cost of capital and the cost of debt capital [i.e., $(K_o - K_d)D/S$].

$$\therefore K_e = K_o + (K_o - K_d)\left(\frac{D}{S}\right)$$

So, in this case, the K_e is a linear function of the leverage (D/S). Though higher leverage may lead to increased earnings per share, it also results in increased K_e. Thus the benefits resulting from the use of cheaper debt capital are just offset by higher cost of equity. So, the market value of the firm would remain unaffected.

Illustration 5.

PQR Ltd. has raised equity capital of ₹ 20,00,000 and 8% Debt of ₹ 10,00,000. It belongs to a risk class having overall cost of capital, K_o of 15%. If however, the company raises additional debt of ₹ 10,00,000 to make debt-equity ratio 1 : 1, calculate the cost of equity capital (K_e) for the firm —

(i) Before raising additional debt
(ii) After raising additional debt.

Solution :

(i) **Before raising additional debt** (Debt-Equity ratio is 1 : 2)

$$K_e = K_o + (K_o - K_d)\left(\frac{D}{S}\right)$$

$$= 0{\cdot}15 + (0{\cdot}15 - 0{\cdot}08) \times \left(\frac{₹\ 10,00,000}{₹\ 20,00,000}\right)$$

$$= 0{\cdot}15 + 0{\cdot}035$$

$$= 0{\cdot}185 \text{ or } 18{\cdot}5\%$$

(ii) **After raising additional debt** (Debt-Equity ratio is 1 : 1)

$$K_e = K_o + (K_o - K_d)\left(\frac{D}{S}\right)$$

$$= 0{\cdot}15 + (0{\cdot}15 - 0{\cdot}08)\left(\frac{₹\ 20,00,000}{₹\ 20,00,000}\right)$$

$$= 0{\cdot}15 + 0{\cdot}07$$

$$= 0{\cdot}22 \text{ or } 22\%$$

Therefore, the overall cost of capital, K_o, remain same, but with the increase in financial leverage i.e. debt-equity ratio, the risk premium of equity shareholders has increased from 3·5% to 7%.

The overall cost of capital, K_o, can also be verified as follows ;

When debt-equity ratio is 1 : 2

$$K_o = \left(\frac{D}{D+S}\right)K_d + \left(\frac{S}{D+S}\right)K_e$$

$$= \left[\frac{₹\ 10,00,000}{₹\ 10,00,000 + ₹\ 20,00,000}\right] \times 0{\cdot}08 + \left[\frac{₹\ 20,00,000}{(₹\ 10,00,000 + ₹\ 20,00,000)}\right] \times 0{\cdot}185$$

$$= 0{\cdot}02667 + 0{\cdot}12333$$

$$= 0{\cdot}15 \text{ or } 15\%$$

When debt-equity ratio is 1 : 1

$$K_o = \left(\frac{D}{D+S}\right)K_d + \left(\frac{S}{D+S}\right)K_e$$

$$= \left[\frac{₹\ 20,00,000}{(₹\ 20,00,000 + ₹\ 20,00,000)}\right] \times 0{\cdot}08 + \left[\frac{₹\ 20,00,000}{(₹\ 20,00,000 + ₹\ 20,00,000)}\right] \times 0{\cdot}22$$

$$= 0{\cdot}04 + 0{\cdot}11$$

$$= 0{\cdot}15 \text{ or } 15\%$$

Thus, the crucial part of the M-M hypothesis is that K_o remains unchanged with higher degree of leverage of any firm.

However, this conclusion is valid only if K_d remains independent of any degree of leverage. In reality, however, K_d is supposed to rise beyond a certain level of leverage. The M-M hypothesis

suggests that at this stage (i.e., when K_d shows a rising trend), K_e may rise at a decreasing rate and may even show a falling trend (Fig.-7). It is argued that with increasing leverage, K_d increases no doubt but the debt-holders may also own some of the assets of the firm and bear some of the business risks of the firm. As a result, the risks of the shareholders are transferred to debt-holders and K_e declines.

So, the overall cost of capital (K_o) would remain unchanged (as shown by the horizontal K_o line in Fig.-7).

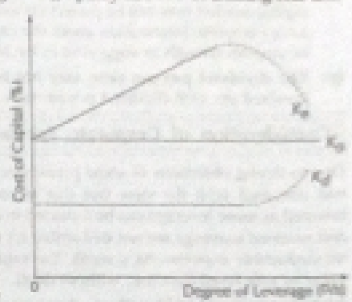


Fig.-7

Interpretation of the M-M hypothesis : When propositions I & II of this hypothesis are blended, we get the flavour of this hypothesis. It shows that although debt capital is less expensive than the equity capital, inclusion of more debt in the capital structure of a firm would not increase the value of the firm. This is because the benefits of the cheaper debt capital are just offset by the increase in the cost of equity capital.

Criticisms of the M-M hypothesis :

The M-M hypothesis has been criticised on the following grounds :

(a) **Assumption of zero transaction costs is unrealistic :** This theory assumes that the process of buying and selling shares by the investors involves no transaction costs. But this is an unrealistic assumption. For instance, the shareholders have to pay brokerage fees when they want to sell shares. Such transaction costs, whatever small amount it may be, affect the efficiency of the arbitrage process.

(b) **Flotation costs cannot also be zero :** Another unrealistic assumption of the M-M theory is the absence of flotation costs. But in reality, a firm has to incur some flotation costs (in the form of underwriting fees, commissions paid to the brokers etc.) whenever it wants to raise funds through floating its shares in the market.

(c) **Corporate taxes cannot be assumed to be nil :** The M-M hypothesis has also come under severe criticism on account of its presumption that corporate taxes are nil. But in reality interest payments on debt capital are deductible under the provisions of corporate tax laws. So, the levered firms are benefited from such provisions. (discussed later on).

(d) **Personal and corporate leverages cannot be perfect substitutes :** The arbitrage process indicated in the M-M hypothesis is based on the assumption that the personal leverage and corporate leverage are perfect substitutes. But in practice, this is a rare possibility because the individual investors and the corporate houses borrow or lend at different interest rates. It implies that an individual cannot borrow or lend funds at the same rate at which a firm can do similar operations. Further, the leverage capacity of a firm will also be higher than that of an individual investor. The risk exposure to an individual borrower is also higher compared to the same of a firm.

(e) **Institutional restrictions may also hinder the arbitrage process :** The M-M hypothesis indicates that an investor can easily switch over from an unlevered to a levered firm and vice versa. But this may not be true for institutional investors (such as LIC, GIC, UTI etc. in India) because of the existence of some institutional restrictions.

(f) Incomplete information regarding the market may also restrict the arbitrage process: The capital market may not be perfect (as assumed in the M-M hypothesis). The investors may not have complete information about the capital market. As a result, the arbitrage process cannot be smooth enough as suggested in the M-M theory.

(g) The dividend pay-out ratio may not be hundred per cent: Though this theory assumes a hundred per cent dividend payout ratio, but this may not always be true in reality.

Consideration of Corporate Tax factor in the M-M Hypothesis

Due to strong objections of some prominent financial theorists, M-M modified their earlier stand and admitted with the view that due to corporate tax factor, the overall cost of capital can be lowered as more leverage can be inducted in capital structure of the firm. This is because dividends and retained earnings are not deductible for tax purposes. On the other hand, interest on debt is a tax-deductible expense. As a result, the value of levered firm (i.e., with debt) is higher than the value of unlevered firm (i.e., without debt).

According to the M-M Hypothesis, the value of an unlevered firm (V_U) may be computed as follows:

$$V_U = \frac{EBIT(1-t)}{K_o}$$

where, V_U = Value of the unlevered firm U

$EBIT$ = Earnings before Interest and Taxes

t = Corporate tax rate

K_o = Overall cost of capital

The value of a levered firm (V_L) can be computed as follows:

$$V_L = V_U + tD$$

Where, V_L = Value of the levered firm L

V_U = Value of the unlevered firm U

t = Corporate tax rate

D = Amount of debt in levered firm L

Illustration 6.

There are two firms P Ltd. and Q Ltd. which are exactly identical except that Q Ltd. has debts in its capital structure. P Ltd. is an unlevered firm having total assets of ₹ 10,00,000, all represented by share capital of ₹ 10,00,000 and equity capitalisation rate K_e of 10% (which is also overall cost of capital, K_o for the unlevered firm). It has an EBIT of ₹ 2,00,000 subject to corporate tax @ 35%. Q Ltd. also having total assets of ₹ 10,00,000 and alike in all respects to P Ltd. except that Q Ltd. has 5% Debt of ₹ 4,00,000.

Using M-M Model with corporate taxes:

(a) Determine the total Market Value of both the firms.

(b) Determine the Cost of Equity (K_e) for both the firms.

(c) Determine the Overall Cost of Capital (K_o) for the firms.

(d) Make suitable comment on the above computations.

Solution:

(a) Determination of total Market Value (V)

For unlevered firm P Ltd. (V_U)

$$V_U = \frac{EBIT(1-t)}{K_o(=K_e)}$$

$$= \frac{₹\ 2,00,000\ (1-0.35)}{0.10}$$

$$= ₹\ 13,00,000$$

For levered firm Q Ltd. (V_L)

$$V_L = V_U + tD$$

$$= ₹\ 13,00,000 + (0.35 \times ₹\ 4,00,000)$$

$$= ₹\ 14,40,000$$

In this case, the market value of equity(s) is ₹ 10,40,000 (₹ 14,40,000 − ₹ 4,00,000)
[∵ $V = S + D$ or, $S = V - D$]

(b) Determination of Cost of Equity (K_e)

For unlevered firm P Ltd.

= 10%

For levered firm Q Ltd.

	₹
EBIT	2,00,000
Less : Interest (5% of ₹ 4,00,000)	20,000
EBT	1,80,000
Less : Tax @ 35%	63,000
EAT/Earnings available to equity share holders/Equity Earning (E_E)	1,17,000

Now, We know that, Market Value of Equity (S) $= \frac{E_E}{K_e}$

$$\text{or, } K_e = \frac{E_E}{S}$$

$$= \frac{₹\ 1,17,000}{₹\ 10,40,000}$$

$$= 0.1125 \text{ or } 11.25\%$$

(c) Determination of Overall Cost of Capital (K_o)

For Unlevered firm P Ltd.

Here, $K_e = K_o = 10\%$ (∵ unlevered)

For levered firm Q Ltd.

$$K_o = \left(\frac{D}{V}\right)K_d(1-t) + \left(\frac{S}{V}\right)K_e$$

$$= \left[\frac{D}{D+S}\right]K_d(1-t) + \left[\frac{S}{D+S}\right]K_e$$

$$= \left[\frac{₹4,00,000}{₹4,00,000 + ₹10,40,000}\right]0.05(1-0.35) + \left[\frac{₹10,40,000}{₹4,00,000 + ₹10,40,000}\right] \times 0.1125$$

$$= [0.2778 \times 0.0325] + [0.7222 \times 0.1125]$$

$$= 0.0090285 + 0.0812475$$

$$= 0.090276 \text{ or, } 9.03\%$$

(d) **The computation of different values for P Ltd. and Q Ltd. can be shown in a summarised form as follows :**

Firm	EBIT (₹)	Corporate Tax Rate (t)	Market Value (V) (₹)	Cost of Equity (K_e)	Overall Cost of Capital (K_o)
P Ltd. (Unlevered)	2,00,000	35%	13,00,000	10%	10%
Q Ltd. (Levered)	2,00,000	35%	14,40,000	11.25%	9.03%

Comment :

It is evident that because of corporate income taxes, the levered firm (i.e., Q Ltd) can lower its cost of capital or increase its market value by continuously increasing leverage in its capitalisation. MM hypothesis suggests that in order to achieve optimal capital structure the firm should strive for the maximum amount of leverage when interest tax-shield is taken into consideration.

LIST OF FORMULAE

	Particulars	Formula
1.	**Net Income (NI) Approach :**	
	(a) Market Value of the Firm	$V = S + D$
	(b) Market Value of the Equity	$S = \frac{E_a}{K_e}$
	(c) Market Value of the Debt	$D = \frac{I}{K_d}$
	(d) Overall Cost of Capital	$K_o = \frac{EBIT}{V}$ Alternatively, $K_o = W_dK_d + W_eK_e = \left(\frac{D}{V}\right)K_d + \left(\frac{S}{V}\right)K_e$
2.	**Net Operating Income (NOI) Approach :**	
	(a) Market Value of the Firm	$V = \frac{EBIT}{K_o}$
	(b) Market Value of the Debt	$D = \frac{I}{K_d}$
	(c) Market Value of the Equity	$S = V - D$
	(d) Cost of Equity Capital	$K_e = \frac{EBIT - I}{S}$ Alternatively, $K_e = K_o + (K_o - K_d)\left(\frac{D}{S}\right)$
3.	**Modigliani-Miller Hypothesis :**	
	(a) Value of the Unlevered Firm	$V_U = \frac{EBIT(1-t)}{K_o}$
	(b) Value of the Levered Firm	$V_L = V_U + tD$
	(c) Overall Cost of Capital	
	(i) For Unlevered Firm	$K_o = K_e$
	(ii) For Levered Firm	$K_o = \left(\frac{D}{V}\right)K_d(1-t) + \left(\frac{S}{V}\right)K_e = \left[\frac{D}{D+S}\right]K_d(1-t) + \left(\frac{S}{D+S}\right)K_e$

Summary

Capital structure of a company generally implies different components of capital and their proportions. In fact, there should be a prudent decision for fixing up a proper mix of debt and equity capital in financing a firm's assets.

Determination of the total requirement of capital of a firm is known as capitalisation. The term "Capitalisation" is relevant only in case of joint stock companies. Capitalisation of a company is the total amount of capital procured from long-term sources and capital structure is the respective proportions of various sources of capital collected from long-term sources. There are two popular theories of capitalisation — (i) Cost theory and (ii) Earnings theory. Both these theories will help us to determine the required or proper capitalisation of a business. If actual capitalisation is more than the required capitalisation, the firm is said to be over-capitalised. On the other hand, if the actual capitalisation falls short of the required capitalisation, the firm is said to be under-capitalised.

The capital structure is important in financial management due to the following reasons:

(a) It helps the management to maximise its return on equity capital ;
(b) It also helps the firm to minimise the cost of capital ;
(c) The business risks can be minimised ;
(d) The liquidity of the firm can be protected ;
(e) The value of the firm can be increased ;
(f) The financing of the long-term development plans of a firm becomes possible ;
(g) The full utilisation of the available capital becomes possible ;
(h) The dilution of control of affairs can be prevented.

The factors which determine the capital structure of a company are as follows :

(a) Ownership rights ;
(b) Repayment requirements ;
(c) Claim on assets ;
(d) Claim on profits ;
(e) Characteristics of a company ;
(f) Stability of earnings ;
(g) Degree of financial risks ;
(h) Cost of capital ;
(i) Surrendering operational control ;
(j) Attitude of the management ;
(k) Trading on Equity ;
(l) Age of a Company ;
(m) General level of business activity ;
(n) Nature of the industry within which the firm operates etc.

The guiding principles of capital structure decision are :

(1) The cost principle ;
(2) The risk principle ;
(3) The control principle ;
(4) The flexibility principle ; and
(5) The timing principle.

It is believed that the optimum capital structure is attained when the market value per equity share becomes maximum. Thus, the objective of any business firm should be to choose such a debt-equity mix in its capital structure that maximises the value of the firm.

There are four major theories which explain the relationship between capital structure, cost of capital and the value of a firm :

(a) Net Income (NI) Approach,
(b) Net Operating Income (NOI) Approach,
(c) Modigliani-Miller (M-M) Approach, and
(d) Traditional Approach (TA)

According to the **NI approach**, the capital structure decisions have an important bearing upon the valuation of the firm. Here, the value (V) of a firm can be ascertained as follows :

$V = S + D$,

where, S = Market value of the equity, and

D = Market value of the debt.

The overall cost of capital (K_o) can be estimated as :

$K_o = \left(\frac{D}{V}\right)K_d + \left(\frac{S}{V}\right)K_e$, where K_d = the cost of debt capital,

K_e = the cost of equity capital.

$$\therefore K_o = \left[\frac{D}{D+S}\right]K_d + \left(\frac{S}{D+S}\right)K_e$$

$$= \frac{DK_d + SK_e}{D+S} = \frac{I+E_e}{V} = \frac{EBIT}{V}$$

$\therefore V = \frac{EBIT}{K_o}$, where, $EBIT$ = Earnings Before Interest and Tax payments,

I = Total interest payments,

E_e = Equity earnings or Earnings available to the equity shareholders.

$SK_e = E_e$ and $DK_d = I$.

Thus, according to the NI approach 'V' is maximum when K_o is minimum.

However, according to the **NOI approach**, the value of the firm is not at all affected by the changes in its capital structure. So, this theory suggests that the capital structure decisions in any firm are irrelevant and hence, there remains no such thing as optimum capital structure. If D/S is treated as the degree of financial leverage, then this theory shows that K_e rises with an increase in D/S since the values of K_o and K_d are assumed to remain constant, i.e.,

$K_e = K_o + (K_o - K_d)$ (D/S).

The **Traditional Approach (TA)** towards the capital structure theory suggests that through a judicious use of both debt and equity capital, the cost of capital of a firm can be minimised and consequently the value of the firm can be maximised.

The first variant of the TA indicates that at the initial stage an increase in the use of debt capital leads to a fall in the cost of capital. However, after a certain stage, the cost of capital tends to rise with an increase in the leverage of the firm.

The second variant of the TA shows that at the first stage, the cost of equity capital remains constant or rises slightly with an increase in debt capital. At this stage, the cost of debt capital also remains almost constant. So, overall cost of capital will fall (or the value of the firm increases) with an increase in the leverage (D/V) of the firm.

$$\therefore V = \frac{EBIT}{K_e} + \frac{D(K_e - K_d)}{K_e}$$

So long as K_e and K_d remain unchanged, V will rise at a constant rate of $\frac{(K_e - K_d)}{K_e}$ with an increase in debt capital (D).

It is also indicated that

$$K_o = K_e - \frac{D}{V}(K_e - K_d)$$

If values of K_e and K_d remain constant and if $K_e > K_d$, then the average cost of capital (K_o) will decline with an increase in leverage (D/V).

At the **second stage**, the cost of equity capital will tend to rise and it would just offset the benefits of using cheaper debt capital. So, the average cost of capital remains unchanged within a range of leverage of the firm.

At the **third stage**, when the degree of leverage still rises the cost of equity capital will increase at a higher pace because of higher risk perceptions among the stakeholders of the firm. So, the average cost of capital will rise.

The **M-M approach**, based on certain assumptions, shows that the change in the capital structure of a firm will have no impact on the cost of capital (or the value of the firm).

Assignment

Objective Type Questions

1. State whether each of the following statement is 'True' or 'False' :
   (i) Capitalisation, capital structure and financial structure are synonymous terms.
   (ii) The optimum capital structure is obtained when the market value per equity share is the maximum.
   (iii) Increased use of debt increases the financial risk of the equity shareholders.
   (iv) Net Income (NI) Approach and Net Operating Income (NOI) Approach are synonymous terms.
   (v) M-M Approach is similar to NOI Approach.
   (vi) According to M-M Approach, the total value of the firm remain constant.
   (vii) The arbitrage mechanism is the behavioural foundation for the M-M Hypothesis.
   (viii) Traditional view of capital structure theory is a compromise between NI and NOI Approach.
   [Ans. (i) False ; (ii) True ; (iii) True ; (iv) False ; (v) False ; (vi) True ; (vii) True ; (viii) True.]

Short Answer Type Questions

1. Explain the term 'Capital Structure'. (See Section 7.2)
2. Differentiate "Capitalisation" and "Capital Structure". (See Section 7.3)
3. Differentiate "Capital Structure" and "Financial Structure". (See Section 7.4)
4. What is optimum capital structure ? (See Section 7.10)
5. Write a note on over capitalisation and under capitalisation. (See Subsection 7.3.2 & 7.3.3)
6. "Neither over-capitalisation nor under-capitalisation is desirable". Elucidate the statement. (See Subsection 7.3.2 & 7.3.3)
7. Write a note on "Arbitrage Process". (See Subsection 7.11.4)
8. What issues are involved in Capital Structure theories ? (See Sections 7.7 & 7.8)

Essay Type Questions

1. What do you understand by 'Capital Structure'? What factors would you consider in planing the capital structure of your company ? (See Sections 7.2. & 7.8)
2. (a) Define and explain the term 'Capital Structure'. What are the key issues involved in Capital Structure Theories ? (See Sections 7.2 & 7.7)
   (b) Explain the Net Income Approach to Capital structure theories and examine its rationality. (See Section 7.11 & Subsection 7.11.1)
3. (a) What factors would you take into consideration in planning the capital structure of a company ? (See Sections 7.7 & 7.8)

(b) Explain in brief the Net Income method and Net Operating Income method of capital structure theories. (See Subsections 7.11.1 & 7.11.2)

Define capital structure. Write a note on the importance of capital structure. (See Sections 7.2 & 7.5)

Explain high-geared, low-geared and evenly-geared capital structure with example. (See Section 7.9)

What is Optimum Capital Structure ? Discuss the features of an optimum capital structure. (See Section 7.10 & Subsection 7.10.1)

What do you understand by Net Income Approach of Capital Structure Theory as advocated by David Durand ? Explain its significance. (See Subsection 7.11.1)

Critically examine the Net Income Approach to Capital structure. (See Subsection 7.11.1)

What do you understand by Net Operating Income Approach of Capital Structure Theory ? Explain its significance. (See Subsection 7.11.2)

Critically discuss the Net Operating Income (NOI) approach of capital structure theory. (See Subsection 7.11.2)

"The Net Income Approach and the Net Operating Income Approach are two extreme Capital Structure Theories". Do you agree ? Critically examine the two. (See Subsections 7.11.1. & 7.11.2.)

Explain the Net Income (NI) Approach to the Theory of Capital Structure. How does it differ from the Net Operating Income (NOI) Approach ? (See Subsections 7.11.1 & 7.11.2)

Explain Traditional Approach to Capital Structure Theory and examine its rationality. (See Subsection 7.11.3)

Critically examine the Traditional Approach to the Capital Structure Theories. (See Subsection 7.11.3)

Critically evaluate M-M theory on capital structure. (See Subsection 7.11.4)

What is the Modigliani/Miller (M-M) view of the effect of capital gearing on the weighted average cost of capital, the value of the firm and the shareholders' wealth ? (See Subsection 7.11.4)

Explain "Arbitrage process" under Modigliani/Miller Theorem. (See Subsection 7.11.4)

"M-M have argued that do affect the value of the firm if corporate tax exist" — Examine the implications. (See Subsection 7.11.4)

"The M-M theory on the issue of optimum capital structure is based on unrealistic assumptions" — Do you agree ? Justify. (See Subsection 7.11.4)

Practical Problems

1. Max Ltd. has EBIT of ₹ 3,00,000. The company employs ₹ 10 lakhs of debt capital carrying 10% interest charge. The equity capitalisation rate applicable to the company is 15%. What is the market value of the company under Net Income (NI) approach ? Assume no corporate tax.

   [Answer : Market value of the company (V) = ₹ 23,33,333]

2. Delta Steel Ltd. is expecting an annual EBIT of ₹ 5,00,000. The company has ₹ 12,00,000 in 15% debentures. The equity capitalisation rate is 16%. Assuming that there is no tax. Calculate the value of the firm and the overall cost of capital under NI approach.

   [Answer : Value of the firm(V) = ₹ 32,00,000, Overall cost of capital (K_o) = 15·625%]

3. A company has annual net operating income of ₹ 5,00,000. It has ₹ 30 lakhs 8% Debentures. The overall capitalisation rate is 10%. You are required to calculate the value of the firm and the equity capitalisation rate according to the Net Operating Income Approach. What will be the effect on the value of the firm and the equity capitalisation rate if the debenture debt is increased to ₹ 40 lakhs ?

   [Answer : When debenture is ₹ 30,00,000 ⇒ V = ₹ 50,00,000 and K_e = 13%.
   When debenture is ₹ 40,00,000 ⇒ V = ₹ 50,00,000 and K_e = 18%.]

4. Ramco Ltd. has an EBIT of ₹ 10,00,000. Its cost of debt is 5% and the outstanding debt amount to ₹ 40,00,000. The overall capitalisation rate is 5%. The company decides to raise a sum of ₹ 10,00,000 through debt at 5% and uses the proceeds to pay off the equity shareholders.

You are required to calculate the total value of the firm and also the equity capitalisation rate under NOI Approach.

[**Answer** : V = ₹ 1,25,00,000 ; K_e = 10%]

Hint :

$EBIT$		₹ 10,00,000
K_o		8%
V	$= \frac{EBIT}{K_o}$	₹ 1,25,00,000
D	= (₹ 40,00,000 + ₹ 10,00,000)	₹ 50,00,000
S	$= V - D$	₹ 75,00,000

$$K_e = \frac{EBIT - I}{S}$$

$$= \frac{₹10,00,000 - ₹[5\% \text{ of } 50,00,00]}{₹\,75,00,000} = 10\%$$

OR

$$K_e = K_o + (K_o - K_d)\left(\frac{D}{S}\right) = 0·08 + (0·08 - 0·05)\left(\frac{₹\,50,00,000}{₹75,00,000}\right)$$

$$= 0·08 + (0·03 \times 0·67)$$
$$= 0·08 + 0·02$$
$$= 0·10 \text{ or } 10\%$$

5. Ugine Ltd. has employed 10% Debentures of ₹ 4,00,000 in its capital structure. The net operating income of the firm is ₹ 1,00,000 and has an equity capitalisation rate of 12·5%.

 (i) The company desires to raise ₹ 1,00,000 by issue of 10% Debentures and use the proceeds thereof to redeem equity shares — Calculate the total value of the firm and also the overall cost of capital.

 (ii) The company desires to redeem debentures of ₹ 1,00,000 by issuing additional equity shares of ₹ 1,00,000 — Calculate the value of the firm and the overall cost of capital.

 Ignore Tax.

[**Answer** : (i) V = ₹ 9,00,000 ; K_o = 11·1%

(ii) V = ₹ 8,60,000 ; K_o = 11·6%]

6. The management of G.D. Ltd., subscribing to the net operating income approach, believes that its cost of debt and overall cost of capital will remain at 8% and 12%, respectively. If the equity shareholders of the firm demand a return of 20%, what should be the proportions of debt and equity in the firm's capital structure ? Assume that there are no taxes.

[**Answer** : Portion of Debt (D/V) = 2/3 and portion of Equity (S/V) = 1/3]

Hint :

We know that, $K_o = W_d K_d + W_s K_e$

or, $K_o = \left(\frac{D}{V}\right)K_d + \left(\frac{S}{V}\right)K_e$

or, $0·12 = \left(\frac{D}{V}\right)0·08 + \left(\frac{S}{V}\right)0·20$

or, $0·12 = \frac{0·08D}{V} + \frac{0·20S}{V}$

or, $0·12 = \frac{0·08D + 0·20S}{V}$

or, $0·12V = 0·08D + 0·20S$

or, $0·12(S + D) = 0·08D + 0·20S$

or, $0·12S + 0·12D = 0·08D + 0·20S$

or, $0·12D - 0·08D = 0·20S - 0·12S$

or, $0·04D = 0·08S$

or, $\frac{D}{S} = \frac{0·08}{0·04} = \frac{2}{1}$

or, $D : S = 2 : 1$

Now, $\frac{D}{V} = \frac{D}{D+S} = \frac{2}{2+1} = \frac{2}{3}$

and $\frac{S}{V} = \frac{S}{D+S} = \frac{1}{2+1} = \frac{1}{3}$

7. From the following information relating to a company determine the optimum capital structure :

Debt as percentage of total capital employed	Before tax cost of debt (%)	Cost of equity (%)
0	10	15
10	10	15
20	10	16
30	11	17
40	12	18
50	14	19
60	15	21
70	18	24

Corporate tax may be taken at 50%.

[**Answer** : Minimum K_o = 12·90 at Debt-Equity Ratio 60 : 40]

8. While considering the most desirable capital structure of a company, the following estimates of the cost of debt and equity capital (after tax) have been made at various level of the debt-equity mix :

Debt as % of total Capital employed	Cost of Debt (%)	Cost of Equity (%)
0	–	15
10	7	15
20	7	16
30	8	17
40	9	18
50	10	21
60	11	24

What is composite cost of capital at different levels of debt-financing ? Can you suggest an optimal debt-equity mix in the above case ?

[**Answer** : Minimum K_o = 14·20%. Optimal debt-equity mix ⇒ option 1 : 10% Debt and 90% Equity and option 2 : 20% Debt and 80% Equity]

9. NRE Ltd. has earnings before interest and taxes (EBIT) of ₹ 4,00,000. It currently has outstanding debts of ₹ 15,00,000 at an average cost, K_d of 10%. Its cost of equity capital K_e is estimated 16%.
   (i) Determine the current value of the firm using the Traditional Valuation Approach.
   (ii) Determine the firm's overall capitalisation rate, K_o.
   (iii) The firm is considering to issue capital of ₹ 5,00,000 in order to redeem ₹ 5,00,000 debt. The cost of debt is expected to be unaffected. However, the firm's cost of equity capital is to be reduced to 14% as a result of decrease in leverage. Would you recommend the proposed action ?

   [Answer : (i) V = ₹ 30,62,500
   (ii) K_o = 13·1%
   (iii) New V = ₹ 31,42,857
   New K_o = 12·73%
   The proposal should be accepted]

10. Aloke Textiles Ltd. furnishes you the following financial data :

Expected net operating income/EBIT	₹ 6,00,000
12% Debentures	₹ 16,00,000
Equity Share Capital	₹ 24,00,000
Debt/Equity Ratio (₹ 16,00,000 : ₹ 24,00,000)	2 : 3
Equity Capitalisation Rate (K_e)	15%

   Discuss the effect of the following actions on the valuation of firm (V) and on overall cost capital (K_o) :
   (i) If the company raises further 12% debentures of ₹ 8,00,000 and $EBIT$ is expected to increase by ₹ 1,20,000, and
   (ii) With the increase in leverage, the equity capitalisation rate increase to 18%.

   [Answer : Existing V = ₹ 43,20,000 and K_o = 13·89%
   (i) V = ₹ 52,80,000 and K_o = 13·64%
   (ii) V = ₹ 48,00,000 and K_o = 15%]

11. Joy Engineering Company and Sad Engineering Company are in the same risk class and are identical in all respects except that Joy Engineering uses debt while Sad Engineering does not resort to debt financing. Joy Engineering has ₹ 10,00,000 debentures, carrying coupon rate of 10%. Both the firms earn 20% before interest and taxes (EBIT) on their total assets of ₹ 30,00,000. Assume perfect capital markets, rational investors and so on. Corporation tax rate is 50% and capitalisation rate is 15% for an all equity company. You are required to compute the value of both the companies using the NI and NOI Approach.

   [Answer :

	NI	NOI
Joy Engg.	₹ 32,00,000	₹ 25,00,000
Sad Engg.	₹ 20,00,000	₹ 20,00,000]

12. Tiny Ltd. and Toy Ltd. are identical in all respects including risk factors except for debt/equity mix. Tiny Ltd. having issued 12% Debentures of ₹ 30,00,000, while Toy Ltd. issued only equity capital. Both the companies earn 24% before interest and taxes on their total assets of ₹ 50,00,000. Assuming the corporate effective tax rate of 35% and capitalisation rate of 18% for an all-equity company. Compute the value of Tiny Ltd. and Toy Ltd. using
   (i) Net Income Approach and (ii) Net Operating Income Approach.

   [Answer :

	NI	NOI
Tiny Ltd.	₹ 60,33,333	₹ 43,33,333
Toy Ltd.	₹ 53,83,333	₹ 43,33,333]

13. X Ltd. and Y Ltd. are two companies in the same industry. They have the same business risk and identical in most respects. The annual profit of both companies is ₹ 20,00,000. The only differences between the companies are in their financial structures and their market values. Details of these are given below :

	X Ltd.	Y Ltd.
Number of Equity Shares	1,00,000	1,50,000
9% Debentures	₹ 50,00,000	—
Market price per share	₹ 130	₹ 100

   All profits after paying debenture interest are distributed as dividends.
   You are required to explain how under Modigliani and Miller Approach, an investor holding 10% of shares in X Ltd. will be better off in switching his holding to Y Ltd.

   [Answer : The investor will reduce his outlay by ₹ 3,00,000
   Alternatively,
   Net income of the investor is more by ₹ 40,000.]

14. B Ltd. and C Ltd. belong to the same risk class. Two companies are identical in all respect except that the C Ltd. has no debt in its capital structure, whereas B Ltd. employs debt in its capital structure. Relevant financial particulars of the two companies are given below :

	B Ltd.	C Ltd.
Net Operating Income	₹ 10,00,000	₹ 10,00,000
Debt Interest	₹ 4,00,000	–
Equity Capitalisation Rate	14%	12%
Debt Capitalisation Rate	8%	

   (a) You own ₹ 10,000 worth of equity of B Ltd. Show what arbitrage you would resort to.
   (b) When will this arbitrage cease according to Miller and Modigliani.

15. Companies A and B belong to the same business-risk class. Average Net Operating Income before interest of each company is ₹ 100 lakhs. Other related information is given below :

(₹ in lakhs)

	Company A	Company B
Market value of equity	400	120
Market value of debentures	–	200
	400	320

   Rate of interest on debentures is 15% p.a. and the same is considered to be certain by all the investors.
   (a) In case the total market values of the two companies are not in equilibrium, explain the process by which equilibrium is restored to according to Modigliani and Miller theory.
   (b) If the cost of equity is 27·78% for company A in equilibrium, what will it be for company B ?

   [Answer : (a) The investor would have earned ₹ 10 lakhs on investments in company A.
   The investor gains by ₹ 2·50 lakhs (i.e., ₹ 12·50 lakhs – ₹ 10·00 lakhs), by switching to company B.
   (b) Equilibrium market value of company A = ₹ 359·97 lakhs
   Equilibrium market value of company B = (₹ 359·97 – ₹ 200 lakhs) i.e., ₹ 159·97 lakhs.

$$\text{Cost of equity of company B} = \frac{\text{Profit available for equity shareholders}}{\text{Equilibrium market value}}$$

$$= \frac{₹\ 70 \text{ lakhs}}{₹\ 159{\cdot}97 \text{ lakhs}}$$

$$= 43{\cdot}758\%]$$

8

DIVIDEND POLICY

CONTENTS

8.1. Introduction

Some portion of the net earnings of a firm is paid out to its shareholders. It is called dividend. While taking decisions regarding the financial management of a company, the decisions regarding dividend payments become relevant. In fact, the firm has to choose between distributing its net profits among the shareholders and ploughing back of net profits into the business. Thus, there remains an inverse relationship between the retained earnings of the company and its cash dividends. In this chapter, we shall discuss the nature and types of dividends and the factors which influence the dividend policies of any company. Generally, a company would follow such a dividend payment policy that maximises the wealth of the owners or the value of the firm. There are, however, conflicting opinions regarding the impact of dividend payments on the value of the firm. In this context, we shall discuss some models developed by Walter, Gordon, Modigliani and Miller with regard to the dividend policy of a firm.

8.2. Meaning of Dividend

The term *dividend* refers to that portion of the profits of a business enterprise which is distributed among the shareholders of the enterprise. According to the Institute of Chartered Accountants of India, dividend is referred to as '*a distribution to shareholders out of profits or reserves available for this purpose*.' Hence, a dividend is a share of profit of the company divided among its shareholders.

8.3. Nature and Types of Dividend

The dividend decision of any business enterprise is taken in the light of the operating and financial position of the firm. The nature of dividend payment will depend upon the dividend policy followed by the firm.

(a) **Constant dividend per share :** A firm may follow a stable dividend policy. In this case, dividend equals a certain fixed amount per share regardless of fluctuations in the levels of earnings per share. Thus, fixed amount of dividend per share (DPS) is paid to the shareholders even when the firm incurs a loss. It does not create stagnation in the D/P ratio of the firm. When the firm attains new levels of earnings and expects to retain it, the DPS is also increased. Generally, during the periods of prosperity, the firm withholds the extraordinary earnings to use them for paying dividends even in the lean season. Thus, in this case, the dividend will not be allowed to fall in periods of falling net profits [or falling trend in the earning per share (EPS)] until it is felt that the firm would not be able to recover from its setback. The nature of such dividend payments can be indicated with the help of a diagram (Fig.-1).

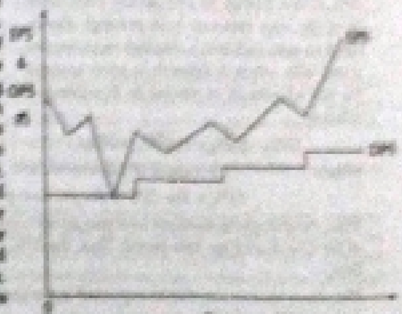


Fig.-1

(b) **Constant pay-out ratio :** Sometimes a firm may pay a constant percentage of net earnings as dividend to its shareholders. So, in this case, dividends would fluctuate in proportion to the earnings of the firm. In this situation, the D/P ratio would remain constant. The nature of the dividend payments under such a framework can also be indicated with the help of a simple diagram (Fig.-2).

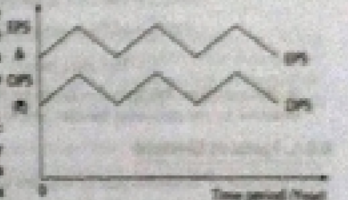


Fig.-2

(c) **A stable cash dividend plus extra dividend :** Some of the business enterprises follow a policy of paying a regular cash dividend plus an extra dividend in the years of high net profits. This extra dividend is paid over an above the regular dividend payments. However, the payments of such extra dividend are stopped whenever the firm stops earning excessive profits. In this case, the nature of the dividend payments is shown in Fig.-3.

The use of such regular plus an extra dividend pattern is generally followed by the firms which experience cyclical shifts in their earnings.

According to **J. Lintner** (in his article '*Distribution of Incomes of Corporations Among Dividends, Retained Earnings and Taxes*', *American Economic Review*, 46, May 1956), if any company follows constant pay-out ratio then

$$DPS_1 = pEPS_1$$

where DPS_1 = Dividend Per Share in the current period,

EPS_1 = Earning Per Share in the current period, and

P = D/P ratio (constant).

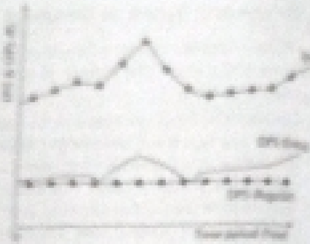


Fig-3

However, the company may not be willing to change the DPS immediately after a change in EPS. It can change its DPS slowly even when there are large increases in its earnings. Hence, the firm may maintain a standard regarding the speed with which it attempts to move towards the full adjustment of pay-out to its earnings. Therefore, Lintner has suggested the following relation :

$$DPS_1 - DPS_0 = b(pEPS_1 - DPS_0)$$

Where, b = speed of adjustment, and

DPS_0 = The DPS of the previous period.

Thus, the changes in dividend over time do not correspond exactly with the changes in earnings of the firm during the time period. Thus, the DPS_1 depends on EPS_1 as well as the DPS_0 of the firm.

The above relation can also be expressed in the form of a regression equation as follows :

$$DPS_t - DPS_{t-1} = a + b(pEPS_t - DPS_{t-1}) + e_t$$

or, $$DPS_t = a + bDPS_t^* + (1-b)DPS_{t-1} + e_t$$

where, DPS_t = The DPS at the period t,

DPS_{t-1} = The DPS at the period t − 1,

$DPS_t^* = pEPS_t$ = desired DPS,

e_t = The error term.

In this equation, the term $(1-b)$ can be interpreted as a safety factor that the management observes by not increasing the dividend to an unsustainable level.

8.3.1. Types of Dividend

Different types of dividends paid by any firm to its shareholders can be classified on the basis of :

(a) The sources from which the dividends are paid : (i) dividend paid from the retained earnings, (ii) dividend paid from the current profits ;

(b) The regularity with which such dividends are paid : (i) interim dividend, and (ii) final dividend ;

(c) The form in which they are paid :

(i) cash dividend,

(ii) bond dividend,

(iii) share dividend (bonus issue),

(iv) property dividend, and

(v) scrip dividend.

(i) **Dividend paid out of the current profits :** Generally a firm pays dividend out of its current profits. For instance, Section 123 of the Companies Act, 2013 indicates that dividend shall be declared or paid by a company out of the profits of the company for that year arrived at after providing for depreciation in accordance with the provision of schedule II.

(ii) **Dividend paid out of the retained earnings :** A business firm can also pay dividend out of its past profits or retained earnings. In this case, the undistributed profits available after providing depreciation might be eligible for such dividend payments. In India, Section 123 of the Companies Act, 2013 indicates that if in any particular year, profits are not adequate to declare a dividend, dividend can be declared out of the 'free reserves' of the company subject to some conditions. This section also indicates that a company before declaring any dividend, can transfer such percentage of its profits for that financial year to the reserves as it may consider appropriate.

(iii) **Interim dividend :** An interim dividend is one which a company pays to its shareholders before the declaration of the final dividend. If the Board of Directors of a company, after taking into account the future prospects of its profits, finds it justified to pay such interim dividend to its shareholders then it can announce such interim dividends. Thus, the order in hand, seasonal elements in business transactions etc. are taken into consideration before declaring such interim dividends.

(iv) **Final dividend :** The final dividend of a company is announced at its annual general meeting. The accounts of the company are prepared to ascertain the amount of profits earned by the company at the end of each financial year. Then the decision regarding the final dividend payment, if any, is taken by the Board of Directors after taking into consideration the provision for reserves, future prospects of the company etc.

(v) **Cash dividend :** Most of the companies pay dividends in cash. Such cash dividend results in an outflow of funds and reduces the net worth of the company. Hence, the company should have enough cash in its bank account to pay such cash dividends. Hence, the company must have adequate liquid resources at its disposal to pay such cash dividends. The shareholders, however, get the opportunity to invest that cash dividend according to their desire. So, the equity shareholders prefer to receive dividends in cash.

(vi) **Bond dividend :** If a company does not have adequate funds to pay dividends in cash, it may issue bonds (bearing a particular interest rate on the face value of the bond) to the shareholders for the amounts due to them. The objective of the scrip dividend is to postpone the immediate outflow of cash (since the bonds would mature after a stipulated period). However, the bond dividend is not popular in India.

(vii) **Share dividend (bonus issue) :** Again, if a company does not have adequate liquid resources to pay cash dividend, it can pay dividend by issuing bonus shares to its shareholders. It is called *share dividend or stock dividend*.

It leads to an increase in the number of outstanding shares of the company. The bonus shares are distributed proportionately to the shareholders. For example, if any shareholder owns 200 shares of a company which announces 10% bonus issue, then the shareholder will receive 20 additional shares. The declaration of bonus issue (or stock dividend) increases the paid-up share capital of the company and reduces its reserves and surplus. Hence, the stock dividend amounts to capitalisation of earnings of the company and distribution of its profits among the existing shareholders without affecting the cash position of the company. However, section 63(3) of the Companies Act, 2013 in India does not allow any company to issue bonus shares in lieu of dividend. The SEBI guidelines towards the issue of bonus shares also indicate the same. Thus, *in Indian context, a bonus share cannot be regarded as a dividend.*

(vi) **Property dividend :** When any company pays dividend in the form of assets other than cash, it is called property dividend. It is paid in the form of such assets which are not required by the company. However, this method of dividend payment is also not popular in India.

(vii) **Scrip dividend :** The scrip dividend refers to the dividend payment in the form of scrips or promissory notes. In this case, the company promises to pay the dividend at a future date. These scrips may be interest-bearing or non-interest-bearing. As in the case of bond dividends, the method of paying scrip dividends is followed by a company when its liquidity position does not permit it to pay cash dividends.

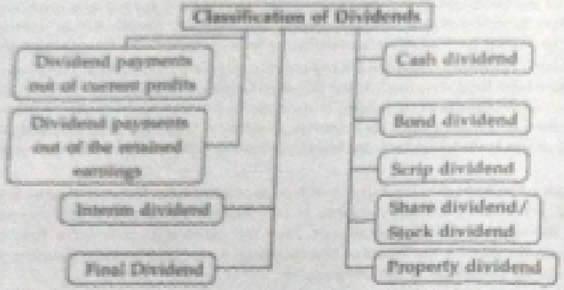


Chart-1

8.4. Dividend Policy

The decisions regarding the dividend payments to the shareholders of a company is a crucial area of financial management. Hence, a firm should frame an appropriate dividend policy that would maximise the wealth of its shareholders. According to **Weston and Brigham**, *'Dividend policy determines the division of earnings between payments to shareholders and retained earnings'*. Thus, the term *'dividend policy'* refers to the policy concerned with the distribution of a portion of profits among the shareholders of the business firm. The power to declare dividends vests completely in the Board of Directors of a company.

8.5. Objectives of Dividend Policy

The principal objectives of the dividend policy of a company are as follows :

(a) **Maximisation of owner's wealth :** The dividend policy of a company aims at the maximisation of the wealth of the shareholders of the company. It is formulated not only to raise the share price during the short-run but also to maximise the owner's wealth in the long-run. Sometimes the shareholders may prefer immediate dividends to future dividends and capital gains. In that case, if the dividend policy that emphasises on future dividends and capital gains to the shareholders, the share prices of that company may indicate a falling trend (since low levels of current dividend may reduce the market demand for the shares). Hence, it is the responsibility of the management to make the owners aware of the objectives and implications of its dividend policy so that the market reactions become favourable.

(b) **Provision of sufficient funds for the future growth of the company :** The future growth of a company depends to a great extent on the availability of long-term finance from the retained earnings of the company. There remains an inverse relationship between the present cash dividends and the retained earnings of the company. Thus, larger cash dividends would mean lesser amount of retained earnings which could be ploughed back for the future growth of the company.

Thus, the management has to evolve an ideal ratio between dividends and retained earnings so that the twin objectives of maintaining short-term interest of the shareholders and the long-term growth of the company are achieved. In fact, the dividend policy of a company has a close interconnection with its retention policy which is concerned with its retained earnings. The retentions or the retained earnings are used to finance the capital projects of the company as well as redeem its debt obligations.

8.6. Nature of Dividend Policy

We have already pointed out some of the important objectives of the dividend policy of any business enterprise. Let us now explain the nature of the dividend policy.

(a) **Tied up with the retention policy of the firm :** We have already shown that the dividend holding of a firm is closely tied up with its retention policy. Higher payments of dividends would mean lesser amount of retained earnings. Since the retained earnings can be used for the future expansion of the firm, so the dividend policy has an intimate relationship with the retention policy.

(b) **Influence on the financial decisions of the firm :** The dividend policy of a firm has also an important bearing upon the financial decisions of a business enterprise. The firm has to depend on external sources of funds if its cash balance becomes insufficient to satisfy its needs after the payments of cash dividends.

However, the cost of funds raised from external sources is relatively higher than that of its retained earnings. Hence, if the firm does not have any profitable investment opportunities, it may take the decision of paying dividends to the shareholders.

(c) **Influence on the share prices :** The dividend policy of a firm has also far reaching impact upon the growth rate of the firm, the share prices of the firm and the wealth of its existing shareholders. We have already noted that the shareholders assign higher weightage to the current dividend payments compared to the future dividends and capital gains. Thus, higher payments of current dividends would attract the investors to purchase the shares of their company and it may lead to an enhancement in the market price of its shares. It would also mean an increase in the wealth of the shareholders.

(d) **Aiming at an optimum dividend policy :** Considering the dividend policy as an active decision variable, the financial manager aims at framing an optimum dividend policy. A dividend policy is said to be at its optimum when, at any particular dividend pay-out ratio, the market price per share attains its maximum value.

Here, the Dividend Pay-out (D/P) Ratio may be described as follows :

Dividend Pay-out (D/P) Ratio :

The first and the most important dimension of a dividend policy is the decision regarding the D/P ratio. It is also known as *Pay-out Ratio*. It measures the relationship between the earnings belonging to the ordinary shareholders and the dividend paid to them. In other words, the D/P ratio shows what percentage share of the net profits after taxes and preference dividend is paid out as dividend to the equity shareholders. Thus it may be calculated as follows :

$$\text{D/P Ratio} = \frac{\text{Dividend paid to equity shareholders}}{\text{Profit available to equity shareholders}} \times 100$$

i.e., PAT less preference dividend

For example, if the net profit after taxes and preference dividends are ₹ 5,00,000 and the dividend paid to the equity shareholders amount to ₹ 2,00,000, the D/P Ratio would be,

$= \frac{₹\ 2,00,000}{₹\ 5,00,000} \times 100 = 40\%$

The profits which are not distributed (i.e., ₹ 3,00,000) are retained and available for financing the investment. This is known as **Retention Ratio**, and in this case, it is 60%. It implies that 60% of the profits of the firm are retained and 40% distributed as dividends.

Alternatively, it can be found out by dividing the Dividend Per Share (*DPS*) by the Earnings Per Share (EPS).

$$\text{D/P Ratio} = \frac{\text{Dividend Per Share (DPS)}}{\text{Earnings Per Share (EPS)}} \times 100$$

For example, if the firm has an *EPS* and *DPS* of ₹ 5 and ₹ 2 respectively, then the D/P Ratio is,

$\frac{₹\ 2}{₹\ 5} \times 100 = 40\%$

The D/P Ratio of a firm should be determined with reference to two basic **objectives** :

(i) Maximising the wealth of the shareholders, and

(ii) Providing sufficient funds to finance growth/expansion.

A high rate of dividend pay-out can be interpreted in two ways —

(a) The firm is positive about future earnings, and therefore a more liberal approach is taken towards dividends and

(b) The firm does not have any meaningful re-investment opportunity.

A low pay-out reflects conservative distribution policy.

In Fig.-4, the curve H shows that with an increase in D/P ratio, the market price per share increases at first but it diminishes after a certain stage. This is due to the fact that higher D/P ratio reduces the retained earnings of the firm and arrests its further growth (as we have already explained). However, if a firm requires outside funds to meet its dividend payments then it leads to higher cost of capital. In this case, curve L shows the falling trend in the market price per share with an increase in D/P ratio. However, curve M shows a combined impact of those two factors (as indicated by curves H and L) on the market price per share. Thus, it is observed that the market price per share is maximum when the D/P ratio = X_0. Hence, the dividend policy of a firm should aim at maintaining that value of D/P ratio at which the market price per share becomes maximum. However, there are different views regarding the impact of dividend policy (or the D/P ratio) on the market price per share or the value of a firm. According to the school of thought headed by Walter, Gordon and others, the D/P ratio has an impact on the value of a firm. However, another group of experts, led by Modigliani and Miller, are of the opinion that the D/P ratio has no impact upon the value of a firm (i.e., the D/P ratio is irrelevant in determining the value of a firm). We shall discuss these theories in our subsequent sections.

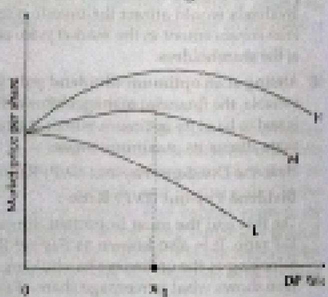


Fig.-4

Formulating a Dividend Policy

When the financial managers alongwith the Board of Directors of any business firm want to design a dividend policy then the following questions become relevant :

What are the preference of the shareholders of the firm ? Do they prefer dividend income or capital gains ?

What are the financial requirements or needs of the business firm ?

Should the firm follow a stable dividend policy ?

How much should be paid out by the firm as dividends ? What should be the form of dividends ?

What are the legal and other constraints on paying dividends ?

We have to take into consideration a number of factors for giving satisfactory answers to those questions. We have already explained that the Board of Directors of any company, while designing a dividend policy, should aim at bringing a balance between the desires of shareholders and the needs of the company.

The factors that should be considered in formulating a dividend policy of any business firm are explained below :

Desires of shareholders : Since the shareholders are the actual owners of a company, so their desires or expectations are given due importance while framing the dividend policy of the company. The shareholders generally expect a return on their investment in the following forms :

(a) *Current dividends* : Generally the equity shareholders expect a regular return on their investments in the form of current dividends.

(b) *Capital gains* : The shareholders also expect an increase in the market value of the equity shares held by them. Thus, the possibility of selling the equity shares at higher prices in future results in capital gains to the shareholders.

In most cases, the shareholders put more emphasis to current dividends because of the following reasons :

(a) *The need for current income* : The receipts from current dividends often supplement the current incomes of the common shareholders. So they prefer current dividend to capital gains to pay for their cost of living.

(b) *Minimising the uncertainty* : The incidence of uncertainty is relatively higher in case of capital gains compared to current dividends.

(c) *An indication of financial strength of the firm* : The current dividends in the form of cash dividends of a firm are taken to be an indicator of its financial strength.

In case of a closely-held company, the expectations of the shareholders are usually known to the management. Thus, it can adopt a dividend policy that serves the interests of all shareholders. However, in case of a widely-held company, there remains a large number of shareholders having diverse desires and expectations. So, it is quite difficult to adopt a dividend policy that satisfies all those shareholders of such a company.

(2) **Dividend Clientele** : A dividend clientele refers to a group of investors who favour a particular type of dividend policy. Low and Zero tax payers seem to prefer high D/P ratios, while the investors falling in higher income tax brackets prefer low dividends and higher capital gains (the dividend payments to individuals are subject to personal income tax payments). The shareholders of a widely-held company can be grouped into such dividend clientele. The dividend policy should give proper weightage to the desires of each of these groups.

(3) **The Dividend Payout (D/P) Ratio :** We have already shown that

$$\text{D/P Ratio} = \frac{\text{Dividend Per Share (DPS)}}{\text{Earnings Per Share (EPS)}}$$

Thus, the dividend pay-out shows the extent of net profits of a firm distributed to its shareholders as dividend. Higher D/P ratio signifies higher amount of cash outflow and availability of lesser amount of fund for the future growth of the firm. An optimum dividend policy should evolve such a D/P ratio that maximises the market prices of the shares of the company. This policy should aim at maintaining a balance between the current dividends and the future growth potential of the company.

(4) **Financial requirements of the company :** The financial needs of the company are also taken into account while designing a dividend policy. If the company has highly profitable investment opportunities, it can convince its shareholders of the need for limiting the D/P ratio. But the financial needs of the company may be in direct conflict with the desires of the shareholders. A prudent management has to give more weightage to the financial needs of the company as opposed to the desires of the shareholders. If the company has better investment opportunities, then its retained earnings can be used to maximise the shareholders' wealth in future.

(5) **Liquidity position of the firm :** A business firm requires cash in order to pay cash dividends to its shareholders. Thus, payment of cash dividends involves an outflow of cash from the business. So it affects the liquidity position of the firm. Even if a firm has sizeable earnings, these funds are generally reinvested in the firm itself or used to meet its debt obligations. Thus, a firm may have good records of profitability, still it may be cash-poor. Hence, the liquidity position of a business firm is taken into account while deciding any dividend pay-out.

(6) **Stability of dividends :** The stability of dividends refers to the consistency in the flow of dividend payments. The shareholder of a firm generally prefers a regular, stable or steady payments of dividends over time. We have already discussed that a firm can follow either (i) constant dividend per share, or (ii) constant or stable D/P ratio, or (iii) constant dividend per share plus extra dividend to maintain stability in dividend pay-out.

Since most of the shareholders generally prefer an assured return on their investment in the form of a fixed dividend with a consistent growth possibility, so out of those three policies the 'constant dividend per share policy' seems to be most appropriate. **The investors prefer a stable dividend due to the following reasons :**

(a) *Desire for current income :* We have already mentioned in our previous discussion that a group of investors (say, retired persons, widows etc.) prefer current dividends or a stable dividend policy to meet their current costs of living. These investors get positive utility for the stable dividend income. Hence, they will be even ready to pay higher prices for the shares which would give them a stable dividend.

(b) *An indicator of the profitability of a firm :* The investors also consider a stable dividend as an indicator of profitability and financial health of a firm. If the dividend payments show an erratic behaviour over time, investment becomes a risky and uncertain proposition for the shareholders. So the investors prefer a stable dividend policy.

(c) *Requirements of institutional investors :* The institutional investors in the stock market (such as the insurance companies and mutual funds in India) also give higher weightage to the stable dividend policy while purchasing the shares of any company. They often create excess demand for the securities of a firm since they invest a huge amount in purchasing securities. This leads to an increase in the market price of the shares of that firm and causes an increase in shareholder's wealth.

On the otherhand, if a firm follows a **constant D/P ratio** or a target pay-out ratio then it implies the payment of dividends according to the ability of the firm, i.e., higher EPS would mean higher DPS and vice versa. Hence, the management can reduce its financial risk by following such dividend policy. But from the view point of the investors, such a policy involves greater incidence of uncertainty or irregularity in dividend earnings. Hence, the investors do not favour the policy of constant D/P ratio.

Thus, if the earnings of a firm are subject to wide fluctuations over time, it can rather follow the policy of **'stable cash dividend plus some extra dividend'**. In this case, the firm can avoid the risk of inability of dividend payment to the shareholders by paying a fixed amount (whatever small it may be) of dividend on a regular basis. Side by side, the investors are assured that they would receive extra dividend with the prosperity of the firm.

Management's attitude towards control : If the management of a company wants that the existing shareholders should retain the control over the company, it generally avoids paying higher dividend pay-outs. Higher dividend pay-outs attract new investors. In that case, if the firm raises additional funds by issuing fresh shares, the control of the existing shareholder over the company becomes diluted. In such cases, the management may rely more upon its retained earnings to meet its fund requirements.

Magnitude and the trend of earnings of the firm : Since dividends can be paid only out of the current or previous year's profits, the earnings of a firm determine the ceiling on dividend pay-outs. The past trend of earnings of the firm is also taken into consideration while preparing its dividend policy.

The age of a firm : Sometimes the age of a firm becomes an important factor in determining its dividend policy. A newly established firm generally sets aside greater portion of its profits as retained earnings to finance its future growth programme. So, the D/P ratio remains at a low level for such younger firms. However an old and established firm having sufficient reserves and surplus, can afford higher D/P ratio.

Growth prospects of the firm : The growth prospects of a firm also influence its dividend policy. The firm should have enough provisions of funds to finance its future expansion plans. Higher costs of availing external sources of funds and the lower costs of using internal sources of funds should be taken into account in framing the dividend policy of a firm having positive growth prospects.

Legal constraints : Several legal restrictions upon the payments of dividend by any company also shape the dividend policy of the company. A company cannot pay dividend out of its paid-up capital. It can pay dividend only out of profits. Section 123(1) indicates that no dividend shall be declared or paid by a company for any financial year, except —

(a) Out of the profits of the company for that year arrived at after providing for depreciation in accordance with the provision of Schedule II or out of the previous financial year(s) profit arrived at after providing for depreciation.

(b) Out of the money provided by the Central Government or a State Government in pursuance of a guarantee given by that Government. A company may before the declaration of any dividend, can transfer such percentage of its profits to the reserves as it may consider appropriate. Owing to inadequacy or absence of profits in any financial year, a company may declare dividend out of the accumulated profits earned by it in previous years and transferred by the company to its reserves. No dividend shall be declared or paid by a company from its reserves other than free reserves.

(12) Contractual requirements : When the business firm raises its funds from external sources, the dividend payments can be constrained by the contractual requirements of those loan agreements, debenture indentures, preference share agreements etc. Lenders of the firm generally put restrictions upon dividend payments to protect their interests at the time of financial crisis faced by the firm. Such restrictions upon the payment of dividends may appear in the following forms:

(a) The firm may be prohibited from paying dividends in excess of a certain percentage of the net profits.

(b) The firm may be prohibited from paying dividends in excess of a certain percentage of the face value of the shares.

(c) The firm may have to maintain a minimum retention rate (i.e., the percentage of earnings to be retained).

(13) Tax policy of the Government : The dividend policy of a business firm is also affected by the tax policy of the Government. For instance, the Government may allow tax incentives to companies which retain larger portion of their earnings. In such cases, the management may be inclined to maintain low D/P ratio. Again, if the capital gains of the shareholders are taxed at lower rates compared to dividend earnings, and if most of the shareholders of a company remain in high tax brackets then also the company may follow a low dividend pay-out ratio. In this case, the dividend policy aims at providing income to its owners in the form capital gains.

(14) Condition of the capital market : A business firm is supposed to follow a liberal dividend policy (i.e., with higher D/P ratio) if it has an easy access to the capital market or if favourable conditions prevail in the capital market. In this situation, it becomes easier for the firm to raise the required funds from the capital market at lower costs. However, if the firm has limited access to the capital market or if the environment of the capital market is not favourable to raise additional funds, the firm would follow a conservative dividend policy (i.e., with lower D/P ratio).

(15) State of the economy : The general state of the economy of a country also influences the dividend policy of a company. For instance, if there is economic depression then a firm cannot expect substantial increase in its earnings in near future. In that case, it wants to retain larger portion of its present earnings to meet its future obligations. On the other hand, during the periods of prosperity or economic boom, the company may not be liberal in paying dividends because it gets better investment opportunities during this period. Again, during the periods of inflation, the general price level increases. As a result, funds saved on account of depreciation would not be adequate to replace the assets or to maintain the capital intact. Hence, the company wants to retain greater part of its current earnings to preserve its earning power and to keep its capital intact.

Factors or Determinants of the Dividend Policy

- Desires of the shareholders
- Dividend clientele
- The D/P ratio
- Financial requirements of the company
- Liquidity position of the firm
- Stability of dividends
- Management's attitude towards control
- Magnitude and trend of earnings of the firm
- The age of the firm
- Growth prospects of the firm
- Legal constraints
- Contractual requirements
- Tax policy of the Government
- Conditions of the capital market
- State of the economy

Chart-1

Various Dividend Policy Theories / Models

There are different theories relating to the impact of dividend decisions on the value of a firm. The financial analysts such as Myron Gordon, James Walter, John Lintner etc. are of the opinion that the dividend policy of a firm has significant impact upon the value of the firm and its position in the stock market. They have justified their views on the basis of some theoretical models. Thus, according to their views, dividend policy is relevant in maximising the net worth of the business enterprise. However, according to another school of thought led by F. Modigliani and M.H. Miller, the dividend policy of a firm has no impact upon the share prices of the company (i.e., upon the value of the firm). In this sense, the dividend policy is irrelevant. According to this view, dividend decision is essentially a financing decision, i.e., whether dividends would be paid out of profits or a substantial portion of earnings be retained will depend upon the investment opportunities before the firm. When such investment opportunities are abundant then the D/P ratio may become zero. On the other hand, when there remains no opportunity to invest the retained earnings of the firm, the D/P ratio would be 100. According to this school of thought, investors remain indifferent between dividend and capital gains. Hence, dividends are considered as a passive residual. It implies that if the firm has some retained earnings 'left over' after financing all possible investment opportunities, then that residual earnings are distributed as dividends among the shareholders.

Hence, the theories on the relationship between the dividend policy and the value of the firm can be grouped into two categories which may be shown in the following chart.

Dividend Theories

- **Relevance Theories** — Here, dividend policy of a firm has an significant impact upon the value of the firm.
  - Walter's Model
  - Gordon's Model
- **Irrelevance Theories** — Here, dividend decision is irrelevant as it does not alter the value of the firm.
  - Modigliani and Miller's Model

Chart-2

8.8.1. Relevance of Dividends : Walter's Model

Walter's model wants to establish the relevance of the dividend policy of a firm. This model shows that the dividend policy chosen by a firm always affects the value of the firm. The distribution of dividend resolves the following issues :

(a) The shareholder's desire for current dividend can be fulfilled ;

(b) Uncertainty of income can be reduced ;

(c) Stock market quickly reacts to the dividend declaration and the market price of the share is determined.

The capital markets are not perfect and hence, the shareholders are not indifferent between dividends and retained earnings. The shareholders may assign higher value of current dividends compared to the future dividends and capital gains because of uncertainty and imperfections in the capital markets. As a result, payments of dividends may significantly affect the market price of the share of a firm.

The Walter model relates the dividend pay-outs (or the retention of earnings) to the investment opportunities available to the firm. If the return on the investment of a firm (or its internal rate of return) is denoted by r and its cost of capital (or the required rate of return) is denoted by K_e, then we get the following possibilities :

(i) If $r > K_e$, *i.e.* if the firm earns higher rate of return on its investment compared to its required rate of return, the firm should retain greater portion of its earnings. Such firms are termed as *growth firms* and for such firms, the optimum dividend pay-out ratio would be zero (i.e. D/P ratio would be zero) ; In this case, the entire earnings of the firm are ploughed back to the business. This step results in the maximisation of the market value of its shares.

(ii) If $r < K_e$, it implies that the firm does not have profitable investment opportunities. These firms may be termed as *declining firms*. In this case, if the firm distributes its earnings as dividends to the shareholders then they will be better off by investing that sum elsewhere. Here, the market price of shares will be maximised through the distribution of entire earning of the firm as dividends. So, the D/P ratio will be 100%.

(iii) If $r = K_e$, the firm would remain indifferent between dividend pay-outs and retention of earnings. This type of firm is termed as *normal firms*. In this case, the market value of the share will remain invariant to changes in the D/P ratio of the firm. Thus, the value of the firm will remain unaffected by the changes in its dividend pay-outs.

The Walter's model is based on the following assumptions :

(a) The firm does not depend on external sources of fund (such as debt or new equity capital) and all investments of the firm are financed through its retained earnings.

(b) The business risks of the firm remain unchanged even when it takes additional investment projects. Thus, the values of r and K_e remain constant.

(c) For a given value of the firm, the dividend per share and the earning per share remain constant.

(d) The firm has an infinite life.

According to this model, the market price of a share is estimated to be the sum of the present value of the future stream of cash dividends and capital gains. The following formula has been evolved by **Walter** to ascertain the market price of a share :

$$P = \frac{D + \frac{r}{K_e}(E - D)}{K_e} \qquad \ldots(1)$$

where, P = Price of equity shares,

D = Initial Dividend Per Share (DPS),

E = Initial Earning Per Share (EPS),

r = Expected rate of return on firm's investment (or the internal rate of return)

K_e = Cost of equity capital (or the *rate of return expected by the shareholders or the capitalisation rate*).

This formula has been derived as follows :

As per the share valuation model,

$$P = \frac{D}{K_e - g} \qquad \ldots(2)$$

where, g = Expected growth rate of earnings.

$$= \frac{\Delta P}{P}$$

Here, $\Delta P = \frac{r(E-D)}{K_e}$ ($\because$ retained earnings are the only source of finance).

In this share valuation formula, the retained earnings are reflected as follows :

$$P = \frac{D}{K_e - rb} \qquad \ldots(3)$$

where, b = Retention rate $= \frac{(E-D)}{E}$

Therefore, rb measures the growth rate in dividends.

We know that the cost of equity capital,

$$K_e = \frac{D}{P} + g \qquad \ldots(4)$$

$$= \frac{D}{P} + \frac{\Delta P}{P} \qquad \left[\because g = \frac{\Delta P}{P}\right]$$

$$= \frac{D}{P} + \frac{\frac{r(E-D)}{K_e}}{P} \qquad \left[\because \Delta P = \frac{r(E-D)}{K_e}\right]$$

$$= \frac{D + \frac{r(E-D)}{K_e}}{P}$$

or, $$P = \frac{D + \frac{r(E-D)}{K_e}}{K_e} = \frac{D + \frac{r}{K_e}(E-D)}{K_e}$$

Illustration 1.

Calculate the prevailing market price of a share using Walter's model from the following information :

Rate of return on investment	10%
Capitalisation rate	8%
Earning per Share	₹ 5
Dividend per Share	₹ 4

Solution :

As per Walter's model, market price of a share (P) is given by,

$$P = \frac{D + \frac{r}{K_e}(E - D)}{K_e}$$

where, D = Dividend per share
= ₹ 4
r = The rate of return on investment
= 10% = 0·10
K_e = Cost of equity capital or capitalisation rate
= 8% = 0·08
and E = Earnings per Share
= ₹ 5

Now, putting the respective values in the model, we get,

$$P = \frac{₹4 + \frac{0.10}{0.08}(₹5 - ₹4)}{0.08} = ₹\ 65{\cdot}63$$

So, the prevailing market price of a share using Walter's model is ₹ 65·63.

Illustration 2.

X Ltd. earns ₹ 6 per share having a capitalisation rate of 10 per cent and has a return on investment of 20%. According to Walter's model, what should be the price of the share at 25% dividend pay-out?

Solution :

According to Walter's model, market price of a share, P is given by,

$$P = \frac{D + \frac{r}{K_e}(E - D)}{K_e}$$

where,

D = Dividend per share i.e., $EPS \times D/P$ Ratio
or, ₹ 6 × 25% = ₹ 1·50.
r = Rate of return on investment i.e., 20% or 0·20.
K_e = Capitalisation rate i.e., 10% or 0·10.

E = Earnings per share i.e., ₹ 6.

Now, putting the values in the model, we get,

$$P = \frac{₹1.50 + \frac{0.20}{0.10}(₹6 - ₹1.50)}{0.10}$$

$= ₹\ 105$

Illustration 3.

From the following information supplied to you, determine the theoretical market value of equity shares of a company as per Walter's Model :

Earnings of the company	₹ 5,00,000
Dividend paid	₹ 3,00,000
Number of shares outstanding	1,00,000
Price-earning ratio	8
Rate of return on investment	15%

Are you satisfied with the current dividend policy of the firm ? If not, what should be the optimal dividend pay-out ratio in this case ?

Solution :

As per Walter's model, the market value of an equity share (P) is given by,

$$P = \frac{D + \frac{r}{K_e}(E - D)}{K_e},$$

where,

P = Market value of an equity share ;
D = Dividend per share

$$= \frac{\text{Dividend paid}}{\text{Number of shares outstanding}}$$

$$= \frac{₹5,00,000}{1,00,000} = ₹\ 3\ ;$$

r = Rate of return on investment
= 0·15 ;
K_e = Capitalisation rate

$$= \frac{1}{\text{Price-earning Ratio (P/E Ratio)}}$$

$$= \frac{1}{8}$$

= 0·125

and E = Earnings per Share

$$= \frac{\text{Total Earnings}}{\text{Number of shares outstanding}}$$

$$= \frac{₹5,00,000}{1,00,000}$$

$$= ₹\,5.$$

Now, putting the values,

$$P = \frac{₹5 + \frac{0.15}{0.125}(₹8 - ₹5)}{0.125} = ₹\,43.20$$

So, the theoretical market value of an equity share is ₹ 43.20.

But we are not satisfied with the current dividend policy. As the firm is a growth firm, where r (0.15) $> K_e$ (0.125), the optimum dividend pay-out ratio should be zero.

This can be shown by considering following four situations.

D = zero

$$P = \frac{0 + \frac{0.15}{0.125}(₹5 - 0)}{0.125}$$

$$= ₹\,48$$

D = ₹ 1

$$P = \frac{₹1 + \frac{0.15}{0.125}(₹5 - ₹1)}{0.125}$$

$$= ₹\,46.40$$

D = ₹ 2

$$P = \frac{₹2 + \frac{0.15}{0.125}(₹5 - ₹2)}{0.125}$$

$$= ₹\,44.80$$

D = ₹ 4

$$P = \frac{₹4 + \frac{0.15}{0.125}(₹5 - ₹4)}{0.125}$$

$$= ₹\,41.60$$

Thus, it is clear from above that the market price of an equity share is maximum (*i.e.*, ₹ 48) when the dividend pay-out ratio becomes zero.

Illustration 4.

Following information relating to Jee Ltd. are given :

Profit after tax	₹ 10,00,000
Dividend pay-out ratio	₹ 50%
Number of Equity Shares	50,000
Cost of Equity	10%
Rate of Return on Investment	12%

(i) What would be the market value per share as per Walter's model?

(ii) What is the optimum dividend pay-out ratio according to Walter's Model and Market value of equity share at that pay-out ratio?

Solution :

(i) **Market value per share (P) as per Walter's Model is given by**

$$P = \frac{D + \frac{r}{K_e}(E - D)}{K_e}$$

where,

D = Dividend per share (*i.e.* 50% of ₹ 10,00,000/50,000 shares)

or, ₹ 10 per share

r = Rate of return on investment *i.e.* 12% or 0.12

K_e = Cost of equity *i.e.*, 10% or 0.10

E = Earnings per share (*i.e.*, ₹ 10,00,000/50,000 shares)

or, ₹ 20

Now, putting the values,

$$P = \frac{₹10 + \frac{0.12}{0.10}(₹20 - ₹10)}{0.10}$$

$$= ₹\,220$$

(ii) **Optimum Dividend Pay-out (D/P) Ratio.**

According to Walter's model when the return on investment (r) is more than the cost of capital (K_e) *i.e.* r (0.12) $> K_e$ (0.10), the firm is considered as a growth firm. In that case, the price per share increases as the D/P ratio decreases. Hence, the optimum dividend pay-out ratio in this case should be Zero or Nil.

Therefore, at D/P ratio of zero, the market price per share (P) will be,

$$P = \frac{0 + \frac{0.12}{0.10}(20 - 0)}{0.10}$$

$$= ₹\,240$$

Illustration 5.

You are requested to find out the approximate dividend payment ratio as to have the share price at ₹56 by using Walter's Model, based on following information available for a company.

Net Profit	₹ 50 lakhs
Outstanding 10% Preference Shares	80 lakhs
Number of Equity Shares	5 lakhs
Return on Investment	15%
Cost of Capital (after tax) (K_e)	12%

Solution :

Calculation of Dividend Pay-out (D/P) Ratio

$$\text{D/P Ratio} = \frac{\text{Dividend Per Share } (DPS)}{\text{Earnings Per Share } (EPS)}$$

Where,

$$EPS = \frac{\text{Earnings available to equity shareholders i.e. PAT – dividend preference}}{\text{Number of equity shares}}$$

$$= \frac{₹50,00,000 \text{ (assuming after tax)} - (10\% \text{ of } ₹80,00,000) \text{ or } ₹8,00,000}{5,00,000}$$

$$= \frac{₹42,00,000}{5,00,000} = ₹ 8.40$$

DPS may be computed as follows :

$$P = \frac{D + \frac{r}{K_e}(E - D)}{K_e} \quad \text{[Using Walter's Model]}$$

Where,

P = Market price i.e., ₹ 56

D = DPS

r = Return on investment i.e., 15% or 0.15

K_e = Cost of equity capital i.e., 12% or 0.12

E = EPS i.e., ₹ 8.40

Now, putting the values,

$$₹ 56 = \frac{D + \frac{0.15}{0.12}(₹ 8.40 - D)}{0.12}$$

or, $56 \times 0.12 = D + 10.50 - 1.25D$

or, $6.72 = 10.50 - 0.25 D$

or, $0.25D = 10.50 - 6.72$

$\therefore \quad D = \frac{3.78}{0.25} = ₹ 15.12$

Now, D/P Ratio $= \frac{₹ 15.12}{₹ 8.40} \times 100$

$= 180\%$

Illustration 6.

The following information is available in respect of a firm :

Capitalisation rate (K_e) = 0.10

Earnings per share (E) = ₹10

Assumed rate of return on investments (r) : (i) 15%, (ii) 10% and (iii) 5%.

Show the effect of dividend policy on the market price of shares, using Walter's model. Assume Dividend Pay-out ratio (D/P Ratio) : 0%, 25%, 50%, 75% and 100%.

Also state the optimum dividend pay-out ratio.

Solution :

According to Walter's Model, market price of a share, P is given by,

$$P = \frac{D + \frac{r}{K_e}(E - D)}{K_e}$$

Where,

D = Dividend per share,

r = Rate of return on investment,

K_e = Capitalisation rate,

E = Earnings per share.

Situation 1 :

When r = 15% or 0.15 and K_e = 0.10

(*i.e.*, $r > K_e$ or Growth Firm)

Value of Shares (Walter's Model) at different D/P Ratio :

D/P ratio = 0%

(*i.e.*, Dividend per Share = zero)

$$P = \frac{0 + \frac{0.15}{0.10}(₹10 - 0)}{0.10}$$

$= ₹ 150$

D/P ratio = 25%

(*i.e.*, Dividend per Share = ₹ 2.50)

$$P = \frac{₹2.50 + \frac{0.15}{0.10}(₹10 - ₹2.50)}{0.10}$$

$= ₹ 137.50$

D/P ratio = 50%

(*i.e.*, Dividend per Share = ₹ 5)

$$P = \frac{₹5 + \frac{0.15}{0.10}(₹10 - ₹5)}{0.10}$$

$= ₹ 125$

D/P ratio = 75%

(*i.e.*, Dividend per Share = ₹ 7.50)

$$P = \frac{₹7.50 + \frac{0.15}{0.10}(₹10 - ₹7.50)}{0.10}$$

$= ₹ 112.50$

D/P ratio = 100%

(*i.e.*, Dividend per Share = ₹10)

$$P = \frac{₹10 + \frac{0.15}{0.10}(₹10 - ₹10)}{0.10}$$

$= ₹ 100$

Interpretation :

From the above calculation, it is quite clear that the value of shares (P) is inversely related to the D/P ratio. As the pay-out ratio increases, the market value of shares declines. This is so, because the firm is a growth firm (where $r > K_e$) and is able to earn a return on investments (r) exceeding the required rate of return (K_e). The market value of shares (₹ 150) is highest when D/P ratio is zero, i.e. the firm retains its entire earnings. When all earnings are distributed, i.e. D/P ratio is 100%, then its market value shows the lowest price (₹ 100).

So, the optimum pay-out ratio is zero.

Situation 2 :

When $r = 10\%$ or 0·10 and $K_e = 0·10$

(i.e., $r = K_e \Rightarrow$ Normal firm)

Value of shares (Walter's Model) at different D/P Ratio :

D/P ratio = 0%

(Dividend per Share = zero)

$$P = \frac{0 + \frac{0·10}{0·10}(₹10 - 0)}{0·10}$$

= ₹ 100

D/P ratio = 25%

(Dividend per Share = ₹ 2·50)

$$P = \frac{₹2·50 + \frac{0·10}{0·10}(₹10 - ₹2·5)}{0·10}$$

= ₹ 100

D/P ratio = 50%

(i.e., Dividend per Share = ₹ 5)

$$P = \frac{₹5 + \frac{0·10}{0·10}(₹10 - ₹5)}{0·10}$$

= ₹ 100

D/P ratio = 75%

(Dividend per Share = ₹ 7·50)

$$P = \frac{₹7·50 + \frac{0·10}{0·10}(₹10 - ₹7·50)}{0·10}$$

= ₹ 100

D/P ratio = 100%

(Dividend per Share = ₹ 10)

$$P = \frac{₹10 + \frac{0·10}{0·10}(₹10 - ₹10)}{0·10}$$

= ₹ 100

Interpretation :

Under this situation, when $r = K_e$, the market value of shares is constant irrespective of the D/P Ratio. It is a matter of indifference whether the firm retains whole of the profits or distribute dividends. So, there is no optimum dividend policy. But this is a hypothetical situation ; r and K_e cannot be the same. Moreover, Walter concludes that dividend policy does matter as a variable in maximising share prices.

Situation 3 :

When $r = 8\%$ or 0·08 and $K_e = 0·10$

(i.e., $r < K_e \Rightarrow$ Declining Firm)

Value of shares (Walter's Model) at different D/P Ratio :

D/P ratio = 0%

(Dividend per Share = zero)

$$P = \frac{0 + \frac{0·08}{0·10}(₹10 - 0)}{0·10}$$

= ₹ 80

D/P ratio = 25%

(Dividend per Share = ₹ 2·50)

$$P = \frac{₹2·50 + \frac{0·08}{0·10}(₹10 - ₹2·50)}{0·10}$$

= ₹ 85

D/P ratio = 50%

(i.e., Dividend per Share = ₹ 5)

$$P = \frac{₹5 + \frac{0·08}{0·10}(₹10 - ₹5)}{0·10}$$

= ₹ 90

D/P ratio = 75%

(Dividend per Share = ₹ 7·50)

$$P = \frac{₹7·50 + \frac{0·08}{0·10}(₹10 - ₹7·50)}{0·10}$$

= ₹ 95

D/P ratio = 100%

(Dividend per Share = ₹ 10)

$$P = \frac{₹10 + \frac{0·08}{0·10}(₹10 - ₹10)}{0·10}$$

= ₹ 100

Interpretation :

When the firm is a declining firm, where $r < K_e$, D/P ratio and the value of share are correlated positively. That is, when pay-out ratio increases, the market value of shares also increases and vice versa. The market value of share is maximum (₹ 100) when D/P ratio is 100%. So, under this situation, it is advisable to distribute the entire earnings as dividend to the shareholders.

Therefore, the optimum D/P Ratio is 100%.

Criticisms of Walter's Model

The model has been criticised on several grounds.

1. In this model, it is assumed that all investments of a firm are financed through retained earnings. However, in real world, the business firms use both retained earnings and external sources of funds for financing their investment plans. Thus, this model is applicable only in case of all-equity firms. At this situation, the investment of the firm or its dividend policy cannot reach at its optimum level. This can be shown with the help of a simple diagram (Fig.-5). In Fig.-5, the earnings, investment and new financing of a firm have been measured along the horizontal axis. The rate of return (r) and the cost of capital (K_e) are measured along the vertical axis. The horizontal line K shows the constant cost of capital which remains independent of the new capital raised. However, the rates of return on investment opportunities available to the firm shows a declining trend (indicated by the downward sloping curve r). When the investment = I*, then $r = K_e$. If the investment is less than I*, then $r > K_e$. So, the firm should invest more to raise its earnings. On the other hand, if the investment is greater than I*, then $r < K_e$. In that situation, the firm should reduce the level of investment to raise it net earnings. Thus, I* is considered to be the optimum level of investment (where the required funds for this investment can be raised either by selling equity shares or bonds/debentures). If the earnings of the firm is at E_1, then (I* − E_1) amount should be raised from external sources. But

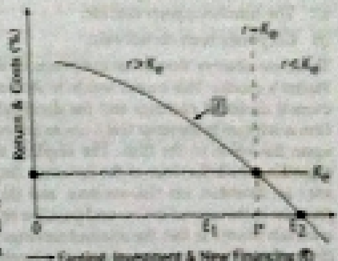


Fig.-5

Walter's model does not permit such external sources of fund. Rather this model shows that owner's wealth can be maximised by retaining and investing that E_1 amount (since $r > K_e$) without paying any dividend to the shareholders. Thus, Walter's model does not allow the firm to reach at the optimum level of investment I* by raising funds from external sources. Similarly, if the earning of the firm equals E_2 then the firm should pay a dividend by an amount of $(E_2 - I^*)$ to reach at the optimum level. But, the Walter's model would suggest that the entire E_2 amount should be distributed as dividend since $r < K_e$ at this stage. This is clearly a wrong policy and would fail to optimise the owner's wealth.

(b) This model also assumes that the internal rate of return (r) remains constant. But this is also not a realistic assumption. In fact, r cannot remain constant with an increased investment undertaken by the firm. The marginal efficiency of investment may diminish with additional investment.

(c) This model has also ignored the impact of business risks on the value of the firm. The business risks have a direct bearing upon the value of a firm. So the cost of capital (K_e) cannot be assumed to remain constant.

8.8.2. Gordon's Model (The Dividend Capitalisation Model)

Myron Gordon has also developed a model to establish the fact that the dividend pay-outs of a firm influence the value of the firm. So, this model also supports the relevance of the dividend policy of a firm.

This model is based on the following assumptions :

(a) The firm is assumed to be an all-equity firm, and all new investments in the firm are financed by its retained earnings.

(b) The return on investment (r) and the cost of equity capital (K_e) remain constant.

(c) The retention ratio (i.e., portion of retained earnings to total earnings of the firm) remains unchanged. So the growth rate ($g = br$) of dividends also remains unchanged.

(d) The cost of equity capital is higher than the growth rate, i.e., $K_e > g$.

(e) The firm has a perpetual life.

(f) Corporate taxes do not exist.

Thus, we observe that the assumptions of the Gordon's model are almost similar to those of the Walter's model. This model wants to indicate that the investors put a positive premium to their current dividend earnings and the dividend policy of a firm is relevant in the sense that it has an important bearing upon the value of the firm. **The implicit assumptions regarding the behaviour of the investors (in this model) are** : (a) Investors are risk-averters, and (b) they put a premium to the assured returns and penalise or discount the uncertain returns. In fact, the retained earnings involve risk (from the view point of investors or shareholders) and hence, the investors discount the future dividends. Since the investors are assumed to be rational in their behaviour, they are supposed to avoid risk and prefer current dividends. Such a preference pattern of the investor, as suggested by Gordon, is referred to as the '*bird-in-hand argument*'. We know that a bird-in-hand is better than two in the bush ! The implication is that the investors would prefer to pay a higher price for the shares which yield current dividend income (other things remaining unchanged). Thus, two stocks may have identical earning records and prospects but if one of them pays larger current dividend than the

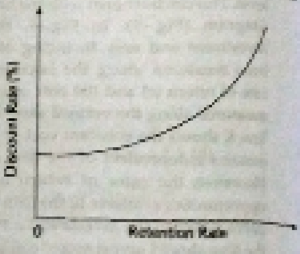


Fig.-6

other, it will undoubtedly command a higher price. This is because the stakeholders or the shareholders prefer present earnings to future earnings. The investors would assign higher discount rates to the value of shares which bring dividends at more distant future. Thus, this discount rate rises with an increase in the retention rate of the firm (Fig.-6).

The market value of a share, according to the Gordon's Model, is estimated with the help of the following formula :

$$P = \frac{E(1-b)}{K_e - br} \qquad \text{.....(5)}$$

where, P = Price of a share,

E = Earning per Share (EPS),

b = Retention ratio (i.e., percentage of earnings retained),

$(1-b)$ = D/P ratio (i.e., percentage of earnings distributed as dividend),

K_e = Cost of equity Capial (or, the capitalisation rate),

$br = g$ = Growth rate of return on investment.

The formula has been derived in the following way :

The present value of the an infinite series of dividends determines the market value of a share.

$$\therefore P_a = \frac{D_1}{(1+K_1)} + \frac{D_2}{(1+K_2)^2} + \frac{D_3}{(1+K_3)^3} + \dots + \frac{D_t}{(1+K_t)^t}$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1+K_t)^t} \qquad \text{.....(6)}$$

where, D_t = Dividend per share at the period t.

Here, the discount rate $K_t > K_{t-1}$, for $t = 1, 2, \dots, \infty$ because of increasing uncertainty in future.

Here, P_a = Present price of the share when $b = 0$, and $K_t > K_{t-1}$. If $b > 0$, then the dividend per share will be $D_t = E_t(1-b)$. Again, the dividend per share (D) is expected to grow at the rate $g = br$ when the retained earnings are reinvested at r rate of return.

Thus, at the first year the dividend per share will be $D_1 = E_0(1-b)$.

Again, $D_1 = D_0(1+g)$ [$\because D$ is expected to grow at a rate of g]

$\therefore$ In the second year, we have $D_0(1+g)^2$; in the third year we have $D_0(1+g)^3$ and so on.

In this case, equation (6) can be expressed as follows :

$$P_b = \frac{D_0(1+g)}{(1+K_e)} + \frac{D_0(1+g)^2}{(1+K_e)^2} + \dots + \frac{D_0(1+g)^t}{(1+K_e)^t} \qquad \text{.....(7)}$$

where, P_b = Price of the share when the retention rate is positive ($b > 0$).

K_e = An uniform discount rate for determining the present value of the dividend stream. (It can be considered as the cost of equity capital which is assumed to remain constant in this model).

$$\therefore P_t = P_b = \frac{D_0(1+g)}{1+K_e} \sum_{t=1}^{\infty} \left(\frac{(1+g)}{1+K_e}\right)^{t-1} = \frac{D_1}{(1+K_e)} \left[\frac{1-\left(\frac{1+g}{1+K_e}\right)^t}{1-\left(\frac{1+g}{1+K_e}\right)} \right]$$

Since $g < K_e$ (by assumption), So $0 < \left(\frac{1+g}{1+K_e}\right) < 1$

Now as $n \to \infty$, $\left(\frac{1+g}{1+K_e}\right)^n \to 0$

$$\therefore \quad P_0 = \frac{D_1}{1+K_e}\left(\frac{1}{1-\frac{1+g}{1+K_e}}\right)$$

$$= \frac{D_1}{1+K_e} \cdot \frac{1}{\frac{1+K_e-1-g}{1+K_e}}$$

$$= \frac{D_1}{1+K_e} \cdot \frac{1+K_e}{K_e-g} = \frac{D_1}{K_e-g}$$

$$= \frac{E_1(1-b)}{K_e-g} = \frac{E_1(1-b)}{K_e-br} \quad [\because g = br]$$

Illustration 7.

The following information is collected from the annual reports of J Ltd. :

Profit before tax	₹ 2·50 crore
Tax rate	40 per cent
Retention ratio	40 per cent
Numbers of outstanding shares	50,00,000
Equity capitalisation rate	12 per cent
Rate of return on investment	15 per cent

What should be the market price per share according to Gordon's Model of dividend policy?

Solution :

According to Gordon's Model, the Market Price (P) of a share, is given by,

$$P = \frac{E(1-b)}{K_e - br}$$

Where,

E = Earnings per share. (i.e., 60% of ₹ 2·50 crore/50,00,000 shares) ₹ 3

b = Retention ratio i.e., 40% or 0·40

K_e = Cost of Capital or Equity Capitalisation rate i.e., 12% or 0·12

r = Rate of return on investment i.e., 15% or 0·15

Now putting the values in the model, we get,

$$P = \frac{₹3(1-0·40)}{0·12-(0·40 \times 0·15)}$$

$$= \frac{1·80}{0·12-0·06}$$

$$= ₹ 30.$$

Illustration 8.

From the following information relating to a company, determine the market price of a share using Gordon's Model :

Total investment in assets	₹ 10,00,000
No. of shares	50,000
Total earnings	₹ 2,00,000
Cost of capital	16%
Pay-out ratio	40%

Solution :

According to Gordon's Model, the Market Price of a share, P, is given by,

$$P = \frac{E(1-b)}{K_e - br}$$

where, P = Market Price of a share

E = Earnings per share

$$= \frac{\text{Total earnings}}{\text{No. of shares}}$$

$$= \frac{₹2,00,000}{50,000}$$

$$= ₹ 4$$

$(1-b)$ = Pay-out ratio = 0·40

K_e = Cost of capital = 0·16

b = Retention ratio = 1 − 0·40 = 0·60

and r = Rate of return on investments

$$= \frac{\text{Total earnings}}{\text{Total Investments}} \times 100$$

$$= \frac{₹2,00,000}{₹10,00,000} \times 100$$

$$= 20\% = 0·20$$

Putting the values in the model, we get,

$$P = \frac{₹4 \times 0·40}{0·16-(0·60 \times 0·20)} = ₹ 40.$$

Illustration 9.

A company's total investment in asset is ₹ 1,00,00,000. It has 1,00,000 shares of ₹ 100 each. Its expected rate of return on investment is 30% and the cost of capital is 18%. The company has a policy of retaining 25% of its profits. Determine the value of the firm using Gordon's Model.

Solution :

As per Gordon's Model, the Market Price of a share,

$$P = \frac{E(1-b)}{K_e - br}$$

where, P = Market price of a share.

E = Earnings per share.

$$= \frac{\text{Return on Investment}}{\text{Number of Shares}}$$

$$= \frac{30\% \text{ of } ₹\,1{,}00{,}00{,}000}{1{,}00{,}000} = ₹\,30$$

b = Retention ratio or percentage of earnings retained.

= 25% or 0·25

K_e = Capitalisation rate = 18% or 0·18

and r = Rate of return on investment.

= 30% = 0·30

Putting the values in the model, we get,

$$P = \frac{₹\,30(1-0.25)}{0.18-(0.25\times0.30)}$$

$= ₹\ 214·28571.$

∴ Market price of a share, P = ₹ 214·28571.

Now, Value of the firm,

$V = n \times P,$

where, n = Number of shares,

and P = Market price of a share,

∴ V = 1,00,000 × ₹ 214·28571

= ₹ 2,14,28,571.

Illustration 10.

The following information is available in respect of the rate of return on investment (r), the capitalisation rate (K_e) and earnings per share (E) of ABC Ltd.

r = 12 per cent, E = ₹ 20.

Determine the value of its shares, assuming the following :

Situation	D/P Ratio (1 – b)	Retention Ratio (b)	K_e (%)
(a)	10	90	20
(b)	20	80	19
(c)	30	70	18
(d)	40	60	17
(e)	50	50	16
(f)	60	40	15
(g)	70	30	14

Solution :

According to Gordon's Model, the value of a share, P, is given by,

$$P = \frac{E(1-b)}{K_e - br}$$

where, P = Value of a share,

E = Earnings per share,

b = Retention ratio,

$(1-b)$ = D/P ratio,

K_e = Capitalisation rate,

r = Rate of return on investment.

The value of shares of ABC Ltd. for different D/P ratios and retention ratios are shown in the following table.

Situation	D/P Ratio (1– b)	Retention Ratio(b)	K_e (%)	Value of Share
(a)	10	90	20	$P = \frac{₹\,20\times0.10}{0.20-(0.90\times0.12)} = ₹\,21.74$
(b)	20	80	19	$P = \frac{₹\,20\times0.20}{0.19-(0.80\times0.12)} = ₹\,42.55$
(c)	30	70	18	$P = \frac{₹\,20\times0.30}{0.18-(0.70\times0.12)} = ₹\,62.50$
(d)	40	60	17	$P = \frac{₹\,20\times0.40}{0.17-(0.60\times0.12)} = ₹\,81.63$
(e)	50	50	16	$P = \frac{₹\,20\times0.50}{0.16-(0.50\times0.12)} = ₹\,100.00$
(f)	60	40	15	$P = \frac{₹\,20\times0.60}{0.15-(0.40\times0.12)} = ₹\,117.65$
(g)	70	30	14	$P = \frac{₹\,20\times0.70}{0.14-(0.30\times0.12)} = ₹\,134.62$

Hence, it can be concluded from the above calculations that the dividend decision has a bearing on the market price of the share. The market price of the share is favourably related with the D/P ratio. As the payment of dividend increases, the market price of the share also increases.

Criticisms of Gordon's Model

This model is also criticised because of its underlying assumptions which are supposed to be unrealistic and restrictive in nature (Please see the criticisms of the Walter's Model).

8.8.3. Modigliani and Miller's Model

We have already noted that the financial analysts like F. Modigliani and M.H. Miller are of the opinion that the dividend policy of a firm has no impact upon the value of the firm. It implies that the dividend policy of a firm does not influence the share prices or the wealth of the shareholders of the firm. Thus, the dividend policy is irrelevant so far as the value of the firm is concerned. In their model, Modigliani and Miller have shown that the earnings of a firm are determined by its investment decisions, and not by its dividend decisions.

This model is based on the following assumptions :

(a) The investors are rational and the capital markets are perfect.

(b) In the capital market, the investors get all information free of cost.

(c) There are no transaction costs either in purchasing or selling the securities. There are also no floatation costs.

(d) The securities are perfectly divisible in smaller units and no investor can influence the market price of shares out of his/her own action.

(e) There are no taxes, or there are no differences between the tax rates applicable to dividends and capital gains.

(f) The firm follows a rigid investment policy. It implies that its investment decisions remain unaffected by the dividend decision. Even if the investments are funded by the retained earnings, it would not cause any change in the pattern of business risks and the rate of return would remain unaffected.

(g) There remains no risk or uncertainty regarding the future movements in the earnings of the firm, dividend payments and the share value of the firm. It is assumed that the investors are able to forecast these future movements (However, this assumption was dropped later on).

On the basis of those assumptions and the *arbitrage* argument, the irrelevance of dividend policy was established by this model. Arbitrage refers to entering simultaneously into two transactions which exactly offset each other. From the view point of a firm, these two transactions might be : (*i*) the dividend payments, and (*ii*) raising of funds from external sources (such as by selling new shares or debentures). Thus, if the firm distributes its earnings among the shareholders as dividends and raises an equal amount by selling equities/bonds, it involves the arbitrage process. Thus, whatever increase in the value of its shares results from the payment of dividends, will be exactly offset by the decline in the market prices of its shares due to external financing. Hence, as a whole the total wealth of the shareholders remains unchanged. Hence, the value of the firm would remain unaffected by such dividend policy of the firm. So, this model suggests that the investors would be indifferent between

the dividend payments and retention of earnings by the firm. As a result, the wealth of the shareholders would not be affected by the current and future dividend decisions of the firm. Rather, it would depend upon the expected future earnings of the firm. Thus, two firms may have different dividend per-out ratios (i.e. D/P ratio) but the market values of their shares would be same if they have similar risk and return profiles.

According to this model, the market price of a share at the beginning of a period is equal to the present value of dividends paid at the end of the period plus the market price of the shares at the end of the period. Symbolically, this relationship can be stated as follows :

$$P_0 = \frac{D_1 + P_1}{(1 + K_e)} \qquad \text{...(8)}$$

where, P_0 = Market price per share at the beginning of the period (i.e. the prevailing market price).

P_1 = Market price per share at the end of the period.

D_1 = Dividend per share at the end of the period.

K_e = Cost of equity capital.

From the above relation, the market price per share at the end of the period can be estimated as follows :

$$P_1 = P_0(1 + K_e) - D_1 \qquad \text{...(9)}$$

If there remains no external financing, then the value of the firm can be ascertained as follows :

$$nP_0 = \frac{(nD_1 + nP_1)}{(1 + K_e)} \qquad \text{...(10)}$$

Where, n = Number of shares outstanding. Thus, in this case the capitalised value of the firm is simply the number of its shares outstanding (n) times the prevailing market price per share (P_0).

Let us now suppose that the firm finances its investment plans from external sources by issuing new shares at the end of the period. In that case, the capitalised value of the firm will be the sum of dividends received at the end of the period and the value of total outstanding shares at the end of the period less the value of the new shares issued.

So, the value of the firm will be :

$$nP_0 \text{ (or } V) = \frac{[nD_1 + (n + m_1)P_1 - m_1P_1]}{(1 + K_e)} \qquad \text{...(11)}$$

where, m_1 = Number of new shares issued.

If the investment requirement of the firm is higher than its retained earnings, then the additional equity capital (m_1P_1) needed can be shown as :

$$m_1P_1 = I - (E - nD_1)$$
$$= I - E + nD_1 \qquad \text{...(12)}$$

where, m_1P_1 = Amount received from the sale of new equities,

I = Total investment requirement,

E = Earnings of the firm during the period.

nD_1 = Total dividends paid by the firm.

$(E - nD_1)$ = Amount of retained earnings.

Now, substituting the value of n_1P_1 in equation (11).

We get the following result :

$$nP_0 = \frac{[nD_1 + (n+n_1)P_1 - (I - E + nD_1)]}{(1+K_e)}$$

$$= \frac{nD_1 + (n+n_1)P_1 - I + E - nD_1}{(1+K_e)}$$

$$= \frac{(n+n_1)P_1 - I + E}{(1+K_e)} \qquad \text{...(13)}$$

Equation (13) shows that the present market value of the shares is not affected by the amount of dividend payments (i.e., nD_1 does not affect nP_0).

Illustration 11.

Exponent Ltd. had 50,000 equity shares of ₹ 10 each outstanding on January 1. The shares are currently being quoted at par in the market. The company now intends to pay a dividend of ₹ 2 per share for the current calendar year. It belongs to a risk class whose appropriate capitalisation rate is 15 per cent. Using Modigliani-Miller model and assuming no taxes, ascertain the price of the company's share as it is likely to prevail at the end of the year (*a*) when dividend is declared and (*b*) when no dividend is declared. (*c*) Also, find out the number of new equity shares that the company must issue to meet its investment needs of ₹ 2 lakh, assuming a net income of ₹ 1.1 lakh (also assuming that the dividend is paid.)

Solution :

According to the Modigliani-Miller model, price of the company's share at the end of the year (P_1) can be calculated from the following formula :

$$P_0 = \frac{D_1 + P_1}{(1+K_e)}$$

where, P_0 = Current market price per share

= ₹ 10.

D_1 = Dividend per share at the year end.

P_1 = Market price per share at the year end.

and K_e = Capitalisation rate.

= 15% = 0·15.

(*a*) When dividend is paid (*i.e.*, D_1 = ₹ 2)

Substituting the values in the model, we get,

$$₹\,10 = \frac{₹\,2 + P_1}{1 + 0{\cdot}15}$$

or, P_1 = ₹ 9·50

(*b*) When dividend is not paid (*i.e.*, $D_1 = 0$),

$$P_0 = \frac{0 + P_1}{(1+K_e)}$$

or, $$₹\,10 = \frac{P_1}{(1+0{\cdot}15)}$$

∴ P_1 = ₹ 11·50

(*c*) Amount required for new financing through the issue of equity shares (n_1P_1) can be shown as

$$n_1P_1 = I - (E - nD_1)$$

where, n_1 = Number of new shares issued.

P_1 = Market price per share at the year end = ₹ 9·50, (when dividend is paid)

I = Total Investment needs = ₹ 2,00,000

E = Total Earnings = ₹ 1,10,000

n = Number of equity shares at the beginning

= 50,000

and D_1 = Dividend per share at the year end = ₹ 2.

Putting the values,

n_1P_1 = ₹ 2,00,000 − (₹ 1,10,000 − 50,000 × ₹ 2)

= ₹ 1,90,000

∴ New financing through the issue of equity shares = ₹ 1,90,000

Hence, number of equity shares to be issued at the year end (n_1) when dividend is paid,

$$n_1 = \frac{₹\,1,90,000}{P_1} = \frac{₹\,1,90,000}{₹\,9{\cdot}50}$$

= 20,000 shares.

Illustration 12.

Omega Company has a cost of equity capital of 10%, the market value of the firm (V) is ₹ 20,00,000 (@ ₹ 20 per share). Assume values for I (new investment), E (earnings) and D (dividends) at the end of the year are I = ₹ 6,80,000, E = ₹ 1,50,000 and D = ₹ 1 per share. Show that under the M-M assumptions, the payment of dividend does not affect the value of the firm.

Solution :

To show the irrelevance of dividend payment on the value of the firm, we have to calculate the value of the firm when dividend is paid and also the value of the firm when dividend is not paid.

Calculation of the value of the firm, when dividends are paid.

(ii) Market price of the share at the end of the year (P_1) can be calculated from the following formula :

$$P_0 = \frac{D_1 + P_1}{(1+K_e)}$$

where, P_0 = Current market price of the share = ₹ 20,

D_1 = Dividend per share at the end of the year = ₹ 1,

P_1 = Market price of the share at the end of the year,

and K_e = Capitalisation rate = 10% = 0·10.

Putting the values in the above formula, we get,

$$₹\,20 = \frac{₹\,1 + P_1}{(1+0{\cdot}10)}$$

or, $₹\,22 = ₹\,1 + P_1$

∴ $P_1 = ₹\,21$

(iii) Amount required for new financing (n_1P_1) :

$= I - (E - nD_1)$,

where, I = New Investment = ₹ 6,80,000

E = Earnings = ₹ 1,50,000

n = Number of shares at the beginning $= \dfrac{\text{Value of the firm}}{\text{Market price per share}}$

$$= \frac{₹\,20,00,000}{₹\,20} = 1,00,000.$$

D_1 = Dividend at the end of the year = ₹ 1.

and n_1 = Number of new shares issued

Substituting the values, we get,

n_1P_1 = ₹ 6,80,0000 − (₹ 1,50,000 − ₹1,00,000 × ₹ 1) = ₹ 6,30,000.

(iii) Number of shares to be for new financing (n_1) :

$$= \frac{I-(E-nD_1)}{P_1} = \frac{₹\,6,30,000}{₹\,21}$$

= 30,000 shares.

(iv) Value of the firm (V) :

$$= \frac{\left[nD_1 + (n+n_1)P_1 - I + E - nD_1\right]}{1+k_e}$$

Where, n = 1,00,000 shares

n_1 = 30,000 shares

D_1 = ₹ 1

P_1 = ₹ 21

I = ₹ 6,80,000

E = ₹ 1,50,000

and K_e = 0·10

Substituting the values, we get,

$$V = \frac{(1,00,000 \times ₹1) + \{(1,00,000 + 30,000) \times ₹21\} - ₹6,80,000 + ₹1,50,000 - (1,00,000 \times ₹1)}{1+0{\cdot}01}$$

$$= \frac{₹27,30,000 - ₹5,30,000}{1{\cdot}10}$$

$$= \frac{₹22,00,000}{1{\cdot}10}$$

= ₹ 20,00,000

Calculation of the value of the firm when dividends are not paid.

(i) Market price of the share at the end of the year (P_1) can be calculated from the following formula :

$$P_0 = \frac{0 + P_1}{(1+K_e)}$$

Notations have usual meaning. So, putting the values, we get,

$$₹\,20 = \frac{P_1}{1{\cdot}10}$$

∴ P_1 = ₹ 22

(ii) Amount required for new financing (n_1P_1) :

$= I - (E - n_1D_1)$

= ₹ 6,80,000 − (₹ 1,50,000 − 1,00,000 × 0) (∵ D_1 = 0)

= ₹ 5,30,000.

(iii) Number of shares to be issued for new financing (n_1) :

$$= \frac{n_1P_1}{P_1}$$

$$= \frac{₹\,5,30,000}{₹\,22}$$

(iv) Value of firm (V) $= \dfrac{n \times 0 + (n+n_1)P_1 - I + E - n \times 0}{1+K_e}$ (∵ D_1 = 0)

$$= \frac{\left(1,00,000 + \frac{5,30,000}{22}\right) \times ₹22 - ₹6,80,000 + ₹1,50,000}{1+0{\cdot}10}$$

$$= \frac{₹27,30,000 - ₹5,30,000}{1{\cdot}10}$$

= ₹ 20,00,000.

From the above calculation it has been seen that the value of the firm (V = ₹ 20,00,000) is same for both the cases i.e., when dividends are paid and when dividends are not paid. Hence it can be concluded that the payment of dividend does not affect the value of the firm under M-M Hypothesis.

Illustration 13.

A company belongs to a risk class for which the approximate capitalisation rate is 10 per cent. It currently has outstanding 25,000 shares selling at ₹ 100 each. The firm is contemplating the declaration of a dividend of ₹ 5 per share at the end of the current financial year. It expects to have a net income of ₹ 2,50,000 and has a proposal for making new investments of ₹ 5,00,000. Show that under the M-M assumptions, the payment of dividend does not affect the value the firm.

Solution :

To show the irrelevance of dividend payment on the value of the firm under the M-M assumptions we have to calculate the value of the firm when dividend is paid and also the value of the firm when dividend is not paid.

The value of the firm (V) is given by,

$$V = \frac{\left[nD_1 + (n+m_1)P_1 - I + E - nD_1\right]}{1+k_e}$$

Where, n = Number of shares at the beginning = 25,000 shares,

m_1 = Number of new shares issued,

D_1 = Dividend per share at the year end,

P_1 = Market price per share at the year end,

I = New Investments = ₹ 5,00,000,

E = Total earnings or net income = ₹ 2,50,000,

and K_e = Capitalisation rate = 10% or 0·10.

When dividend is paid (i.e., D_1 = ₹ 5) :

(i) The market price per share at the year end (P_1) can be calculated from the following formula –

$$P_0 = \frac{D_1 + P_1}{1+K_e}$$

or, $P_1 = P_0(1 + K_e) - D_1$

Where, P_0 = Current Market Price per share = ₹ 100

Now, putting the values in the above formula, we get,

P_1 = ₹ 100(1 + 0·10) – ₹ 5 = ₹ 105.

(ii) The number of new shares to be issued (n_1) is given by,

$$n_1 = \frac{I - (E - nD_1)}{P_1}$$

$$= \frac{₹5,00,000 - \left[₹2,50,000 - (25,000 \times ₹5)\right]}{₹105}$$

$$= \frac{3,75,000}{105} \text{ Shares.}$$

Now putting the values, we get the value of the firm (V) as follows :

$$V = \frac{\left[(25,000 \times ₹5) + \left(25,000 + \frac{3,75,000}{105}\right) \times ₹105 - ₹5,00,000 + ₹2,50,000 - (25,000 \times ₹5)\right]}{1+0·10}$$

$$= \frac{₹30,00,000 - ₹2,50,000}{1·10} = ₹\ 25,00,000.$$

When dividend is not paid (i.e., D_1 = 0) :

(i) $P_1 = P_0(1 + k_e) - D_1$

= ₹ 100(1 + 0·10) – 0

= ₹ 110

(ii) $$n_1 = \frac{I - \left[E - nD_1\right]}{P_1}$$

$$= \frac{₹5,00,000 - (₹2,50,000 - 25,000 \times 0)}{₹110} = \frac{2,50,000}{110} \text{ shares.}$$

Now, putting the values, we get the value of the firm (V) as follows :

$$V = \frac{\left[(25,000 \times 0) + \left(25,000 + \frac{2,50,000}{110}\right) \times ₹110 - ₹5,00,000 + ₹2,50,000 - (25,000 \times 0)\right]}{1+0·10}$$

$$= \frac{₹30,00,000 - ₹2,50,000}{1·10} = ₹\ 25,00,000.$$

From the above calculation, it has been seen that the value of the firm, V, is ₹ 25,00,000 for both the cases i.e., when dividend is paid and when dividend is not paid. Hence, it can be concluded that the payment of dividend does not affect the value of the firm under M-M assumptions.

Criticisms of M-M Hypothesis

Modigliani and Miller have expressed in the most comprehensive manner the theory of irrelevance. According to them investors are indifferent between dividend and retention of earnings in the sense that the value of the firm is independent of it. This hypothesis is based on a number of simplifying assumptions. But these assumptions are unrealistic and untenable in practice. It has only theoretical relevance. The assumptions are critically evaluated in the following paragraphs.

(1) **Non-existence of perfect capital market :** The assumption of perfect capital market is theoretical in nature. It is rarely found in practice.

(2) **Existence of flotation cost :** External financing in many cases involves cost, trouble and time gap. So, the company will prefer internal financing to external financing as it does not involve cost, trouble and time gap to fulfill a lot of legal formalities. The firm will therefore, prefer to pay low dividend or no dividend.

(3) **Existence of transaction cost and desire for current income :** Transaction cost means brokerage, commission, stamp duty etc. payable when the investors want to sell the shares in future. Again, sale of shares may be inconvenient. In such a situation, a shareholder would prefer to have current dividend than to have capital gains in future by selling of shares in future if dividends are not paid.

(4) **Tax Effect :** This hypothesis assumes 'no tax', which is also questionable. When the rate of tax is same for current dividend and capital gains, then this assumption is true. But in the real world this is not same. For example, if tax rate is lower in case of current dividend than in the case of capital gains, investors would desire for current dividend.

(5) **Desires to diversify investment portfolio :** Shareholders may like to invest in other firms with their dividend in order to diversify their investment. As such, they would prefer to get current dividend.

(6) **Legal constraints to raise capital :** Some firms have legal restrictions to raise capital from the market. In such a situation financing of projects can be done through retained earnings. In that case firms may prefer to raise retentions by lowering dividend pay-out ratio.

(7) **Informational value of dividend :** It is contended that dividends are relevant as they contain some important information for the shareholders. The payment of dividend conveys information about the profitability and prospects of the firm. A change in dividend policy signals to investors about the firm's earning position. Accordingly, the market price of the shares may be affected. In the words of **Ezra Solomon**, in an uncertain world, dividend action speaks louder than a thousand words.

(8) **Uncertainty :** Dividends are also relevant under conditions of uncertainty. The payment of dividend reduces the uncertainty perceived by investors and therefore, they do prefer current dividends to future capital gains. As a result, shares with higher current dividends, other things being equal, may have a higher price in the market.

(9) **Discount Rate :** As uncertainty increases with the length of the time period, discount rate for discounting future cash inflows at different time periods also increase. Thus, future dividend is discounted at higher rate than near dividends. As a result, investors prefer present dividends to future dividends.

(10) **Sale of additional stock :** In order to tempt new investors or existing ones to buy new shares, the company may offer lower price. But as per this theory, a firm distributing all of its earnings will be able to sell its fresh stocks at current prices. As this does not happen, retention of profit is a better option than paying dividends to shareholders.

(11) **Irrational behaviour of investors :** The assumption that the investors always act rationally may not be true always. An investor may buy underpriced stock even if he expects that the price of share will fall down further and may sell overpriced stock even though share prices show rising tendency.

(12) **Risk aversion :** Lastly, we can conclude that investors always like to avert risk which may arise due to uncertain and unpredictable future. Hence, they are more interested in short-run income which is more certain and assured than the long-run earnings which are highly unpredictable.

Thus, M-M hypothesis is not a practical proposition. It will not hold good if the assumptions underlying this hypothesis are relaxed.

LIST OF FORMULAE

S. No.	Particulars	Formula
1.	Walter's Model	$P = \dfrac{D + \frac{r}{K_e}(E - D)}{K_e}$ where, P = Market price of a share, D = Dividend per share (DPS), E = Earnings per share (EPS), r = Rate of return on investment, and K_e = Cost of equity capital or Capitalisation rate.
2.	Gordon's Model	$P = \dfrac{E(1-b)}{K_e - br}$ where, P = Market price of a share, E = Earnings per share (EPS), b = Retention ratio and K_e = Capitalisation Rate.
3.	M-M Hypothesis (i) Market price of the share at the end of the year (P_1) :	$P_0 = \dfrac{D_1 + P_1}{(1 + K_e)}$ or, $P_1 = P_0(1 + K_e) - D_1$ where, P_0 = Market price of a share at the beginning, D_1 = Dividend per share at the year end, P_1 = Market price of a share at the year end, and K_e = Capitalisation rate.
	(ii) Amount required for new financing (n_1P_1) :	$n_1P_1 = I - (E - nD_1)$ where, I = New investment, E = Total earnings, n = Number of share at the beginning, and D_1 = Dividend per share at the year end.
	(iii) Number of new shares (n_1) to be issued for new financing :	$n_1 = \dfrac{I - (E - nD_1)}{P_1}$ [Abbreviations have similar meaning as above]

LIST OF FORMULAE

S. No.	Particulars	Formula
	(iii) Value of the firm (V) :	$V = \frac{[nD_1 + (n+n_1)P_1 - I + E - nD_1]}{1+K_e}$ $= \frac{(n+n_1)P_1 - I + E}{1+K_e}$ [Abbreviations have similar meaning as above]

Summary

Dividend policy refers to the policy concerned with the distribution of dividend among the shareholders of the business firm.

The financial managers along with the Board of Directors has to frame an optimum dividend policy. A dividend policy is said to be optimum when, at any particular dividend pay-out ratio, the market price per share attains its maximum value.

Dividend pay-out ratio is obtained by dividing Cash Dividend Per Share by the Earnings Per Share (DPS/EPS).

Dividend may be of different kinds, such as interim dividend, final dividend, cash dividend, bonus dividend (issue of bonus share), bond dividend etc.

There are different factors which determines dividend policy, such as, desires of the shareholders, liquidity position of the company, financial requirement of the firm, growth aspect etc.

There are different dividend policy models relating to the impact of dividend decisions on the value of a firm.

According to Walter's Model, dividend policy is relevant in maximising the net worth of the business. To maximise the value per share D/P ratio should be zero in case of growth firms where $r > K_e$. In case of declining firms where $r < K_e$, D/P ratio should be 100% and in case of normal firms where $r = K_e$, the firm would remain indifferent.

Gordon's Model also depicts the fact of relevancy of the dividend policy in maximising shareholders wealth. Dividend policy depends upon the availability of profitable investment opportunities and the relationship between r and K_e. Under this model, when $r > K_e$, the firm should distribute lesser dividend, when $r < K_e$, retention of profit becomes undesirable and when $r = K_e$, the firm would remain indifferent as the value of share is not affected by the dividend policy.

Dividend policy is irrelevant according to M-M hypothesis. It does not affect the wealth of shareholders. This hypothesis is based on certain unrealistic assumptions which are untenable. It only has theoretical relevance.

The discussion on different models indicates the fact that investors do prefer current dividend to retained earnings.

To satisfy risk oriented investors, pay-out ratio should be low and consequently retention ratio will be high with higher expected growth rate. But high pay-out and low retention and low growth rate attracts risk averse and conservative investors.

To conclude, it can be said that neither 100% pay-out nor 0% pay-out will bring the maximum market price. The optimum point lies somewhere in between.

Assignment

Objective Type Questions

1. State whether the following statements are true or false.
   (i) Dividend is a portion of the profits kept by the management.
   (ii) The main objective of dividend policy is to maximise shareholders wealth.
   (iii) Payment of dividend involves legal as well as financial considerations.
   (iv) Stock dividend affects liquidity position of the company.
   (v) Walter Model suggests that dividend decision does not affect the value of the firm.
   (vi) According to Gordon's model, the dividend policy will not affect the market price of shares, if $r = K_e$.
   (vii) When $r > K_e$, it is said that the firm is a growth firm.
   (viii) M-M model suggests that dividend decision does not affect the value of the firm.
   [**Answer** : (i) False ; (ii) True ; (iii) True ; (iv) False ; (v) False ; (vi) True ; (vii) True ; (viii) True]

Short Answer Type Questions

1. Explain the term 'Dividend' and 'Dividend Policy'. *(See Sections 8.2, 8.4)*
2. What are the objectives of dividend policy ? *(See Section 8.5)*
3. What are the two main theories of dividend ? *(See Section 8.8)*
4. What is dividend pay-out ratio ? *(See Section 8.7)*
5. What do you mean by 'Interim Dividend' and 'Final Dividend' ? *(See Subsection 8.3.1)*
6. Is there any difference between cash dividend and share dividend ? *(See Subsection 8.3.1)*
7. Write five determinants of dividend policy. *(See Section 8.7)*
8. What are the assumptions underlying Walter's Model ? *(See Subsection 8.8.1)*
9. Write in brief criticisms of M-M Hypothesis. *(See Subsection 8.8.3)*

Essay Type Questions

1. What do you mean by 'Dividend' and 'Dividend Policy'? What are the objectives and nature of a dividend policy ? *(See Sections 8.2, 8.4, 8.5, 8.6)*
2. Classify dividends according to (i) sources, (ii) medium of payment and (iii) regularity with which they are paid and explain the legal position, if any in India in these respects. *(See Subsection 8.3.2)*
3. What are the determinants of the dividend policy of a corporate enterprise ? *(See Section 8.7)*
4. Give five important factors that a firm should consider in formulating a dividend policy. *(See Section 8.7)*
5. (i) Explain, giving suitable illustrations, the following formula given by Walter for determining dividend policy :

$$V_c = \frac{D + \frac{R_a}{R_c}(E-D)}{R_c}$$

where, V_c = Theoretical market value of ordinary share.
R_a = Internal productivity of retained earnings.

k_e = Market capitalisation Rate

E = Earnings per share

D = Dividends per share

(ii) What are the merits and limitations of this formula in designing the dividend policy for a company? (See Subsection 8.8.1)

6. What are the essentials of Walter's Dividend Model? Do you subscribe to the view that under Walter's Model the pay-out ratio can be either zero or 100%? (See Subsection 8.8.1)

7. In Walter's Model, the dividend policy of the firm depends on the availability of investment opportunity and the relation between the firm's internal rate of return and its cost of capital. Discuss what are the shortcomings of his views? (See Subsection 8.8.1)

8. Explain the significance of Walter's Model along with examples. (See Subsection 8.8.1)

9. Critically discuss Prof. James E. Walter's dividend model. To what extent are the shortcomings of this model taken care of by Prof. Gordon's dividend model? (See Subsections 8.8.1 & 8.8.2)

10. What are the implications of the Walter's model of dividend policy? (See Subsection 8.8.1)

11. Compare Walter's model with Gordon's model of dividend policy and examine their rationality. (See Subsections 8.8.1 & 8.8.2)

12. According to Gordon the value of the firm is applied by its dividend policy. Discuss critically. (See Subsection 8.8.2)

(a) Explain the Gordon's model in respect of Dividend pay-out.

13. Explain the significance of Gordon's model along with examples. (See Subsection 8.8.2)

14. (a) Indicate the use of the following formula in the relevant dividend policy models:

(i) $$P_0 = \frac{D_1}{K_e - g}$$

where, P_0 = Price per share today.

D_1 = Dividend per share at the end of the first year.

K_e = Capitalisation rate.

g = Growth rate of dividends.

(ii) $g = br$

where, g = Growth rate of dividends

b = Retention ratio

r = Internal rate of return.

(b) What are the assumptions underlying Gordon's dividend theory? Does dividend policy affect the value of the firm according to Gordon? Explain fully. (See Subsection 8.8.2)

15. Explain clearly Modigliani-Miller's Hypothesis of "Irrelevance of dividends". Under what assumptions do they hold? (See Section 8.8.3)

16. What is substance of Miller and Modigliani "dividend irrelevance theorem"? (See Section 8.8.3)

17. Prove that under M-M model:

$$V = nP_0 = \frac{(n+m)P_1 - I + E}{(1+K)}$$

where, V = Value of the firm.

n = Number of shares.

m = Number of new shares

P_1 = Market price per share at time 1.

I = Total new investment during period 1

E = Earnings of the firm for period

K = Cost of capital. (See Subsection 8.8.3)

P_0 = Market price per share at time 0

Practical Problems

1. The Apex Company which earns ₹ 5 per share is capitalised at 10% and has a return on investment of 12%. Using Walter's dividend Policy model, determine (i) the optimum pay-out, (ii) the price of share at this pay-out.

[Answer : (i) Zero, (ii) ₹ 60]

2. X Company earns ₹ 5 per share, is capitalised at a rate of 10 per cent and has a rate of return on investment of 18 per cent.

According to Walter's model, what would be the price per share at 25 per cent dividend pay-out ratio? Is this the optimum pay-out ratio according to Walter?

[Answer : ₹ 80, No, optimum D/P ratio = zero.

Market price per share at zero D/P ratio = ₹ 90]

3. A closely held plastic manufacturing company has been following a dividend policy which can maximise the market value of the firm as per Walter's model. Accordingly, each year, at dividend time, the capital budget is reviewed in conjunction with the earnings for the period and alternative investment opportunities for the shareholders. In the current year, the firm reports net earnings of ₹ 5,00,000. It is estimated that the firm can earn ₹ 1,00,000 if the amounts are retained. The investors have alternative investment opportunities that will yield them 10%. The firm has 50,000 shares outstanding.

What should be the D/P ratio if the final wishes to maximise the wealth of the shareholders?

[Answer : D/P ratio = zero, Market value of share = ₹ 200]

4. Following are the details regarding 3 companies A Ltd., B Ltd. and C Ltd.

	A Ltd.	B Ltd.	C Ltd.
Internal rate of return (%)	15	5	10
Cost of equity capital (%)	10	10	10
Earnings per share (₹)	8	8	8

Calculate the value of an equity share of each of these companies applying Walter's formulae when dividend payment ratio (D/P ratio) is (a) 0·50, (b) 0·75 and (c) 0·25. What conclusions do you draw?

[Answer : A Ltd. (a) ₹ 100, (b) ₹ 90, (c) ₹ 110

B Ltd. (a) ₹ 60, (b) ₹ 70, (c) ₹ 50

C Ltd. (a) ₹ 80, (b) ₹ 80 (c) ₹ 80]

5. Zee company has 1,00,000 equity shares of ₹ 10 each fully paid. The company expects it earnings at ₹ 12,00,000 and cost of capital at 10% for the next financial year. Using the Walter's model, what dividend policy would you recommend when the rate of return on investment of the company is estimated at 8% and 12% respectively? What will be the price of equity share if your recommendations are accepted?

[Answer : D/P Ratio = 100%, Market price = ₹ 120·00

D/P Ratio = 0%, Market price = ₹ 144·00]

6. ABC Ltd. was started a year back with a paid-up equity capital of ₹ 40,00,000. The other details are as under:

Earnings of the company	₹ 4,00,000
Dividend paid	₹ 3,20,000
Price-earning ratio	12.5
Number of shares	40,000

You are required to find out whether the company's dividend pay-out ratio is optimal, using Walter's formula.

[Answer : P = ₹ 131.25, i.e., pay-out ratio is 80%. P will be maximum (₹ 156.25) if pay-out ratio is zero]

7. Saha & Co. earns ₹ 6 per share having capitalisation rate of 10 per cent and has a return on investment at the rate of 20 per cent. According to Walter's model, what should be the price per share at 30 per cent dividend pay-out ratio? Is this optimum pay-out ratio as per Walter?

[Answer : P = ₹ 102, No — D/P Ratio should be zero]

8. The following data are available for KPI Ltd. :

Earnings per share	₹ 8.00
Rate of Return on Investment	16%
Rate of Return required by shareholders	12%

If Gordon's basic valuation formula holds, what will be price per share when the dividend pay-out ratio is 30% and 60%?

[Answer : P = ₹ 300 and ₹ 85.71 (approx.)]

9. A company has a total investment of ₹ 10,00,000 in assets and 10,000 outstanding shares of ₹ 100 each. Its rate of return is 24% and it has a policy of retaining 50% of the earnings. If the cost of capital is 18%, determine the market price of the share using Gordon's model. How would the market price change if the pay-out ratio is 90% or 10%? What should be the optimum dividend policy and why?

[Answer : ₹ 200, ₹ 139.46 ; (₹ 67) ;

optimum dividend policy is 50%

Hint : Since $r > K_e$, earnings should be retained more to have maximum value of share. When retained earnings b is 10%, P = ₹ 130 and when $b = 50\%$, P = ₹ 120. But earnings should be retained so long as the value of b does not exceed K_e/r (which is $\frac{0.18}{0.24} = 0.75$). For any value of b exceeding K_e/r but less than 1, $(K_e - br)$ becomes negative thus giving negative value for P. When $b = 90\%$, it exceeds the limit of K_e/r which is 0.75 but less than 1. For this reason, P shows negative figure of ₹ 67.00.

So, among the alternatives given, optimum dividend pay-out ratio is 50%]

10. A textile company belongs to a risk class for which the appropriate P/E ratio is 10. It currently has 50,000 outstanding shares selling at ₹ 100 each. The firm is contemplating the declaration of ₹ 8 dividend at the end of the current year which has started. Given the assumptions of Modigliani-Miller, answer the following questions :

(a) What will be the price of share at the end of the year (i) if a dividend is not declared, (ii) if it is declared ?

(b) Assuming that the firm pays the dividend, has net income of ₹ 5,00,000 and makes new investment of ₹ 10,00,000 during the period, how many new shares must be issued ?

(c) What will be the value of the firm (i) if a dividend is declared, (ii) if a dividend is not declared ?

[Answer : (a) (i) ₹ 110, (ii) ₹ 102 ;

(b) 8824 shares (approx.) ;

(c) (i) ₹ 50,00,000 ; (ii) ₹ 50,00,000]

11. The Apex Company which belongs to a risk class for which the appropriate capitalisation rate is 10%. It currently has 1,00,000 shares selling at ₹ 100 each. The firm is contemplating the declaration of ₹ 5 as dividend at the end of the current financial year, which has just begun. What will be the price of a share at the end of the year, if a dividend is not declared? What will it be if dividend is declared ? Answer these on the basis of M-M model and assume no taxes.

[Answer : ₹ 110, ₹ 105]

12. A company belongs to a risk class for which the appropriate capitalisation rate is 10%. It currently has outstanding 25,000 shares selling at ₹ 100 each. The firm is contemplating the declaration of dividend of ₹ 5 per share at the end of the current financial year. The company expects to have a net income of ₹ 2.5 lakh and has a proposal for making a new investments of ₹ 5 lakh. Show that under the M-M assumptions, the payment of dividend does not affect the value of the firm.

[Answer : When dividends are paid, P_1 = ₹ 105,

$$n_1 = \frac{3,75,000}{105} \text{ shares}, V = ₹\ 25 \text{ lakhs}$$

when dividends are not paid, P_1 = ₹ 110,

$$n_1 = \frac{2,50,000}{110} \text{ shares and } V = ₹\ 25 \text{ lakhs}]$$

13. An engineering company has a cost of equity capital of 15 per cent. The current market value of the firm is ₹ 30,00,000 (@ ₹ 30 per share). Assume values for I (new investment ₹ 9,00,000), E (Earnings ₹ 5,00,000) and total dividends D, ₹ 3,00,000. Show that under the M-M assumptions the payment of dividend does not affect the value of the firm.

[Answer : When dividends are not paid, P_1 = ₹ 34.50

$$n_1 = \frac{4,00,000}{34.50} \text{ shares}, V = ₹\ 30 \text{ lakhs.}$$

when dividends are paid, P_1 = ₹ 31.50

$$n_1 = \frac{7,00,000}{31.50} \text{ shares and } V = ₹\ 30 \text{ lakhs}]$$

14. Bestbuy Auto Ltd. has outstanding 1,20,000 shares selling at ₹ 20 per share. The company hopes to make a net income of ₹ 3,50,000 during the year ended 31st March, 2006. The Company is considering to pay a dividend of ₹ 2 per share at the end of current year. The capitalisation rate for risk class of this company has been estimated to be 15%.

Assuming no taxes, answer the questions listed below on the basis of the Modigliani-Miller dividend valuation model :

(i) What will be the price of a share at the end of 31st March, 2006

— if the dividend is paid ; and

— if the dividend is not paid ?

(ii) How many new shares must the company issue if the dividend is paid and company needs ₹ 7,40,000 for an approved investment expenditure during the year ?

[Answer : (i) ₹ 21, ₹ 23 ; (ii) 30,000 shares.]

15. X Ltd. has 8 lakhs equity shares outstanding at the beginning of the year 2005. The current market price per share is ₹ 120. The Board of Directors of the company is contemplating ₹ 6.4 per share as dividend. The rate of capitalization, appropriate to the risk class to which the company belongs, is 9.6%.

(i) Based on M-M. approach, calculate the market price of the share of the company, when the dividend is — (a) declared and (b) not declared.

(ii) How many new shares are to be issued by the company, if the company desired to fund an investment budget of ₹ 3.20 crores by the end of the year assuming net income for the year will be ₹ 1.60 crores.

[Answer : (i) ₹ 129.13, ₹ 131.92 ;

(ii) 1,48,798 shares, when dividend is declared and 1,21,408 shares when dividend is not declared.]

58. D Ltd. has 50 lakh equity shares outstanding at the beginning of the accounting year 2006. The appropriate P/E ratio for the industry in which D Ltd. belongs is 8.35. The earning per share is ₹ 15 in the last twelve months and current P/E ratio for the company is 10. The EPS is expected to be ₹ 20 at the end of the accounting year and company has an investment budget of ₹ 4 crores. Based on M-M approach calculate the market price of the share of the company

(a) When the Board of Directors of the company has recommended ₹ 8 per share as dividend is (i) declared and (ii) not declared.

(b) How many new shares are to be issued by the company at the end of the accounting year when (i) the above dividends are distributed ; and (ii) dividends are not declared?

(c) Show that the market value of the shares at the end of accounting year will remain the same whether dividends are distributed or not declared.

APPENDIX 1

CONTENTS

TABLE - 1

The Compound Sum of One Rupee

Year	1%	2%	3%	4%	5%	6%	7%	8%
1	1.010	1.020	1.030	1.040	1.050	1.060	1.070	1.080
2	1.020	1.040	1.061	1.082	1.102	1.124	1.145	1.166
3	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260
4	1.041	1.082	1.126	1.170	1.216	1.262	1.311	1.360
5	1.051	1.104	1.159	1.217	1.276	1.338	1.403	1.469
6	1.062	1.126	1.194	1.265	1.340	1.419	1.501	1.587
7	1.072	1.149	1.230	1.316	1.407	1.504	1.606	1.714
8	1.083	1.172	1.267	1.369	1.477	1.594	1.718	1.851
9	1.094	1.195	1.305	1.423	1.551	1.689	1.838	1.999
10	1.105	1.219	1.344	1.480	1.629	1.791	1.967	2.159
11	1.116	1.243	1.384	1.539	1.710	1.898	2.105	2.332
12	1.127	1.268	1.426	1.601	1.796	2.012	2.252	2.518
13	1.138	1.294	1.469	1.665	1.886	2.133	2.410	2.720
14	1.149	1.319	1.513	1.732	1.980	2.261	2.579	2.937
15	1.161	1.346	1.558	1.801	2.079	2.397	2.759	3.172
16	1.173	1.373	1.605	1.873	2.183	2.540	2.952	3.426
17	1.184	1.400	1.653	1.948	2.292	2.693	3.159	3.700
18	1.196	1.428	1.702	2.026	2.407	2.854	3.380	3.996

TABLE - 1 (Contd.)

Year	1%	2%	3%	4%	5%	6%	7%	8%
19	1.208	1.457	1.753	2.107	2.527	3.026	3.616	4.316
20	1.220	1.486	1.806	2.191	2.653	3.207	3.870	4.661
21	1.232	1.516	1.860	2.279	2.786	3.399	4.140	5.034
22	1.245	1.546	1.916	2.370	2.925	3.603	4.430	5.436
23	1.257	1.577	1.974	2.465	3.071	3.820	4.740	5.871
24	1.270	1.608	2.033	2.563	3.225	4.049	5.072	6.341
25	1.282	1.641	2.094	2.666	3.386	4.292	5.427	6.848
30	1.348	1.811	2.427	3.243	4.322	5.743	7.612	10.062
35	1.417	2.000	2.814	3.946	5.516	7.686	10.676	14.785
40	1.489	2.208	3.262	4.801	7.040	10.285	14.974	21.724
45	1.565	2.438	3.781	5.841	8.985	13.764	21.002	31.920
50	1.645	2.691	4.384	7.106	11.467	18.419	29.456	46.901

Year	9%	10%	11%	12%	13%	14%	15%	16%
1	1.090	1.100	1.110	1.120	1.130	1.140	1.150	1.160
2	1.188	1.210	1.232	1.254	1.277	1.300	1.322	1.346
3	1.295	1.331	1.368	1.405	1.443	1.482	1.521	1.561
4	1.412	1.464	1.518	1.574	1.630	1.689	1.749	1.811
5	1.539	1.611	1.685	1.762	1.842	1.925	2.011	2.100
6	1.677	1.772	1.870	1.974	2.082	2.195	2.313	2.436
7	1.828	1.949	2.076	2.211	2.353	2.502	2.660	2.826
8	1.993	2.144	2.305	2.476	2.658	2.853	3.059	3.278
9	2.172	2.358	2.558	2.773	3.004	3.252	3.518	3.803
10	2.367	2.594	2.839	3.106	3.395	3.707	4.046	4.411
11	2.580	2.853	3.152	3.479	3.836	4.226	4.652	5.117
12	2.813	3.138	3.498	3.896	4.334	4.818	5.350	5.936
13	3.066	3.452	3.883	4.363	4.898	5.492	6.153	6.886
14	3.342	3.797	4.310	4.887	5.535	6.261	7.076	7.987
15	3.642	4.177	4.785	5.474	6.254	7.138	8.137	9.265
16	3.970	4.595	5.311	6.130	7.067	8.137	9.358	10.748
17	4.328	5.054	5.895	6.866	7.986	9.276	10.761	12.468
18	4.717	5.560	6.543	7.690	9.024	10.575	12.375	14.462
19	5.142	6.116	7.263	8.613	10.197	12.055	14.232	16.776
20	5.604	6.727	8.062	9.646	11.523	13.743	16.366	19.460
21	6.109	7.400	8.949	10.804	13.021	15.667	18.821	22.574
22	6.658	8.140	9.933	12.100	14.713	17.861	21.644	26.186
23	7.258	8.954	11.026	13.552	16.626	20.361	24.891	30.376

TABLE - 1 (Contd.)

Year	9%	10%	11%	12%	13%	14%	15%	16%
24	7.911	9.850	12.239	15.178	18.788	23.212	28.625	35.236
25	8.623	10.834	13.585	17.000	21.230	26.461	32.918	40.874
30	13.267	17.449	22.892	29.960	39.115	50.949	66.210	85.849
35	20.413	28.102	38.574	52.799	72.066	98.097	133.172	180.311
40	31.409	45.259	64.999	93.049	132.776	188.876	267.856	378.715
45	48.327	72.888	109.527	163.985	244.629	363.662	538.752	795.429
50	74.357	117.390	184.559	288.996	450.711	700.197	1083.619	1670.669

Year	17%	18%	19%	20%	21%	22%	23%	24%
1	1.170	1.180	1.190	1.200	1.210	1.220	1.230	1.240
2	1.369	1.392	1.416	1.440	1.464	1.488	1.513	1.538
3	1.602	1.643	1.685	1.728	1.772	1.816	1.861	1.907
4	1.874	1.939	2.005	2.074	2.144	2.215	2.289	2.364
5	2.192	2.288	2.386	2.488	2.594	2.703	2.815	2.932
6	2.565	2.700	2.840	2.986	3.138	3.297	3.463	3.635
7	3.001	3.185	3.379	3.583	3.797	4.023	4.259	4.508
8	3.511	3.759	4.021	4.300	4.595	4.908	5.239	5.590
9	4.108	4.435	4.785	5.160	5.560	5.987	6.444	6.931
10	4.807	5.234	5.695	6.192	6.727	7.305	7.926	8.594
11	5.624	6.176	6.777	7.430	8.140	8.912	9.749	10.657
12	6.580	7.288	8.064	8.916	9.850	10.872	11.991	13.215
13	7.699	8.599	9.596	10.699	11.918	13.264	14.749	16.386
14	9.007	10.147	11.420	12.839	14.421	16.182	18.141	20.319
15	10.539	11.974	13.589	15.407	17.449	19.742	22.314	25.195
16	12.330	14.129	16.171	18.488	21.113	24.085	27.446	31.242
17	14.426	16.672	19.244	22.186	25.547	29.384	33.759	38.740
18	16.879	19.673	22.900	26.623	30.912	35.848	41.523	48.038
19	19.748	23.214	27.251	31.948	37.404	43.735	51.074	59.567
20	23.105	27.393	32.429	38.337	45.259	53.357	62.821	73.863
21	27.033	32.323	38.591	46.005	54.762	65.095	77.268	91.591
22	31.629	38.141	45.923	55.205	66.262	79.416	95.041	113.573
23	37.005	45.007	54.648	66.247	80.179	96.887	116.901	140.831
24	43.296	53.108	65.031	79.496	97.017	118.203	143.788	174.630
25	50.658	62.667	77.387	95.395	117.389	144.210	176.859	216.542
30	111.061	143.367	184.672	237.375	304.417	389.758	497.904	634.820
35	243.495	327.988	440.691	590.657	789.736	1053.370	1401.749	1861.020

TABLE - 1 (Contd.)

Year	17%	18%	19%	20%	21%	22%	23%	24%
40	533.846	750.353	1051.642	1469.740	2048.309	2846.941	3946.340	5455.797
45	1170.425	1716.619	2509.563	3657.176	5312.758	7694.418	11110.121	15994.316
50	2566.080	3927.169	5988.730	9100.191	13779.844	20795.680	31278.301	46889.207

Year	25%	26%	27%	28%	29%	30%	31%	32%
1	1.250	1.260	1.270	1.280	1.290	1.300	1.310	1.320
2	1.562	1.588	1.613	1.638	1.664	1.690	1.716	1.742
3	1.953	2.000	2.048	2.097	2.147	2.197	2.248	2.300
4	2.441	2.520	2.601	2.684	2.769	2.856	2.945	3.036
5	3.052	3.176	3.304	3.436	3.572	3.713	3.858	4.007
6	3.815	4.001	4.196	4.398	4.608	4.827	5.054	5.290
7	4.768	5.042	5.329	5.629	5.945	6.275	6.621	6.983
8	5.960	6.353	6.767	7.206	7.669	8.157	8.673	9.217
9	7.451	8.004	8.595	9.223	9.893	10.604	11.362	12.166
10	9.313	10.086	10.915	11.806	12.763	13.786	14.884	16.060
11	11.642	12.708	13.862	15.112	16.462	17.921	19.498	21.199
12	14.552	16.012	17.605	19.343	21.236	23.298	25.542	27.982
13	18.190	20.175	22.359	24.759	27.395	30.287	33.460	36.937
14	22.737	25.420	28.395	31.691	35.339	39.373	43.832	48.756
15	28.422	32.030	36.062	40.565	45.587	51.185	57.420	64.358
16	35.527	40.357	45.799	51.923	58.808	66.541	75.220	84.953
17	44.409	50.850	58.165	66.461	75.862	86.503	98.539	112.138
18	55.511	64.071	73.869	85.070	97.862	112.454	129.086	148.022
19	69.389	80.730	93.813	108.890	126.242	146.190	169.102	195.389
20	86.736	101.720	119.143	139.379	162.852	190.047	221.523	257.913
21	108.420	128.167	151.312	178.405	210.079	247.061	290.196	340.446
22	135.525	161.490	192.165	228.358	271.002	321.178	380.156	449.388
23	169.407	203.477	244.050	292.298	349.592	417.531	498.004	593.192
24	211.758	256.381	309.943	374.141	450.974	542.791	652.385	783.013
25	264.698	323.040	393.628	478.901	581.756	705.627	854.623	1033.577
30	807.793	1025.904	1300.477	1645.488	2078.208	2619.936	3297.081	4142.008
35	2465.189	3258.053	4296.547	5653.840	7423.988	9727.598	12719.918	16598.536
40	7523.156	10346.879	14195.051	19426.418	26520.723	36117.754	49072.621	66519.525
45	22958.844	32859.457	46897.973	66748.500	94739.937	134102.187		
50	70064.812	104354.562	154942.687	229345.875	338440.000	497910.125		

TABLE - 1 (Contd.)

Year	33%	34%	35%	36%	37%	38%	39%	40%
1	1.330	1.340	1.350	1.360	1.370	1.380	1.390	1.400
2	1.769	1.796	1.822	1.850	1.877	1.904	1.932	1.960
3	2.353	2.406	2.460	2.515	2.571	2.628	2.686	2.744
4	3.129	3.224	3.321	3.421	3.523	3.627	3.733	3.842
5	4.162	4.320	4.484	4.653	4.826	5.005	5.189	5.378
6	5.535	5.789	6.053	6.328	6.612	6.907	7.213	7.530
7	7.361	7.758	8.172	8.605	9.058	9.531	10.025	10.541
8	9.791	10.395	11.032	11.703	12.410	13.153	13.935	14.758
9	13.022	13.930	14.894	15.917	17.001	18.151	19.370	20.661
10	17.319	18.666	20.106	21.646	23.292	25.049	26.924	28.925
11	23.034	25.012	27.144	29.439	31.910	34.567	37.425	40.495
12	30.635	33.516	36.644	40.037	43.716	47.703	52.020	56.694
13	40.745	44.912	49.469	54.451	59.892	65.830	72.308	79.371
14	54.190	60.182	66.784	74.053	82.051	90.845	100.509	111.119
15	72.073	80.643	90.158	100.712	112.410	125.366	139.707	155.567
16	95.857	108.061	121.713	136.968	154.002	173.005	194.192	217.793
17	127.490	144.802	164.312	186.277	210.983	238.747	269.927	304.911
18	169.561	194.035	221.822	253.337	289.046	329.471	375.198	426.875
19	225.517	260.006	299.459	344.537	395.993	454.669	521.525	597.625
20	299.937	348.408	404.270	468.571	542.511	627.443	724.919	836.674
21	398.916	466.867	545.764	637.256	743.240	865.871	1007.637	1171.343
22	530.558	625.601	736.781	866.668	1018.238	1194.900	1400.615	1639.878
23	705.642	838.305	994.653	1178.668	1394.986	1648.961	1946.854	2295.829
24	938.504	1123.328	1342.781	1602.988	1911.129	2275.564	2706.125	3214.158
25	1248.210	1505.258	1812.754	2180.063	2618.245	3140.275	3761.511	4499.816
30	5194.516	6503.285	8128.426	10142.914	12636.086	15716.703	19517.969	24201.043
35	21617.363	28097.695	36448.051	47190.727	60983.836	78660.188	101276.125	130158.687
40	89962.188	121388.437	163433.875	219558.625	294317.937	393684.687	525508.312	700022.688

TABLE - 2

The Compound Value of an Annuity of One Rupee

Year	1%	2%	3%	4%	5%	6%	7%	8%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.010	2.020	2.030	2.040	2.050	2.060	2.070	2.080
3	3.030	3.060	3.091	3.122	3.152	3.184	3.215	3.246

TABLE - 2 (Contd.)

Year	1%	2%	3%	4%	5%	6%	7%	8%
4	4.060	4.122	4.184	4.246	4.310	4.375	4.440	4.506
5	5.101	5.204	5.309	5.416	5.526	5.637	5.751	5.867
6	6.152	6.308	6.468	6.633	6.802	6.975	7.153	7.336
7	7.214	7.434	7.662	7.898	8.142	8.394	8.654	8.923
8	8.286	8.583	8.892	9.214	9.549	9.897	10.260	10.637
9	9.369	9.755	10.159	10.583	11.027	11.491	11.978	12.488
10	10.462	10.950	11.464	12.006	12.578	13.181	13.816	14.487
11	11.567	12.169	12.808	13.486	14.207	14.972	15.784	16.645
12	12.682	13.412	14.192	15.026	15.917	16.870	17.888	18.977
13	13.809	14.680	15.618	16.627	17.713	18.882	20.141	21.495
14	14.947	15.974	17.086	18.292	19.598	21.015	22.550	24.215
15	16.097	17.293	18.599	20.023	21.579	23.276	25.129	27.152
16	17.258	18.639	20.157	21.824	23.657	25.672	27.888	30.324
17	18.430	20.012	21.761	23.697	25.840	28.213	30.840	33.750
18	19.614	21.412	23.414	25.645	28.132	30.905	33.999	37.450
19	20.811	22.840	25.117	27.671	30.539	33.760	37.379	41.446
20	22.019	24.297	26.870	29.778	33.066	36.785	40.995	45.762
21	23.239	25.783	28.676	31.969	35.719	39.992	44.865	50.422
22	24.471	27.299	30.536	34.248	38.505	43.392	49.005	55.456
23	25.716	28.845	32.452	36.618	41.340	46.995	53.435	60.893
24	26.973	30.421	34.426	39.082	44.501	50.815	58.176	66.764
25	28.243	32.030	36.459	41.645	47.726	54.864	63.248	73.105
30	34.784	40.567	47.575	56.084	66.438	79.057	94.459	113.282
35	41.659	49.994	60.461	73.651	90.318	111.435	138.234	172.314
40	48.885	60.401	75.400	95.024	120.797	154.758	199.633	259.052
45	56.479	71.891	92.719	121.027	159.695	212.737	285.743	386.497
50	64.462	84.577	112.796	152.664	209.31	290.325	406.516	573.756

Year	9%	10%	11%	12%	13%	14%	15%	16%
1	1.000	1.000	1.000	1.000	1.00	1.000	1.000	1.000
2	2.090	2.100	2.110	2.120	2.130	2.140	2.150	2.160
3	3.278	3.310	3.342	3.374	3.407	3.440	3.472	3.506
4	4.573	4.641	4.710	4.779	4.850	4.921	4.933	5.066
5	5.985	6.105	6.228	6.353	6.480	6.610	6.742	6.877
6	7.523	7.716	7.913	8.115	8.323	8.535	8.754	8.977
7	9.200	9.487	9.783	10.089	10.405	10.730	11.067	11.414
8	11.028	11.436	11.859	12.300	12.757	13.233	13.727	14.240

TABLE - 2 (Contd.)

Year	9%	10%	11%	12%	13%	14%	15%	16%
9	13.021	13.579	14.164	14.776	15.416	16.085	16.786	17.518
10	15.193	15.937	16.722	17.549	18.420	19.337	20.304	21.321
11	17.560	18.531	19.561	20.655	21.814	23.044	24.349	25.733
12	20.141	21.384	22.713	24.133	25.650	27.271	29.001	30.850
13	22.953	24.523	26.211	28.029	29.984	32.088	34.351	36.786
14	26.019	27.975	30.095	32.392	34.882	37.581	40.504	43.672
15	29.361	31.772	34.405	37.280	40.417	43.842	47.580	51.659
16	33.003	35.949	39.190	42.753	46.671	50.980	55.717	60.925
17	36.973	40.544	44.500	48.883	53.738	59.117	65.075	71.673
18	41.301	45.598	50.396	55.749	61.724	68.393	75.836	84.140
19	46.018	51.158	56.939	63.439	70.748	78.968	88.211	98.603
20	51.159	57.274	64.202	72.051	80.946	91.024	102.443	115.379
21	56.754	65.002	72.264	81.698	92.468	104.767	118.809	134.840
22	62.872	71.402	81.213	92.502	105.489	120.434	137.630	157.414
23	69.531	79.542	91.147	104.602	120.203	138.295	159.274	183.600
24	76.789	88.496	102.173	118.154	136.829	158.656	184.166	213.976
25	84.699	98.346	114.412	133.333	155.616	181.867	212.790	249.212
30	136.305	164.491	199.018	241.330	293.192	356.778	434.738	530.306
35	215.705	271.018	341.583	431.658	546.663	693.552	881.152	1120.699
40	337.872	442.580	581.812	767.080	1013.667	1341.979	1779.048	2360.724
45	525.846	718.881	986.613	1358.208	1874.086	2590.464	3585.031	4965.191
50	815.051	1163.865	1668.732	2399.975	3459.364	4994.301	7217.488	10435.449

Year	17%	18%	19%	20%	21%	22%	23%	24%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.170	2.180	2.190	2.200	2.210	2.220	2.230	2.240
3	3.539	3.572	3.606	3.640	3.674	3.708	3.743	3.778
4	5.141	5.215	5.291	5.368	5.446	5.524	5.604	5.684
5	7.014	7.154	7.297	7.442	7.589	7.740	7.893	8.048
6	9.207	9.442	9.683	9.930	10.183	10.442	10.708	10.980
7	11.772	12.141	12.523	12.916	13.321	13.740	14.171	14.615
8	14.773	15.327	15.902	16.499	17.119	17.762	18.430	19.123
9	18.285	19.086	19.923	20.799	21.714	22.670	23.669	24.712
10	22.393	23.521	24.709	25.959	27.274	28.657	30.113	31.643
11	27.200	28.755	30.403	32.150	34.001	35.962	38.039	40.238
12	32.824	34.931	37.180	39.580	42.141	44.873	47.787	50.895
13	38.936	42.218	45.244	48.496	51.991	55.745	59.778	64.109

TABLE - 2 *(Contd.)*

Year	17%	18%	19%	20%	21%	22%	23%	24%
14	47.102	50.818	54.841	59.196	63.909	69.009	74.528	80.496
15	56.109	60.965	66.260	72.035	78.330	85.191	92.669	100.815
16	66.648	72.938	79.850	87.442	95.779	104.935	114.983	126.010
17	78.978	87.067	96.021	105.930	116.892	129.019	142.428	157.252
18	93.404	103.739	115.265	128.116	142.439	158.403	176.187	195.993
19	110.283	123.413	138.165	154.739	173.353	194.253	217.710	244.031
20	130.031	146.626	165.417	186.687	210.755	237.986	268.783	303.598
21	153.136	174.019	197.846	225.024	256.013	291.343	331.603	377.461
22	180.169	206.342	236.436	217.028	310.775	356.438	408.871	469.052
23	211.798	244.483	282.359	326.234	377.038	435.854	503.911	582.624
24	248.803	289.490	337.007	392.480	457.215	532.741	620.810	723.453
25	292.099	342.598	402.038	471.976	554.230	650.944	764.596	898.082
30	647.423	790.932	966.698	1181.865	1445.121	1767.044	2160.459	2640.881
35	1426.448	1816.607	2314.173	2948.294	3755.334	4783.520	6090.227	7750.894
40	3134.412	4163.094	5529.713	7343.715	9749.141	12936.141	17153.693	22728.107
45	6879.808	9531.258	13203.105	18280.914	25294.223	34970.230	48300.663	66638.937
50	15088.805	21812.273	31514.492	45496.094				

Year	25%	26%	27%	28%	29%	30%	31%	32%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.250	2.260	2.270	2.280	2.290	2.300	2.310	2.320
3	3.813	3.848	3.883	3.918	3.954	3.990	4.026	4.062
4	5.766	5.848	5.931	6.016	6.101	6.187	6.274	6.362
5	8.207	8.368	8.533	8.700	8.870	9.043	9.219	9.398
6	11.259	11.544	11.837	12.136	12.442	12.756	13.077	13.406
7	15.073	15.546	16.032	16.534	17.051	17.583	18.131	18.696
8	19.842	20.588	21.361	22.163	22.995	23.858	24.752	25.678
9	25.802	26.940	28.129	29.369	30.664	32.015	33.425	34.895
10	33.253	34.945	36.723	38.592	40.556	42.619	44.786	47.062
11	42.566	45.030	47.639	50.398	53.318	56.405	59.670	63.121
12	54.208	57.738	61.501	65.510	69.780	74.326	79.167	84.320
13	68.760	73.750	79.106	84.853	91.016	97.624	104.709	112.302
14	86.949	93.925	101.465	109.611	118.411	127.912	138.169	149.239
15	109.687	119.346	129.860	141.302	153.750	167.285	182.001	197.996
16	138.109	151.375	165.922	181.867	199.337	218.470	239.421	262.354
17	173.636	191.733	211.721	233.790	258.145	285.011	314.642	347.307
18	218.045	242.583	269.885	300.250	334.006	371.514	413.180	459.445

TABLE - 2 *(Contd.)*

Year	25%	26%	27%	28%	29%	30%	31%	32%
19	273.556	306.654	343.754	385.321	431.868	483.968	542.266	607.467
20	342.945	387.384	437.568	494.210	558.110	630.157	711.368	802.856
21	429.681	489.104	556.710	633.589	720.962	820.204	932.891	1060.769
22	538.101	617.270	708.022	811.993	931.040	1067.265	1223.087	1401.215
23	673.626	778.760	900.187	1040.351	1202.042	1388.443	1603.243	1850.603
24	843.032	982.237	1144.237	1332.649	1551.634	1805.975	2101.247	2443.795
25	1054.791	1238.617	1454.180	1706.790	2002.608	2348.765	2753.631	3226.808
30	3227.172	3941.953	4812.891	5873.172	7162.785	8729.805	10632.543	12940.672
35	9856.746	12527.160	15909.480	20188.742	25596.512	32422.090		
40	30088.621	39791.957	52570.707	69376.562	91447.375	120389.375		
45	91831.312	126378.937	173692.875	238384.312	326686.375	447005.062		

Year	33%	34%	35%	36%	37%	38%	39%	40%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.330	2.340	2.350	2.360	2.370	2.380	2.390	2.400
3	4.099	4.136	4.172	4.210	4.247	4.284	4.322	4.360
4	6.452	6.542	6.633	6.725	6.818	6.912	7.008	7.104
5	9.581	9.766	9.954	10.146	10.341	10.539	10.741	10.946
6	13.742	14.086	14.438	14.799	15.167	15.544	15.930	16.324
7	19.277	19.876	20.492	21.126	21.779	22.451	23.142	23.853
8	26.638	27.633	28.664	29.732	30.837	31.982	33.167	34.395
9	36.429	38.028	39.696	41.435	43.247	45.135	47.103	49.152
10	49.451	51.958	54.590	57.351	60.248	63.287	66.473	69.813
11	66.769	70.624	74.696	78.998	83.540	88.337	93.397	98.739
12	89.803	95.636	101.840	108.437	115.450	122.903	130.822	139.234
13	120.438	129.152	138.484	148.474	159.166	170.606	182.842	195.928
14	161.183	174.063	187.953	202.925	219.058	236.435	255.151	275.299
15	215.373	234.245	254.737	276.978	301.109	327.281	355.659	386.418
16	287.446	314.888	344.895	377.690	413.520	452.647	495.366	541.985
17	383.303	422.949	466.608	514.658	567.521	625.652	689.558	759.779
18	510.792	567.751	630.920	700.935	778.504	864.399	959.485	1064.689
19	680.354	761.786	852.741	954.271	1067.551	1193.870	1334.683	1491.563
20	905.870	1021.792	1152.200	1298.809	1463.544	1648.539	1856.208	2089.188
21	1205.807	1370.201	1556.470	1767.380	2006.055	2275.982	2581.128	2925.862
22	1604.724	1837.068	2102.234	2404.636	2749.294	3141.852	3588.765	4097.203
23	2135.282	2462.668	2839.014	3271.304	3767.532	4336.750	4989.379	5737.078
24	2840.924	3300.974	3833.667	4449.969	5162.516	5985.711	6936.238	8032.906

TABLE - 2 (Contd.)

Year	33%	34%	35%	36%	37%	38%	39%	40%
25	3779.020	4624.301	5176.445	6052.957	7073.645	8261.779	9642.593	11247.062
30	15737.945	19124.436	23221.258	28172.036	34148.906	41357.227	50043.625	60500.207

TABLE - 3

The Present Value of One Rupee

Year	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.990	.980	.971	.962	.952	.943	.935	.926	.917	.909
2	.980	.961	.943	.925	.907	.890	.873	.857	.842	.826
3	.971	.942	.915	.889	.864	.840	.816	.794	.772	.751
4	.961	.924	.888	.855	.823	.792	.763	.735	.708	.683
5	.951	.906	.863	.822	.784	.747	.713	.681	.650	.621
6	.942	.888	.837	.790	.746	.705	.666	.630	.596	.564
7	.933	.871	.813	.760	.711	.665	.623	.583	.547	.513
8	.923	.853	.789	.731	.677	.627	.582	.540	.502	.467
9	.914	.837	.766	.703	.645	.592	.544	.500	.460	.424
10	.905	.820	.744	.676	.614	.558	.508	.463	.422	.386
11	.896	.804	.722	.650	.585	.527	.475	.429	.388	.350
12	.887	.789	.701	.625	.557	.497	.444	.397	.356	.319
13	.879	.773	.681	.601	.530	.469	.415	.368	.326	.290
14	.870	.758	.661	.577	.505	.442	.388	.340	.299	.263
15	.861	.743	.642	.555	.481	.417	.362	.315	.275	.239
16	.853	.728	.623	.534	.458	.394	.339	.292	.252	.218
17	.844	.714	.605	.513	.436	.371	.317	.270	.231	.198
18	.836	.700	.587	.494	.416	.350	.296	.250	.212	.180
19	.828	.686	.570	.475	.396	.331	.277	.232	.194	.164
20	.820	.673	.554	.456	.377	.312	.258	.215	.178	.149
21	.811	.660	.538	.439	.359	.294	.242	.199	.164	.135
22	.803	.647	.522	.422	.342	.278	.226	.184	.150	.123
23	.795	.634	.507	.406	.326	.262	.211	.170	.138	.112
24	.788	.622	.492	.390	.310	.247	.197	.158	.126	.102
25	.780	.610	.478	.375	.295	.233	.184	.146	.116	.092
30	.742	.552	.412	.308	.231	.174	.131	.099	.075	.057
35	.706	.500	.355	.253	.181	.130	.094	.068	.049	.036
40	.672	.453	.307	.208	.142	.097	.067	.046	.032	.022
45	.639	.410	.264	.171	.111	.073	.048	.031	.021	.014
50	.608	.372	.228	.141	.087	.054	.034	.021	.013	.009

TABLE - 3 (Contd.)

Year	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	.901	.893	.885	.877	.870	.862	.855	.847	.840	.833
2	.812	.797	.783	.769	.756	.743	.731	.718	.706	.694
3	.731	.712	.693	.675	.658	.641	.624	.609	.593	.579
4	.659	.636	.613	.592	.572	.552	.534	.516	.499	.482
5	.593	.567	.543	.519	.497	.476	.456	.437	.419	.402
6	.535	.507	.480	.456	.432	.410	.390	.370	.352	.335
7	.482	.452	.425	.400	.376	.354	.333	.314	.296	.279
8	.434	.404	.376	.351	.327	.305	.285	.266	.249	.233
9	.391	.361	.333	.308	.284	.263	.243	.225	.209	.194
10	.352	.322	.295	.270	.247	.227	.208	.191	.176	.162
11	.317	.287	.261	.237	.215	.195	.178	.162	.148	.135
12	.286	.257	.231	.208	.187	.168	.152	.137	.124	.112
13	.258	.229	.204	.182	.163	.145	.130	.116	.104	.093
14	.232	.205	.181	.160	.141	.125	.111	.099	.088	.078
15	.209	.183	.160	.140	.123	.108	.095	.084	.074	.065
16	.188	.163	.141	.123	.107	.093	.081	.071	.062	.054
17	.170	.146	.125	.108	.093	.080	.069	.060	.052	.045
18	.153	.130	.111	.095	.081	.069	.059	.051	.044	.038
19	.138	.116	.098	.083	.070	.060	.051	.043	.037	.031
20	.124	.104	.087	.073	.061	.051	.043	.037	.031	.026
21	.112	.093	.077	.064	.053	.044	.037	.031	.026	.022
22	.101	.083	.068	.056	.046	.038	.032	.026	.022	.018
23	.091	.074	.060	.049	.040	.033	.027	.022	.018	.015
24	.082	.066	.053	.043	.035	.028	.023	.019	.015	.013
25	.074	.059	.047	.038	.030	.024	.020	.016	.013	.010
30	.044	.033	.026	.020	.015	.012	.009	.007	.005	.004
35	.026	.019	.014	.010	.008	.006	.004	.003	.002	.002
40	.015	.011	.008	.005	.004	.003	.002	.001	.001	.001
45	.009	.006	.004	.003	.002	.001	.001	.001	.000	.000
50	.005	.003	.002	.001	.001	.001	.000	.000	.000	.000

Year	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	.826	.820	.813	.806	.800	.794	.787	.781	.775	.769
2	.683	.672	.661	.650	.640	.630	.620	.610	.601	.592
3	.564	.551	.537	.524	.512	.500	.488	.477	.466	.455
4	.467	.451	.437	.423	.410	.397	.384	.373	.361	.350
5	.386	.370	.355	.341	.328	.315	.303	.291	.280	.269

TABLE - 3 (Contd.)

Year	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
6	.319	.303	.289	.275	.262	.250	.238	.227	.217	.207
7	.263	.249	.235	.222	.210	.198	.188	.178	.168	.159
8	.218	.204	.191	.179	.168	.157	.148	.139	.130	.123
9	.180	.167	.155	.114	.134	.125	.116	.108	.101	.094
10	.149	.137	.126	.116	.107	.099	.092	.085	.078	.073
11	.123	.112	.103	.094	.086	.079	.072	.066	.061	.056
12	.102	.092	.083	.076	.069	.062	.057	.052	.047	.043
13	.084	.075	.068	.061	.055	.050	.045	.040	.037	.033
14	.069	.062	.055	.049	.044	.039	.035	.032	.028	.025
15	.057	.051	.045	.040	.035	.031	.028	.025	.022	.020
16	.047	.042	.036	.032	.028	.025	.022	.019	.017	.015
17	.039	.034	.030	.026	.023	.020	.017	.015	.013	.012
18	.032	.028	.024	.021	.018	.016	.014	.012	.010	.009
19	.027	.023	.020	.017	.014	.012	.011	.009	.008	.007
20	.022	.019	.016	.014	.012	.010	.008	.007	.006	.005
21	.018	.015	.013	.011	.009	.008	.007	.006	.005	.004
22	.015	.013	.011	.009	.007	.006	.005	.004	.004	.003
23	.012	.010	.009	.007	.006	.005	.004	.003	.003	.002
24	.010	.008	.007	.006	.005	.004	.003	.003	.002	.002
25	.009	.007	.006	.005	.004	.003	.003	.002	.002	.001
30	.003	.003	.002	.002	.001	.001	.001	.001	.000	.000
35	.001	.001	.001	.001	.000	.000	.000	.000	.000	.000
40	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
45	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
50	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Year	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	.763	.758	.752	.746	.741	.735	.730	.725	.719	.714
2	.583	.574	.565	.557	.549	.541	.533	.525	.518	.510
3	.445	.435	.425	.416	.406	.398	.389	.381	.372	.364
4	.340	.329	.320	.310	.301	.292	.284	.276	.268	.260
5	.259	.250	.240	.231	.223	.215	.207	.200	.193	.186
6	.198	.189	.181	.173	.165	.158	.151	.145	.139	.133
7	.151	.143	.136	.129	.122	.116	.110	.105	.100	.095
8	.115	.108	.102	.096	.091	.085	.081	.076	.072	.068
9	.088	.082	.077	.072	.067	.063	.059	.055	.052	.048
10	.067	.062	.058	.054	.050	.046	.043	.040	.037	.035

TABLE - 3 (Contd.)

Year	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
11	.051	.047	.043	.040	.037	.034	.031	.029	.027	.025
12	.039	.036	.033	.030	.027	.025	.023	.021	.019	.018
13	.030	.027	.025	.022	.020	.018	.017	.015	.014	.013
14	.023	.021	.018	.017	.015	.014	.012	.011	.010	.009
15	.017	.016	.014	.012	.011	.010	.009	.008	.007	.006
16	.013	.012	.010	.009	.008	.007	.006	.006	.005	.005
17	.010	.009	.008	.007	.006	.005	.005	.004	.004	.003
18	.008	.007	.006	.005	.005	.004	.003	.003	.003	.002
19	.006	.005	.004	.004	.003	.003	.003	.002	.002	.002
20	.005	.004	.003	.003	.002	.002	.002	.002	.001	.001
21	.003	.003	.003	.002	.002	.002	.001	.001	.001	.001
22	.003	.002	.002	.002	.001	.001	.001	.001	.001	.001
23	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001
24	.002	.001	.001	.001	.001	.001	.001	.001	.000	.000
25	.001	.001	.001	.001	.001	.001	.000	.000	.000	.000
30	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
35	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
40	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
45	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
50	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE - 4

The Present Value of an Annuity of One Rupee

Year	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.990	.980	.971	.962	.952	.943	.935	.926	.917	.909
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868
8	7.652	7.326	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495

TABLE - 4 (Contd.)

Year	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.560	8.061	7.606
16	14.718	13.578	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824
17	15.562	14.292	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.372	8.756	8.201
19	17.226	15.679	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365
20	18.046	16.352	14.878	13.590	12.462	11.470	10.594	9.818	9.129	8.514
21	18.857	17.011	15.415	14.029	12.821	11.764	10.836	10.017	9.292	8.649
22	19.662	17.658	15.937	14.451	13.163	12.042	11.061	10.201	9.422	8.772
23	20.456	18.292	16.444	14.857	13.489	12.303	11.272	10.371	9.580	8.883
24	21.244	18.914	16.936	15.247	13.799	12.550	11.469	10.529	9.707	8.985
25	22.023	19.524	17.413	15.622	14.094	12.783	11.654	10.675	9.823	9.077
30	25.808	22.397	19.601	17.292	15.373	13.765	12.409	11.258	10.274	9.427
35	29.409	24.999	21.487	18.665	16.374	14.498	12.948	11.655	10.567	9.644
40	32.835	27.356	23.115	19.793	17.159	15.046	12.332	11.925	10.757	9.779
45	36.095	29.490	24.519	20.720	17.774	15.456	13.606	12.108	10.881	9.863
50	39.197	31.424	25.730	21.482	18.256	15.762	13.801	12.234	10.962	9.915

Year	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	.901	.893	.885	.877	.870	.862	.855	.847	.840	.833
2	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528
3	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106
4	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589
5	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991
6	4.231	4.111	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326
7	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605
8	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837
9	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031
10	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192
11	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.487	4.327
12	6.492	6.194	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439
13	6.750	6.424	6.122	5.842	5.583	5.842	5.118	4.910	4.715	4.533
14	6.982	6.628	6.303	6.002	5.724	5.468	5.229	5.008	4.802	4.611
15	7.191	6.811	6.462	6.142	5.847	5.675	5.324	5.092	4.876	4.675
16	7.379	6.974	6.604	6.265	5.954	5.669	5.405	5.162	4.938	4.730

TABLE - 4 (Contd.)

Year	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
17	7.549	7.120	6.729	6.373	6.047	5.749	5.475	5.222	4.990	4.775
18	7.702	7.250	6.840	6.467	6.128	5.818	5.534	5.273	5.033	4.812
19	7.839	7.366	6.938	6.550	6.198	5.877	5.585	5.316	5.070	4.843
20	7.963	7.469	7.024	6.623	6.259	5.929	5.628	5.353	5.101	4.870
21	8.075	7.562	7.102	6.687	6.312	5.973	5.665	5.384	5.127	4.891
22	8.176	7.645	7.170	6.743	6.359	6.011	5.696	5.410	5.149	4.909
23	8.266	7.718	7.230	6.792	6.399	6.044	6.723	5.432	5.167	4.925
24	8.348	7.784	7.283	6.835	6.434	6.073	5.747	5.451	5.182	4.937
25	8.422	7.843	7.330	6.873	6.464	6.097	5.766	5.467	5.195	4.948
30	8.694	8.055	7.496	7.003	6.566	6.177	5.829	5.517	5.235	4.979
35	8.855	8.176	7.586	7.070	6.617	6.215	5.858	5.539	5.251	4.992
40	8.951	8.244	7.634	7.105	6.642	6.233	5.871	5.548	5.258	4.997
45	9.008	8.283	7.661	7.123	6.654	6.242	5.877	5.552	5.261	4.999
50	9.042	8.305	7.675	7.133	6.661	6.246	5.880	5.554	5.262	4.999

Year	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	8.26	.820	.813	.806	.800	.794	.787	.781	.775	.769
2	1.509	1.492	1.474	1.457	1.440	1.424	1.407	1.392	1.376	1.361
3	2.074	2.042	2.011	1.981	1.952	1.923	1.896	1.868	1.842	1.816
4	2.540	2.494	2.448	2.404	2.362	2.320	2.280	2.241	2.203	2.166
5	2.926	2.864	2.803	2.745	2.689	2.635	2.583	2.532	2.483	2.436
6	3.245	3.167	3.092	3.020	2.951	2.885	2.821	2.759	2.700	2.643
7	3.508	3.416	3.327	3.242	3.161	3.083	3.009	2.937	2.868	2.802
8	3.726	3.619	3.518	3.421	3.329	3.241	3.156	3.076	2.999	2.925
9	3.905	3.786	3.673	3.566	3.463	3.366	3.273	3.184	3.100	3.019
10	4.054	3.923	3.799	3.682	3.570	3.465	3.364	3.269	3.178	3.092
11	4.177	4.035	3.902	3.776	3.656	3.544	3.437	3.335	3.239	3.147
12	4.278	4.127	3.985	3.751	3.752	3.606	3.493	3.387	3.286	3.190
13	4.362	4.203	4.053	3.912	3.780	3.656	3.538	3.427	3.322	3.223
14	4.432	4.265	4.108	3.962	3.824	3.695	3.573	3.459	3.351	3.249
15	4.489	4.315	4.153	4.001	3.859	3.726	3.601	3.483	3.373	3.268
16	4.336	4.357	4.189	4.033	3.887	3.751	3.623	3.503	3.390	3.283
17	4.576	4.391	4.219	4.059	3.910	3.771	3.640	3.518	3.403	3.295
18	4.608	4.419	4.243	4.080	3.928	3.786	3.654	3.529	3.413	3.304
19	4.635	4.442	4.263	4.097	3.942	3.799	3.664	3.539	3.421	3.311
20	4.657	4.460	4.279	4.110	3.954	3.808	3.673	3.546	3.427	3.316
21	4.675	4.476	4.292	4.121	3.963	3.816	3.679	3.551	3.432	3.320
22	4.690	4.488	4.302	4.130	3.970	3.822	3.684	3.556	3.436	3.323
23	4.703	4.499	4.311	4.137	3.976	3.827	3.689	3.559	3.438	3.325

TABLE - 4 (Contd.)

Year	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
24	4.713	4.507	4.318	4.143	3.981	3.831	3.692	3.562	3.441	3.327
25	4.721	4.514	4.323	4.147	3.985	3.834	3.694	3.564	3.442	3.329
30	4.746	4.534	4.339	4.160	3.995	3.842	3.701	3.569	3.447	3.332
35	4.756	4.541	4.345	4.164	3.998	3.845	3.703	3.571	3.448	3.333
40	4.760	4.544	4.347	4.166	3.999	3.846	3.703	3.571	3.448	3.333
45	4.761	4.545	4.347	4.166	4.000	3.846	3.704	3.571	3.448	3.333
50	4.762	4.545	4.348	4.167	4.000	3.846	3.704	3.571	3.448	3.333

Year	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	.763	.758	.752	.746	.741	.735	.730	.725	.719	.714
2	1.346	1.331	1.317	1.303	1.289	1.276	1.263	1.250	1.237	1.224
3	1.791	1.766	1.742	1.719	1.696	1.673	1.652	1.630	1.609	1.589
4	2.130	2.096	2.062	2.029	1.997	1.966	1.935	1.906	1.877	1.849
5	2.390	2.345	2.302	2.260	2.220	2.181	2.143	2.106	2.070	2.035
6	2.588	2.534	2.483	2.433	2.385	2.339	2.294	2.251	2.209	2.168
7	2.739	2.677	2.619	2.562	2.508	2.455	2.404	2.355	2.308	2.263
8	2.854	2.786	2.721	2.658	2.598	2.540	2.485	2.432	2.380	2.331
9	2.942	2.868	2.798	2.730	2.665	2.603	2.544	2.487	2.432	2.379
10	3.009	2.930	2.855	2.784	2.715	2.649	2.587	2.527	2.469	2.414
11	3.060	2.978	2.899	2.824	2.752	2.683	2.618	2.555	2.496	2.438
12	3.100	3.013	2.931	2.853	2.779	2.708	2.641	2.576	2.515	2.456
13	3.129	3.040	2.956	2.876	2.799	2.727	2.658	2.592	2.529	2.469
14	3.152	3.061	2.974	2.892	2.814	2.740	2.670	2.603	2.539	2.477
15	3.170	3.076	2.988	2.905	2.825	2.750	2.679	2.611	2.546	2.484
16	3.183	3.088	2.999	2.914	2.834	2.757	2.685	2.616	2.551	2.489
17	3.193	3.097	3.007	2.921	2.840	2.763	2.690	2.621	2.555	2.492
18	3.201	3.104	3.012	2.926	2.844	2.767	2.693	2.624	2.557	2.494
19	3.207	3.109	3.107	2.930	2.848	2.770	2.596	2.626	2.559	2.496
20	3.211	3.113	3.020	2.933	2.850	2.772	2.698	2.627	2.561	2.497
21	3.215	3.116	3.023	2.935	2.852	2.773	2.699	2.629	2.562	2.498
22	3.217	3.118	3.025	2.936	2.853	2.775	2.700	2.629	2.562	2.498
23	3.219	3.120	3.026	2.938	2.854	2.755	2.701	2.630	2.563	2.499
24	3.221	3.121	3.027	2.939	2.855	2.776	2.701	2.630	2.563	2.499
25	3.222	3.122	3.028	2.939	2.856	2.776	2.702	2.631	2.563	2.499
30	3.225	3.124	3.030	2.941	2.857	2.777	2.702	2.631	2.564	2.500
35	3.226	3.125	3.030	2.941	2.857	2.778	2.703	2.632	2.564	2.900
40	3.226	3.125	3.030	2.941	2.857	2.778	2.703	2.632	2.564	2.500
45	3.226	3.125	3.030	2.941	2.857	2.778	2.703	2.632	2.564	2.500
50	3.226	3.125	3.030	2.941	2.857	2.778	2.703	2.632	2.564	2.500

Question Paper

2020

ECONOMICS — HONOURS

Paper : DSE-B-2

(Financial Economics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any *ten* questions :

(a) A person keeps ₹ 4,500 in each of investment options, I_1 and I_2, for 5 years. I_1 provides 8% simple interest rate per annum where as I_2 provides 6% interest rate compounded yearly. What will be the maturity values of these two investments ? 2

(b) Suppose, you got ₹ 1,070 on maturity of a deposit of ₹ 1,000 for one year. If the inflation rate for that year was 5%, what was the rate of interest that you got actually on your deposit ? 2

(c) Differentiate between Bid price and Ask price of a bond. 2

(d) What is yield curve ? 2

(e) Determine the present value of a perpetuity that pays ₹ 7,200 per year with 15% interest rate. 2

(f) How could a risk-averse individual minimize risk of portfolio return when there are n mutual funds that are (i) uncorrelated, (ii) positively correlated ? 1+1

(g) If the spot rates for 1 and 2 years are $S_1 = 6.3\%$ and $S_2 = 6.9\%$, what is the forward rate f_{12}? 2

(h) If the premium on a call option has declined recently, does this decline indicate that the option is a better buy than it was previously ? 2

(i) State the one-fund theorem. 2

(j) What is the difference between simple and compound interest ? 2

(k) What is a commercial paper ? 2

(l) What is amortization ? 2

(m) Define price-yield curve. 2

(n) State Forward price formula. 2

(o) Define Debt Equity Ratio. 2

Group - B

2. Answer any *three* questions.

(a) (i) Consider the following informations for two assets :

Asset	$\bar{r}$	σ	
A	12%	20%	$\sigma_{AB} = 0.01$
B	15%	18%	

A portfolio is formed with weights $\omega_A = 0.2$ and $\omega_B = 0.8$.

Calculate the mean and variance of the portfolio.

(ii) Show the feasible set of two assets in a diagram. (1 + 2) + 2

(b) Discuss the factors that affect stock option prices. 5

(c) Explain the dividend payment process of corporates. 5

(d) State and prove the portfolio diagram lemma. 5

(e) Two stocks are believed to satisfy the two-factor model

$$r_1 = \alpha_1 + 2f_1 + f_2$$

$$r_2 = \alpha_2 + 3f_1 + 4f_2$$

In addition, there is a risk-free asset with a rate of return of 10%. It is known that $\bar{r}_1 = 15\%$ and $\bar{r}_2 = 20\%$. What are the values of λ_0, λ_1 and λ_2 for this model ? 5

Group - C

Answer any three questions.

3. Assume that the expected rate of return on the market portfolio is 23% and the risk-free return is 7%. The standard deviation of the market is 32%. Assuming that the market portfolio is efficient.
   (a) Derive the equation of the capital market line. Interpret the slope of the line.
   (b) What will be the standard deviation of this position if an expected return of 39% is desired?
   (c) If you invest ₹ 600 in the risk-free asset and ₹ 1,400 in the market portfolio, how much money should you expect to have at the end of the year ?
   (d) Consider an asset with expected pay-off ₹ 1,000 and covariance of 0·154 with the market. Determine the current value of the asset. (2+2)+1+2+3

4. What is futures? How could you create a synthetic futures contract with purchase of a European call option and sale of a European put option, having same exercise price and same expiration date ? 2+8

5. 'The CAPM is derived directly from the condition that the market portfolio is a point on the edge of the feasible region that is tangent to the capital market line.'— Discuss the statement. 10

6. Explain three standard explanations (or theories) for the Term Structure. 10

7. Show that points on the efficient frontier can be characterised by an optimisation problem, formulated by Markowitz. 10